## CAE Oxford

Aviation Academy

## GENERAL NAVIGATION <br> ATPL GROUND TRAINING SERIES

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## Introduction

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## The Shape of the Earth

The simple view of the shape of the Earth is that it is a sphere, and this is nearly true. In fact, the Earth's shape is commonly described as an oblate spheroid, that is, a sphere which is slightly flattened at its poles. This shape developed when the Earth formed from a gas-cloud as the spin of the cloud caused higher centrifugal forces at the equatorial region than in regions nearer the poles. The flattening is called compression and in the case of the Earth is approximately $0.3 \%(1 / 300 \mathrm{th})$. More simply put, the Earth's polar diameter is or 23 nautical miles or 43 km less than its equatorial diameter. Recent satellite surveys of the Earth have also shown it to be slightly pear-shaped with its maximum diameter occurring south of the Equator. This Southern hemisphere distortion is considerably less than the compression distortion and is measured in tens of metres rather than kilometres.

If the Earth were perfectly spherical, then its cross-section would be a circle. Mathematicians could deal with that quite easily in producing the equations necessary to produce map projections. If the compression were completely symmetrical, then the cross-section would be a perfect ellipse, which would also not be too mathematically complicated.

However, the Earth is not quite either of these, and so the only word to describe it is "Earthshaped". This is what the word geoid (of Greek origin) means.

## Geodosy and Geoid Models

A number of different agencies have measured and modelled the Earth (and produced the equations to define their geoids). Each agency has tended to optimize its geoid to give the best fit to the actual shape of the Earth over the area in which it is interested in mapping. This may mean that the geoid does not fit the actual Earth in another part of the world. For example, the UK Ordnance Survey uses a geoid based on a survey of 1936 (OS36), France has tended to use the Nouvelle Triangulation de France (NTF) 1970 model, some other European countries use the European Datum 1950 model (ED50), and the USA uses the World Geodetic System 1984 (WGS 84).

Use of different geoids can result in arriving at different values for defining latitude and longitude. There can be differences of up to the order of 200 metres for positions on the extremities of the European ED50 and the UK OS36. This may not sound much to an airline pilot - though it would to the programmer of a cruise missile which was guided to latitude and longitude coordinates! Until recently, these differences have not been considered significant, but two recent developments, however, have changed this. These are the arrival of the Global Positioning System (GPS) and the widespread use of Flight Management Systems (FMS).

GPS is an electronic navigation system in which aircraft receivers compare signals from several of the 24 transmitters in the satellites which make up the GPS constellation. It can be received over the whole globe and its accuracy is of the order of tens of metres. The system accuracy is such that the differences in geoids becomes significant, and the system has world-wide application. The US government adopted WGS 84 for GPS .

FMS compares the output of Inertial Reference Systems (IRS) with positions derived from range information received from Distance Measuring Equipment (DME). The positions of the DME stations are stored in latitude and longitude held in the data base of the FMS computer. DME is a very accurate system and any inaccuracy in the datum positions would degrade the position calculation of FMS. If the data base held the positions of all the UK DMEs in OS36 and the French DMEs in NTF70, it could cause large discontinuities in the calculation of FMS position as the aircraft crossed the English Channel.

For these reasons, ICAO has adopted WGS 84 as the world standard.
In modern navigation systems, position information is corrected for the distortions of the Earth's shape automatically in the navigation computers.

For any calculations that you may be required to do for the EASA examination syllabus, the Earth may be considered to be a true sphere with a circumference of 21600 NM or 40000 km .

## The Poles

The Poles are defined as the extremities of the axis about which the Earth spins. The axis of the Poles is inclined to the axis of the Earth's orbit around the Sun at an angle of $231 / 2^{\circ}$. This topic will be covered more fully in the chapter on "Time". However, in this chapter the polar axis will be drawn upright.

## Basic Direction on the Earth

To start to define directions on the Earth, a datum must be selected. The simplest datum is the direction in which the Earth is spinning. This is then defined as East, hence sunrise in the East. West is then defined as the opposite of East. Facing East, the pole on the left is called the North Pole and the direction North is defined as the direction towards the North Pole. The pole diametrically opposite the North Pole is called the South Pole and the direction South is defined as being opposite to North.


Figure 1.1 Earth's rotation shown in 'elevation'

Figure 1.2 Cardinal and quadrantal points

These directions, North, South, East and West are known as Cardinal Points. The midway directions between North(N), East(E), South(S), West(W) and North(N) are North-East (NE), South-East (SE), South-West (SW), and North-West (NW). These directions are known as the Quadrantal directions.

To solve navigation problems, the student may occasionally need to consider the Earth when viewed from above either the North Pole or the South Pole. When viewed from above the North Pole, the Earth appears to rotate in an anticlockwise (counter- clockwise) direction. When viewed from above the South Pole, the Earth appears to rotate in a clockwise direction.


Figure 1.3 Earth's rotation 'plan view'
Figure 1.4

The system of Cardinal and Quadrantal directions was widely used in maritime navigation. Further sub-divisions were made to give more precision but the system is too cumbersome and does not allow for the precise definition of direction required in air navigation. To achieve the required precision, air navigation terminology retains the cardinal points as a reference but uses the Sexagesimal system for measuring directions.

## Sexagesimal System / True Direction

The Sexagesimal system utilizes the fact that a clockwise rotation of direction from North through East, South and West and back to North is a circle of 360 degrees. If North is defined as $000^{\circ}$, East becomes $090^{\circ}$, South becomes $180^{\circ}$ and West becomes $270^{\circ}$. Finally, continuing the rotation back to North makes North $360^{\circ}$. In practice, the use of $000^{\circ}$ or $360^{\circ}$ for North is a personal choice (but a northerly runway would be labelled ' 36 ').


Figure 1.5 A sexagesimal compass

Direction on the Earth is measured in degrees clockwise from North. When the North datum is the direction of the North geographic pole, it is referred to as True direction. The abbreviation ( $T$ ) is used, thus:-

North is referred to as $000^{\circ}(\mathrm{T})$
East is referred to as $090^{\circ}(\mathrm{T})$
South is referred to as $180^{\circ}(\mathrm{T})$
West is referred to as $270^{\circ}(\mathrm{T})$
Note: A 3-figure group is always used $-000^{\circ}(\mathrm{T})$, not $0^{\circ}(\mathrm{T})$ and $090^{\circ}(\mathrm{T})$, not $90^{\circ}(\mathrm{T})$
The use of 3-figure groups for directions is to avoid any ambiguity, particularly in transmitting messages by R/T or other means. Any direction of less than a 3 -figure group should be treated as suspect. For example, $27^{\circ}$ could be a corruption of $027^{\circ}$ or $270^{\circ}$, or even $127^{\circ}, 227^{\circ}, 327^{\circ}$, $271^{\circ}, 272^{\circ}$ etc. There is one exception to this rule; runway directions are given at $10^{\circ}$ intervals (to the nearest $10^{\circ}$ ). For example a RW orientated $273^{\circ}$ is called RW 27 and a RW orientated $078^{\circ}$ becomes RW 08. (But beware, Runway Directions are normally given with reference to Magnetic North, not True North).

Reciprocal directions. In navigation subjects, it is necessary frequently to calculate the reciprocal of a given direction, i.e. that direction which is $180^{\circ}$ opposite to the direction given. Thus, if you are travelling in a direction of $060^{\circ}(\mathrm{T})$, the reciprocal direction is $240^{\circ}(\mathrm{T})$, i.e. $060 \pm 180$. If a runway has a direction of $353^{\circ}(\mathrm{T})$, the reciprocal runway direction is $173^{\circ}(\mathrm{T})-353 \pm 180$.

## Position Reference Systems

Navigation is a fundamental process whereby an aircraft is directed from one position to another; it is therefore necessary to use a Position Reference System which defines position accurately and unambiguously on the Earth's surface.

On a plane surface (such as graph paper), position can be defined accurately and unambiguously using Cartesian co-ordinates $\pm x$ and $\pm y$ from 2 mutually perpendicular axes, the $X$ and $Y$ axes. For instance, a Cartesian system is used for the UK National Grid System, e.g. Ordnance Survey Landranger Maps.

The Cartesian system works well on a flat surface but must be modified to work on a spherical surface, i.e. the Earth. In practice, the linear co-ordinates $x$ and $y$ are replaced by angular coordinates known as Longitude and Latitude. The first requirement for this spherical system is to define the 2 mutually perpendicular axes (the equivalents of the $X, Y$ axes). The definition and use of these axes involves circles on the Earth - Great Circles and Small Circles.

## Circles on the Earth

Great Circle: A circle on the surface of the Earth whose centre and radius are those of the Earth itself is called a Great Circle. It is called 'great' because a disc cut through the Earth in the plane of the Great Circle would have the largest area that can be achieved.


Figure 1.6 Great circle
The shortest distance between two points on the Earth's surface is the shorter arc of the Great Circle joining the two points.

Given two points on the Earth's surface, there will be only one Great Circle joining them (unless the points are diametrically opposed).

## The Equator

The Great Circle whose plane is at $90^{\circ}$ to the axis of rotation of the Earth (the polar axis) is called the Equator. It lies in an East-West direction and divides the Earth equally into two hemispheres. For the definition of position on the Earth, the Equator is the datum for defining Latitude and is the equivalent of the $X$ axis of the Cartesian system.

## The Meridians

Meridians are semi-Great Circles joining the North and South poles. All meridians indicate True North-South direction. Every Great Circle passing through the poles forms a meridian and its Anti-meridian. The meridians cross the Equator at $90^{\circ}$.

## The Prime (or Greenwich) Meridian

The meridian passing through Greenwich is known the Prime Meridian . The Prime Meridian is the datum for defining Longitude and is the equivalent of the Y axis of the Cartesian system.

## Small Circle

A circle on the surface of the Earth whose centre and radius are not those of the Earth is called a Small Circle. The main small circles of relevance to position are the Parallels of Latitude.

## Parallels of Latitude

The parallels of latitude are small circles on the surface of the Earth whose planes are parallel to the Equator. They lie in an East-West direction. Their function is to indicate position North or South of the Equator.

## Graticule

The network formed on a map or the surface of a globe by the Prime Meridian, the meridians, the Equator and the parallels of latitude is called the Graticule. The graticule is an analogy on the Earth's surface of the $\mathrm{X}-\mathrm{Y}$ grid on graph paper.


Figure 1.7 Graticule on a globe
Using the graticule, the position of any point can be accurately and unambiguously defined. However, position is defined by angular measurement (degrees, minutes, and seconds of arc), not by distance, as on Cartesian co-ordinates. This is the foundation of the Latitude/ Longitude system.

## Angular Measurements

The fundamental unit of angular measurement is the degree $\left({ }^{\circ}\right)$ which is defined as the angle subtended at the centre of a circle by an arc equal to the 360th part of the circumference (hence $360^{\circ}$ in a circle).

For more accuracy A degree is sub-divided into 60 minutes of arc (') A minute can be further sub-divided into 60 seconds of arc (").

The above units are mainly used for angular measurement of position but for angular measurement of direction, it is more common to use degrees and decimals of degrees.

## Latitude

The latitude of any point is the arc (angular distance) measured along the meridian through the point from the Equator to the point. It is expressed in degrees, minutes, and seconds of arc and is annotated North or South according to whether the point lies North or South of the Equator. In Figure 1.8, you are looking at the Earth 'sideways on' from space, i.e. the N pole is at the top of the diagram. The angular distance between where the meridian of A crosses the Equator (point P) and point $A$ is $40^{\circ}$ and the latitude of point $A$ is $40^{\circ} \mathrm{N}$. (It can also be said that A lies on the Parallel of Latitude of $40^{\circ} \mathrm{N}$ ).


Figure 1.8 Latitude
On Figure 1.8, come in from the Equator to the centre of the Earth. Then, for a Northerly latitude of, say, $40^{\circ} \mathrm{N}$, construct a line from the centre of the Earth with an elevation angle from the Equator of $40^{\circ}$ (depression angle for Southerly latitudes). Where this line touches the surface of the Earth is the $40^{\circ} \mathrm{N}$ parallel of latitude.

The range of latitude values covers the Equator $\left(0^{\circ} \mathrm{N} / \mathrm{S}\right)$ to the geographic poles - the North Pole is $90^{\circ} \mathrm{N}$ and the South Pole is $90^{\circ} \mathrm{S}$.

## Geocentric and Geodetic Latitude

The definition of latitude given above is based on the centre of the Earth, namely it is the smaller angle between the line joining the point to the centre of the Earth and the plane of the Equator. This is Geocentric Latitude. Because the Earth is actually an oblate spheroid, a different definition of latitude can be used, namely, Geodetic (or Geographic) Latitude. Geodetic (or Geographic) Latitude is the smaller angle between the normal ( $90^{\circ}$ ) to the meridian at the point on the spheroid and the plane of the Equator. This normal line does not necessarily pass through the centre of the spheroid as illustrated in the diagram below. Note that the shapes used in the diagram are exaggerated for illustration purposes and the real spheroid is much closer to a sphere than shown. The latitudes plotted on navigation charts are Geodetic Latitudes. The maximum difference between Geocentric and Geodetic Latitudes occurs at approx $45^{\circ} \mathrm{N} / \mathrm{S}$ and is about 11.6 minutes of arc.


Figure 1.9 Geodetic and geocentric latitude

## Special Cases of Parallels of Latitude

There are several 'special' cases of the parallels of latitude (other than the Equator) which will be explained in the chapter on "Time". These relate to the seasons and the periods of day and night throughout the year. Briefly these are:

| Arctic Circle | the parallel of $6612^{\circ} \mathrm{N}$ (note that $661^{\circ}{ }^{\circ}$ is the value of the Earth's <br> tilt) <br> the parallel of $6611^{\circ} \mathrm{S}$ |
| :--- | :--- |
| Antarctic Circle | the parallel of $22^{\circ} 1^{\circ} \mathrm{N}$ <br> (the Sun is overhead the Tropic of Cancer on mid-summer's day in <br> the Northern hemisphere) |
| Tropic of Cancer Capricorn | the parallel of $231^{\circ} 2^{\circ} \mathrm{S}$ <br> (the Sun is overhead the Tropic of Capricorn on mid-winter's day <br> in the Northern hemisphere) |

## Longitude

The longitude of any point is the shorter distance in the arc along the Equator between the Prime Meridian and the meridian through the point. Longitude is measured in degrees and minutes of arc and is annotated East (E) or West (W) depending whether the point lies East or West of the Prime Meridian (Greenwich). In Figure 1.10, you are looking 'down' on the North Pole from a point out in space. The meridian of point $B$ crosses the Equator (at point Q) at, say, $40^{\circ}$ of arc to the East of the Prime Meridian. Point B is said to be at longitude $040^{\circ} \mathrm{E}$.

## Difference in Longitude

When calculating the difference between the longitude of two positions if those positions are the same East or West longitude then the one is subtracted from the other ie $100^{\circ} \mathrm{W}-080^{\circ} \mathrm{W}$ $=20^{\circ}$ difference or sometimes called change of longitude (ch long). If the two longitudes are East and West then the two are added together ie $020^{\circ} \mathrm{W}+010^{\circ} \mathrm{E}=30^{\circ}$ difference or $30^{\circ} \mathrm{ch}$ long. One anomaly might occur when the two longitudes are East and West and 'close' to the Greenwich Anti Meridian.

Example. What is the difference in longitude between $163^{\circ} \mathrm{E}$ and $152^{\circ} \mathrm{W}$ ? If the principle above was followed the two would be added together to give $315^{\circ}$ however we need to consider the 'shortest' value of difference. In this case if you take the $315^{\circ}$ away from $360^{\circ}$ you will get $45^{\circ}$ ch long.


Figure 1.10 Longitude
Longitude can be measured up to $180^{\circ} \mathrm{E}$ or $180^{\circ} \mathrm{W}$ of the Prime Meridian. These 2 meridians are coincident and are known as the Greenwich Anti-Meridian, normally labelled $180^{\circ} \mathrm{E}$ or $180^{\circ} \mathrm{W}$.

## Reversal of the Apparent Sense of Longitude at the Greenwich Antimeridian ( $180^{\circ} \mathrm{E}$ )

Look at Figure 1.11 and Figure 1.12.


Figure 1.11
Figure 1.12

On the Greenwich Meridian, the Eastern Longitudes are East of you. The Western Longitudes are West of you. But now travel round the globe to the $180^{\circ}$ E longitude (the Greenwich antimeridian). The direction of East has not changed. It is still $090^{\circ}(\mathrm{T})$, the direction of the Earth's spin. Nor has the direction of West. It is still $270^{\circ}(\mathrm{T})$, the opposite direction to the Earth's spin. But in Figure 1.12, the Eastern Longitudes are out to your left (West of you) and the Western Longitudes are out to the right of you (East of you). So you get a situation where the Eastern hemisphere is to your West and the Western hemisphere is to your East. It can cause confusion in solving some navigation problems.

## Difference in Principle between Latitude and Longitude

These 2 angular co-ordinate systems are quite different in principle. Lines of latitude are all parallel to each other. Indeed, it is normal to talk of 'parallels of latitude'. Lines of longitude, however, emanate from a point (the North Pole), reach a maximum separation at the Equator, and then converge back to a point again, at the South Pole.

You may find it a helpful illustration to think of latitude as like slicing a pineapple (parallels of latitude) whilst longitude is like segmenting an orange (meridians of longitude).

## Positions in Latitude and Longitude

When giving a position in latitude and longitude, the latitude is always quoted first and the longitude second: thus New York's position is $41^{\circ} \mathrm{N} 074^{\circ} \mathrm{W}$. Alternative forms are: $41^{\circ} 00^{\prime} \mathrm{N}$ $074^{\circ} 00^{\prime} \mathrm{W}$, or 4100 N 07400 W which are used to allow more precision by the inclusion of minutes of arc. For example, the Aerodrome Reference Point (ARP) for Oxford Airport is approximately $51^{\circ} 50^{\prime} \mathrm{N}, 001^{\circ} 19^{\prime} \mathrm{W}$ or 5150 N 00119 W . (This is the mid-point of runway $01 / 19$ ).

Even greater precision can be achieved by quoting position in seconds of arc although this is not normally required in these notes. For example, the precise position for the Oxford ARP is $51^{\circ} 50^{\prime} 12^{\prime \prime} \mathrm{N}, 001^{\circ} 19^{\prime} 11^{\prime \prime} \mathrm{W}$ or 515012 N 0011911 W .

## Conversion of Latitude and Longitude to Distance on the Earth

The definition of a nautical mile has been chosen to make it relate to angular arc on a Great Circle. It so happens that the mean radius of the Earth is approximately 20.9 million feet in length. This means that on a Great Circle, 1 minute of arc corresponds to a distance on the Earth's surface of 6080 feet. This distance of 6080 feet can be defined as 1 nautical mile, however the ICAO definition of a nautical mile is 1852 metres.

Therefore you can relate angle along a Great Circle arc to distance on the Earth. For instance, all meridians (lines of longitude) are Great Circles, so the distance up or down a meridian (which is change of latitude) has this relationship. The difference in latitude between positions 5000 N 00100 W and 5005 N 00100 W is exactly 5 minutes, so they are exactly 5 nautical miles apart. This is true of differences in longitude only at the Equator, because the Equator is the only parallel of latitude which is also a Great Circle.

## Resolution Accuracy Using Latitude and Longitude

Suppose a position is quoted as $51^{\circ} 49^{\prime} \mathrm{N}, 001^{\circ} 19^{\prime} \mathrm{W}$ or 5149 N 00119 W . By writing 5149 , we are making a declaration of the accuracy of the resolution we are interested in. We are saying that it is nearer to 5149 than 5148 or 5150 . This means that we are quoting to the nearest 1 minute of latitude, which is an accuracy to the nearest one nautical mile. In longitude, if we write 00119 W , we are also quoting to the nearest minute, but in this case, because at latitude 50 N the meridians are closer together than at the Equator, we are quoting to about the nearest 0.6 of a nautical mile. To avoid this complication at this stage, in discussing resolution accuracy, we will consider latitude only.

There are 2 ways of quoting positions to a greater accuracy than the nearest whole minute. It can either be given as decimal minutes ( 5150.2 N ) or as degrees, minutes, seconds (DMS) ( $51^{\circ} 50^{\prime} 12^{\prime \prime} \mathrm{N}$ ). In decimal minutes, only the first decimal place is used. If we need greater accuracy than that, we go to DMS.

Normally, modern automatic navigation systems in which positions are entered by the pilot use decimal minutes. If a position is entered into an Inertial Navigation System, Inertial Reference System, Flight Management Computer, or Global Positioning System, it will be to the nearest decimal minute, i.e. 5150.2 N . This is a statement that the position is nearer 5150.2 than 5150.3 or 5150.1. By quoting to decimal minutes, you are declaring an accuracy of 0.1 of a nautical mile, or 608 feet, or 185 metres.

However, where greater accuracy is required, on large-scale maps and charts, positions will be given in DMS, i.e. 515013 N . In this case, we are declaring an accuracy of the nearest second, i.e. the position is nearer 515013 than 515014 or 515012 . This is an accuracy of $1 / 60$ th of a minute, or about 101 feet or 30 metres. The Aerodrome Reference Point (ARP) for Oxford Airport, which is the midpoint of Runway $01 / 19$, is quoted in the official CAA publication, the UK Aeronautical Information Package (AIP), as 515013N 0011912W.

Even greater accuracy can be achieved by use of decimals of a second. The first decimal place implies an accuracy of $1 / 10$ th of a second, which is 10 feet, or 3 metres, whilst the use of 2 decimal places of a second implies an accuracy of $1 / 100$ th of a second, i.e. 1 foot, or 30 centimetres. THE UK AIP quotes the position of the OX locator beacon, our low-powered homing beacon at Oxford as 515000.28 N 001924.45 W , which is an overall accuracy, taking into account both latitude and longitude, of within 50 cm . This is perhaps a bit more accurate than we really need for a homing aid, but is an appropriate level of accuracy for a landing aid such as
an ILS (Instrument Landing System) localizer. Accuracy at this level would certainly be needed for inspection and flight calibration of an ILS.

It says much for the flexibility of this system of position reference that, whilst for most of the time in flight we are interested in our position accuracy only to the nearest mile or so, and can simply write our position as 5150 N 00119 W - a total of 11 fairly simple numbers or letters, we can also use the system to establish a position to within half a metre by writing 515000.28 N 001924.45 W , which is really not too long-winded considering the level of accuracy achieved.

A summary of accuracy resolution is given in the following table:

| How Written | Level of Accuracy | Typical Application |
| :---: | :---: | :---: |
| 5321 N | $1 \mathrm{NM}=6080$ feet | En route navigation |
| 5321.3 N | 600 feet or 185 m | INS, IRS, FMS, GPS displays |
| $53^{\circ} 21^{\prime} 17^{\prime \prime} \mathrm{N}$ | 100 feet or 30 m | Airfield diagram chart |
| $53^{\circ} 21^{\prime} 17.3^{\prime \prime} \mathrm{N}$ | 10 feet or 3 metres | Location of precision navaid (ILS) |
| $53^{\circ} 21^{\prime} 17.32^{\prime \prime} \mathrm{N}$ | 1 foot or 30 cm | Calibration of precision navaid |

## Great Circle Vertices

The northern vertex of a Great Circle is simply the most northerly point on that Great Circle. Similarly, the southern vertex is the most southerly point on the Great Circle. The vertices are antipodal and the Great Circle distance between them is 10800 NM.


Figure 1.13
The vertices lie on meridian and anti-meridian and have latitude values of equal value but of opposite sign. For example, if the southern vertex of a Great Circle is 63S 170W, its northern vertex will be 63N 010E.

At either of its vertices, the direction of the Great Circle will be East $090^{\circ}(\mathrm{T})$ or West $270^{\circ}(\mathrm{T})$. Knowing the co-ordinates of either vertex enables calculation of where and at what angle the Great Circle crosses the Equator.

A Great Circle will cross the Equator at two points whose longitude is $90^{\circ}$ change of longitude from either of its vertices. Thus the Great Circle in the example in Figure 1.13 would cross the Equator at $080^{\circ} \mathrm{W}$ and $100^{\circ} \mathrm{E}$.

The track angle at which the Great Circle would cross the Equator is based on the latitude of each vertex. The track angle would therefore depend on which direction the Great Circle was travelling from the vertex ie East or West. Thus the Great Circle in the example above would cross the Equator at an angle of $63^{\circ}$ so the track angle measured at the Equator, if travelling Eastbound, would be $90^{\circ}+63^{\circ}=153^{\circ}$ (see Figure 1.14).

Having passed the Southern Vertex, still travelling East, the track angle at the second crossing of the Equator would be $90^{\circ}-63^{\circ}=027^{\circ}$ (see Figure 1.15).

If the original direction had been West from the Northern Vertex the track angles crossing the Equator would be the reciprocal ie $207^{\circ}$ and $333^{\circ}$.

## Two special cases:

A great circle with vertices at 90N/S (a meridian) crosses the Equator at $90^{\circ}$ in direction $180^{\circ}(\mathrm{T})$ $(090+90)$ or $000^{\circ}(\mathrm{T})(090-90)$.

A great circle with vertices at $0 \mathrm{~N} / \mathrm{S}$ crosses the Equator at $0^{\circ}(\mathrm{T})$ (it is the Equator) and has direction $090^{\circ}(\mathrm{T})$ or $270^{\circ}(\mathrm{T})$.


Figure 1.14 First crossing (tracking EAST)


Figure 1.15 Second crossing (tracking EAST)

## Questions

1. What is the approximate compression of the Earth?
a. $3 \%$
b. $0.03 \%$
c. $0.3 \%$
d. $1 / 3000$
2. A Graticule is the name given to:
a. a series of lines drawn on a chart
b. a series of Latitude and Longitude lines drawn on a chart or map
c. a selection of small circles as you get nearer to either pole
3. A great circle has its North vertex at 70N 130E. What is the position of its South vertex?
4. In Question 3, at what longitudes and in what direction would the Great Circle cross the Equator assuming:
a. initial direction is east from the northern vertex?
b. initial direction is west from the northern vertex?
5. Given that the compression value of the Earth is $1 / 297$ and that the semi-major axis of the Earth, measured at the axis of the Equator, is 6378.4 km , what is the semi-minor (i.e. radius) axis of the Earth measured at the axis of the Poles?
a. $\quad 6399.9 \mathrm{~km}$
b. $\quad 6356.9 \mathrm{~km}$
c. $\quad 6378.4 \mathrm{~km}$
d. $\quad 6367.0 \mathrm{~km}$

1

## Answers

| 1 | 2 | 3 | 4 a | 4 b | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| c | b | 70 S 050 W | $140 \mathrm{~W} 160^{\circ}(\mathrm{T})$ | $040 \mathrm{E} 200^{\circ}(\mathrm{T})$ | b |

## Chapter <br> 2

## Great Circles, Rhumb Lines \& Directions on the Earth

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## A Reminder about Great Circles

Great Circle: A circle on the surface of the Earth whose centre and radius are those of the Earth itself is called a Great Circle.

It is called 'great' because a disc cut through the Earth in the plane of the Great Circle would have the largest area that can be achieved.


Figure 2.1 Great circle
The shortest distance between two points on the Earth's surface is the shorter arc of the Great Circle joining the two points.

Given two points on the Earth's surface, there will be only one Great Circle joining them (unless the points are diametrically opposed).

## The Rhumb Line

Consider the figure below.


Figure 2.2 Moscow - Vancouver rhumb line track
A Rhumb Line is a regularly curved line on the surface of the Earth which cuts all meridians at the same angle - a line of constant direction.

This constant track direction was an enormously important property for all mariners and aviators until about 40 years ago, when cheap powerful computing started to become widely available. Prior to that, one calculated a constant straight line track and then held the compass heading which would give that track. This made establishing the track a relatively simple problem although, unlike the Great Circle route, it did not give the shortest distance over the Earth between the two points.

However, in the case of our Moscow to Vancouver route, you can see that the Great Circle track between those points passes very close to the North Pole, and is much shorter.


Figure 2.3 Moscow - Vancouver great circle track

Comparing the Great Circle and the Rhumb Line together:


Figure 2.4 Moscow - Vancouver great circle track compared with rhumb line

Let us now re-plot the Great Circle track from the globe, taking its position where it crosses various coastlines, and transfer it to, for example, a Mercator chart, where Rhumb Lines are represented by straight lines


Figure 2.5 Mercator projection with great circle track added
This is a dramatic illustration of the extent to which projections can introduce distortions. It does not appear that way on the Mercator Chart, but we know from plotting it on the globe, that the Great Circle track is actually the shortest distance over the Earth between Moscow and Vancouver. However, notice also that the track direction changes.

Compare the Great Circle track direction on the Mercator projection with the meridians. The track starts out as it leaves Moscow with a direction of about $330^{\circ}$. As it passes near the North Pole the track direction is about $270^{\circ}$. As it comes into Vancouver, the direction is about $210^{\circ}$. The Great Circle track has changed direction - in this case, quite significantly.

Modern computer-based navigation systems can direct the aircraft along a Great Circle route and cope with the constantly changing direction of the Great Circle quite easily. However, in the early days of air and maritime navigation, compass and navigation equipment was less capable and it was found easier to navigate along lines of constant direction - Rhumb Lines.

As with Great Circles, there is only one Rhumb Line that can be drawn between 2 points.

## Examples of common Rhumb Lines are:

- Parallels of Latitude (because they cut all meridians at $90^{\circ}$ ).
- Equator (a special case because the Equator is also a Great Circle).
- Meridians (are also Great Circles and the cut angle involved is $0^{\circ}$ ).

The Rhumb Line between 2 points will always lie nearer to the Equator than the corresponding Great Circle. Conversely, the Great Circle between the $\mathbf{2}$ points will always lie nearer the Pole than the Rhumb Line.

In some explanations the shape of the Rhumb Line may be stated as convex to the Equator or concave to the nearer Pole.

## Lines Which Are Both Great Circles and Rhumb Lines

The only lines which are both Great Circles and Rhumb Lines are the Equator and any meridian (along with its associated anti-meridian). There are no other lines that are both Great Circles and Rhumb Lines. The parallels of latitude are Rhumb Lines because they cut all meridians at $90^{\circ}$, but they are Small Circles as they do not have the same radius and centre as the Earth.

## Great Circle Direction

The direction of the Great Circle over the Earth's surface changes, as we have seen. The Great Circle track is always nearer to the nearer pole. In Figure 2.6, which again represents a Mercator chart, we have 2 Rhumb Line tracks, each in an E-W direction, one in the Northern hemisphere and one in the Southern. Both are at fairly high latitudes (say, 50 N or 50 S ) in order to show a reasonable curvature on their Great Circle tracks, but the effect occurs, to some extent, at any latitude other than the Equator.


Figure 2.6 Change in direction of great circle tracks
Northern hemisphere. Consider the track left to right in the Northern hemisphere - a Rhumb Line track of $090^{\circ}$.

If heading in an Easterly direction, the Great Circle track starts with an initial direction of about $030^{\circ}$, then curves round to $090^{\circ}$ and finishes up on about $150^{\circ}$. In other words, the track direction is increasing.

If heading in a Westerly direction, the Great Circle track starts with an initial direction of about $330^{\circ}$, then curves round to $270^{\circ}$ and finishes up on about $210^{\circ}$. In other words, the track direction is decreasing.

Southern hemisphere. Consider the track left to right in the Southern hemisphere - a Rhumb Line track of $090^{\circ}$.

If heading in an Easterly direction, the Great Circle track starts with an initial direction of about $150^{\circ}$, then curves round to $090^{\circ}$ and finishes up on about $030^{\circ}$. In other words, the track direction is decreasing.

If heading in a Westerly direction, the Great Circle track starts with an initial direction of about $210^{\circ}$, then curves round to $270^{\circ}$ and finishes up on about $330^{\circ}$. In other words, the track direction is increasing.

This can be summarized in the following diagram. North and South represent the hemispheres. Left and right indicate that the track direction is Easterly (right) or Westerly (left). This gives a diagram which spells out DIID - (decreasing - increasing - increasing - decreasing).


In all the above cases, note that the Great Circle direction always changes towards the Equator.

## Distance on the Earth

Both metric and Imperial measures are in common use in aviation. Because many students will not have used the smaller Imperial measures of distance, the conversions below should be remembered.

| 1 metre $(\mathrm{m})$ | $=100$ centimetres $(\mathrm{cm})$ | $=1000$ millimetres $(\mathrm{mm})$ |
| :--- | :--- | :--- |
| 1 centimetre $(\mathrm{cm})$ | $=10$ millimetres $(\mathrm{mm})$ |  |
| 1 metre $(\mathrm{m})$ | $=3.28$ feet $(\mathrm{ft})$ |  |
| $1 \mathrm{foot}(\mathrm{ft})$ | $=12$ inches ( 'in' or ") |  |
| $1 \mathrm{inch}(\mathrm{in})$ | $=2.54$ centimetres $(\mathrm{cm})$ |  |
| 1 yard $(\mathrm{yd})$ | $=3$ feet $(\mathrm{ft})$ |  |

The Navigation Computer is accurate enough for any conversion for the purposes of aviation.

The Kilometre (km). The definition of the kilometre is $\mathbf{1 / 1 0 0 0 0 t h}$ of the average distance on the Earth between the Equator and either Pole. Thus there are 10000 km between the Equator and either Pole, and the circumference of the Earth is 40000 km .

For conversions between kilometres and imperial units:

| 1 kilometre (km) | $=$ | 3280 feet (ft) |
| :--- | :--- | :--- |
| 1 metre $(\mathrm{m})$ | $=$ | 3.28 ft |

The Statute Mile (stat.m). Although the statute mile ( 5280 feet) is widely used on the ground it is hardly ever used in aviation nowadays. Older airspeed indicators used to be calibrated in mph , and still are for some American light aircraft, but this is now rare. Also, visibility is still given in statute miles in the USA. Otherwise, the only time a modern pilot will need to know about mph is to inform passengers in UK or US aircraft how fast the aircraft is travelling in terms of the same units they use in their cars.

The Nautical Mile (NM). The nautical mile is the most important large measure of distance used in aviation because it can be related directly to the angular measurements of the Latitude/ Longitude graticule of the Earth.

The ICAO definition of the nautical mile is that it is a measure of distance of $\mathbf{1 8 5 2}$ metres.
The Standard Nautical Mile is defined as a length of 6080 feet.
(You may have noticed that 1852 metres converted to feet at a factor of 3.28 does not give exactly 6080 feet. For all EASA ATPL practical use, either 1852 metres or 6080 feet will give close enough answers to any problems which you will have to solve).

Because they all use Great Circle arcs, the definition of the nautical mile is similar to the definitions of Latitude and Longitude. Remember the following:

One minute of latitude $=1$ nautical mile $(\mathrm{NM})$ One degree of latitude $=60$ minutes $=\mathbf{6 0} \mathrm{NM}$

## But

One minute of longitude $=1$ NM AT THE EQUATOR ONLY.
As there are $90^{\circ}$ of latitude change between the Equator and the Poles, the distance between the Equator and the Poles is $90 \times 60=5400$ NM

The circumference of the Earth at the Equator is $\mathbf{3 6 0 \times 6 0} \mathbf{= 2 1 6 0 0}$ NM

## Variations in the Length of a Nautical Mile

The full definition of the length of a nautical mile is that length of arc of a Great Circle which subtends an angle of one minute at the centre of curvature of the Earth's surface.

It might seem initially, therefore, that the longer the Earth radius, the longer the nautical mile, and this would be true if we used geocentric latitude. However, our system of latitude and longitude is based on geodetic, not geocentric, latitude.

Because the Earth is flattened at the Poles, the radius of curvature is increased and a greater arc is required to subtend an angle of one minute at the centre of curvature, hence a nautical
mile is longer at the Poles, at about 6108 feet. The amount of curvature is greatest at the Equator (the radius of curvature is decreased) and therefore the length of arc required to generate an angle of one minute at the centre of curvature is less. A nautical mile is shortest at the Equator and measures about 6048 feet. The average value, 6076.1 feet, is known as the International Nautical Mile and it is approximately 1852 metres. However, as stated before, for navigation calculations on this course, the Standard (or Admiralty) Nautical Mile of 6080 feet will be used.


Figure 2.8 Geodetic and geocentric latitude

## Conversion Factors

5400 nautical miles (from the Equator to either pole) $=10000$ kilometres
21600 nautical miles (circumference of the Earth) $=40000$ kilometres

## Great Circle Distances

The Great Circle distance between any 2 points on the Earth can be calculated using a general equation involving spherical geometry - this is not part of the EASA syllabus. Consequently, ATPL problems involving the calculation of Great Circle distances are limited to those where the 2 points lie on the special Great Circles, i.e. on the same meridian, on meridian and antimeridian, or on the Equator.

There are 5 general cases illustrated in the following examples. In all the examples, consider the direction that is flown - there may be some surprises!

The latitudes/longitudes for the named points are chosen to illustrate the example and are only in the approximate geographic 'ball-park'.

## Example 1 - Same meridian, same hemisphere

What is the shortest (great circle) distance between (A) London (5137N 00012W) and (B) Accra (0648N 00012W)?

Note: Both positions are on the same meridian (00012W).
Draw a simple sectional diagram of the Earth as below using the Great Circle formed by the 00012W meridian and its anti-meridian (17948E) although the latter is not needed for this example.


Figure 2.9
The positions of $B\left(06^{\circ} 48^{\prime}\right.$ north of the Equator measured at the centre of the Earth, $\left.C\right)$ and $A$ ( $51^{\circ} 37^{\prime}$ north of the Equator) are shown both on the $00012^{\prime} \mathrm{W}$ meridian. The angular distance between them (Change in Latitude or ch.lat) is $51^{\circ} 37^{\prime}-06^{\circ} 48^{\prime}=44^{\circ} 49^{\prime}$ South from A to $B$.

To calculate the Great Circle distance from A to B, convert the angular distance into linear distance , i.e. multiply the degrees by 60 and add the extra minutes.

Thus $44^{\circ} 49^{\prime}=(44 \times 60)+49=2689$ minutes of latitude

$$
=2689 \mathrm{NM}
$$

Be prepared to convert this answer to km or stat.m as required by the question. Always read the question carefully because although the basic calculation is done in NM, the Examiner may want the answer in different units.

## Example 2 - Same meridian, different hemispheres

What is the shortest distance between (D) Durban (2930S 03030E) and (E) Leningrad (5947N 03030E)?

Again, note that both positions are on the same meridian but in this case they are in opposite hemispheres. Draw a sectional diagram as below.


Figure 2.10
The ch.lat is now $29^{\circ} 30^{\prime}$ plus $59^{\circ} 47^{\prime}$ so the angular distance between $D$ and $E$ is $89^{\circ} 17^{\prime}$ north from D. The linear distance is $(89 \times 60)+17=5357 \mathrm{NM}$

Example 3 - Meridian and anti-meridian, same hemisphere
What is the shortest distance between (F) Rome (4155N 01110E) and (G) Honolulu (2117N 16850W)?

Both positions are in the same hemisphere but they are on meridian and anti-meridian - the two longitude values are of opposite sign and add up numerically to $180^{\circ}$.

$$
011^{\circ} 10^{\prime} \mathrm{E}+168^{\circ} 50^{\prime} \mathrm{W}=180^{\circ}
$$

Draw a sectional diagram as follows:


Figure 2.11
The Great Circle between F (Rome) and G (Honolulu) is over the North Pole - a surprising but important result if you wanted to travel in one stage. By inspection, the simplest way to calculate the angular distance in this case is to add the 2 latitudes and subtract their total from $180^{\circ}$, i.e. Angular distance $=180^{\circ}-\left(41^{\circ} 55^{\prime}+21^{\circ} 17^{\prime}\right)=180^{\circ}-63^{\circ} 12^{\prime}=116^{\circ} 48^{\prime}$.

An alternative method is to calculate the ch.lat from F to the Pole ( $90^{\circ}-41^{\circ} 55^{\prime}=48^{\circ} 05^{\prime}$ ) and from the Pole to $\mathrm{G}\left(90^{\circ}-21^{\circ} 17^{\prime}=68^{\circ} 43^{\prime}\right)$ and add the two to give the total angular distance. ( $48^{\circ} 05^{\prime}+68^{\circ} 43^{\prime}=116^{\circ} 48^{\prime}$ ).

Both methods are mathematically the same.
Angular distance $116^{\circ} 48^{\prime}=(116 \times 60)+48=7008$ NM.
The initial direction from F to the pole is north and then south from the Pole to G .

## Example 4. Meridian and anti-meridian, different hemispheres.

What is the shortest distance between (J) Tokyo (3557N 13535E) and (K) Rio de Janeiro (2210S 04425 W )?

This is the most difficult of the cases discussed. Note again that the longitudes have opposite signs and add up to $180^{\circ}$ - another meridian and anti-meridian case but with the latitudes in different hemispheres. Draw the sectional diagram as follows:


Figure 2.12
If the angles are drawn reasonably accurately, it is possible by inspection to see that the shortest distance between J and K is again via the North Pole and, therefore, to base the calculation on that route. Travelling to the North Pole from J is an angular distance of $90^{\circ}-35^{\circ} 57^{\prime}=54^{\circ} 03^{\prime}$ north; from the North Pole to the Equator is a further $90^{\circ}$ but now south; from the Equator to K is $22^{\circ} 10^{\prime}$ south. Adding these 3 stages together gives a total angular distance of $166^{\circ} 13^{\prime}$ which equates to a linear distance of $(166 \times 60)+13=9973$ NM.

If the initial decision had been to route via the South Pole, the values would have been $35^{\circ} 57^{\prime}$ south $+90^{\circ}$ south $+\left(90^{\circ}-22^{\circ} 10^{\prime}=67^{\circ} 50^{\prime}\right)$ north $=193^{\circ} 47^{\prime}$. Because the angular distance is greater than $180^{\circ}$, this solution is the longer way around the Earth. There is no need to recalculate but simply subtract the longer angular distance from $360^{\circ}$ to give the correct answer, i.e. $360^{\circ}$ $193^{\circ} 47^{\prime}=166^{\circ} 13^{\prime}$. Alternatively, continue the wrong calculation to give a linear distance of $(193 \times 60)+47=11627$ NM and subtract this distance from the circumference of the Earth, 21600 NM. Thus the correct answer is $21600-11627=9973$ NM. Remember also to change the directions involved.

## Example 5 - Two points on the Equator

What is the shortest distance between (L) Dakar (0000N 01635W) and (M) Singapore (0000N 10355E)?

This example uses the fact that a minute of longitude equals one nautical mile but only at the Equator. The sectional diagram is drawn in the plane of the Equator viewed from above the North Pole and, for reference, the Prime Meridian (00E/W) and the Anti-Prime Meridian (180E/W) are drawn.


Figure 2.13
The change of longitude (ch.long) from $L$ to $M$ is $016^{\circ} 35^{\prime}$ in an easterly direction to the Prime Meridian and a further $103^{\circ} 55^{\prime}$ in an easterly direction to M giving a total easterly ch.long of $120^{\circ} 30^{\prime}$. At the Equator only, an angular measurement of $120^{\circ} 30^{\prime}$ equals a linear measurement of $(120 \times 60)+30=7230$ NM.

## Example 6-A special case

What is the shortest distance between (N) Greenwich (5130N 00000E) and (P) Antipodes Island (5130S 18000E)?


Figure 2.14
The points N and P are on meridian and anti-meridian and have equal latitudes north and south of the Equator. They are diametrically opposite each other on the same great circle and are said to be antipodal , hence Captain Cook named the nearest land diametrically opposite London as Antipodes Island. The great circle distance between any point and its antipodes is $180^{\circ}$ in angular terms, 10800 NM in linear measure, i.e. the semi-circumference of the Earth.

## Mean Latitude

To calculate the mean latitude between two positions add the two latitudes together and divide the result by 2 .

## Example 1

Find the mean latitude between $52^{\circ} 17^{\prime} \mathrm{N}$ and $17^{\circ} 57^{\prime} \mathrm{N}$ ?
$52^{\circ} 17^{\prime} \mathrm{N}+17^{\circ} 57^{\prime} \mathrm{N}=70^{\circ} 14^{\prime} \div 2=35^{\circ} 07^{\prime} \mathrm{N}$

## Example 2

Find the mean latitude between $35^{\circ} 25^{\prime} \mathrm{N}$ and $13^{\circ} 38^{\prime} \mathrm{S}$ ?
Because the latitudes are in opposite hemispheres to find the mean latitude you must take the $24^{\circ} 31.5^{\prime}$ away from $35^{\circ} 25^{\prime} \mathrm{N}$ to give $10^{\circ} 53.5^{\prime} \mathrm{N}$

## Questions

1. What is the change of latitude between the following positions:-
a. $\quad 52^{\circ} 15^{\prime} \mathrm{N}$ to $39^{\circ} 35^{\prime} \mathrm{N}$
b. $\quad 49^{\circ} 35^{\prime} \mathrm{N}$ to $60^{\circ} 20^{\prime} \mathrm{S}$
c. $\quad 74^{\circ} 20^{\prime} \mathrm{S}$ to $34^{\circ} 30^{\prime} \mathrm{S}$
d. $\quad 71^{\circ} 20^{\prime} \mathrm{N}$ to $86^{\circ} 45^{\prime} \mathrm{N}$ over the North Pole.
2. What is the difference in nautical miles and in kilometres from position $A\left(41^{\circ} 25^{\prime} N\right)$ to position $B\left(79^{\circ} 30^{\prime} N\right)$ ? Both are on the same meridian.
3. What is the change of longitude between the following positions:-
a. $\quad 075^{\circ} 40^{\prime} \mathrm{W}$ to $125^{\circ} 35^{\prime} \mathrm{W}$
b. $\quad 001^{\circ} 20^{\prime} \mathrm{E}$ to $004^{\circ} 20^{\prime} \mathrm{W}$
c. $\quad 150^{\circ} 40^{\prime} \mathrm{E}$ to $179^{\circ} 30^{\prime} \mathrm{E}$
d. $\quad 162^{\circ} 36^{\prime} \mathrm{W}$ to $140^{\circ} 42^{\prime} \mathrm{E}$
4. Give the direction and change of latitude and longitude from $X$ to $Y$ in each case:

## X

a. $\quad 50^{\circ} 31^{\prime} \mathrm{N} 006^{\circ} 30^{\prime} \mathrm{W}$
b. $\quad 47^{\circ} 32^{\prime} \mathrm{N} 002^{\circ} 46^{\prime} \mathrm{W}$
c. $\quad 61^{\circ} 47^{\prime} \mathrm{N} 003^{\circ} 46^{\prime} \mathrm{W}$
d. $\quad 31^{\circ} 27^{\prime} \mathrm{S} 091^{\circ} 47^{\prime} \mathrm{E}$
e. $\quad 51^{\circ} 05^{\prime} \mathrm{N} 177^{\circ} 42^{\prime} \mathrm{E}$

## Y

to $\quad 52^{\circ} 00^{\prime} \mathrm{N} 008^{\circ} 35^{\prime} \mathrm{W}$
to $43^{\circ} 56^{\prime} \mathrm{N} 001^{\circ} 33^{\prime} \mathrm{W}$
to $\quad 62^{\circ} 13^{\prime} \mathrm{N} 001^{\circ} 36^{\prime} \mathrm{E}$
to $\quad 35^{\circ} 57^{\prime} \mathrm{N} 096^{\circ} 31^{\prime} \mathrm{E}$
to $\quad 51^{\circ} 06^{\prime} \mathrm{N} 167^{\circ} 42^{\prime} \mathrm{W}$
5. Give the shortest distance in nautical miles and kilometres between the following positions:

| a. | $52^{\circ} 06^{\prime} \mathrm{N} 002^{\circ} 32^{\prime} \mathrm{E}$ | and | $53^{\circ} 36^{\prime} \mathrm{N} 002^{\circ} 32^{\prime} \mathrm{E}$ |
| :--- | :--- | :--- | :--- |
| b. | $04^{\circ} 41^{\prime} \mathrm{S} 163^{\circ} 36^{\prime} \mathrm{W}$ | and | $03^{\circ} 21^{\prime} \mathrm{N} 163^{\circ} 36^{\prime} \mathrm{W}$ |
| c. | $62^{\circ} 00^{\prime} \mathrm{N} 093^{\circ} 00^{\prime} \mathrm{E}$ | and | $62^{\circ} 00^{\prime} \mathrm{N} 087^{\circ} 00^{\prime} \mathrm{W}$ |
| d. | $00^{\circ} 00^{\prime} \mathrm{N} 176^{\circ} 00^{\prime} \mathrm{E}$ | and | $00^{\circ} 00^{\prime} \mathrm{N} 173^{\circ} 00^{\prime} \mathrm{W}$ |
| e. | $43^{\circ} 57^{\prime} \mathrm{N} 071^{\circ} 37^{\prime} \mathrm{W}$ | and | $43^{\circ} 57^{\prime} \mathrm{S} 108^{\circ} 23^{\prime} \mathrm{E}$ |

6. An aircraft is to fly from position $72^{\circ} 00^{\prime} \mathrm{N} 002^{\circ} 30^{\prime} \mathrm{E}$ to position $72^{\circ} 00^{\prime} \mathrm{N} 177^{\circ} 30^{\prime} \mathrm{W}$ on the shortest possible route.
a. Give the initial (True) track direction
b. Will the track direction remain the same for the whole flight?
c. Give a reason for the answer given in $b$ above.
7. You are at position A at $54^{\circ} 20^{\prime} \mathrm{N} 002^{\circ} 30^{\prime} \mathrm{W}$. Given a ch.lat of $16^{\circ} 20^{\prime} \mathrm{N}$ and a ch.long of $20^{\circ} 30^{\prime} \mathrm{W}$ to B , what is the position of $B$ ?
8. You are at position $C$ at $36^{\circ} 47^{\prime} S 179^{\circ} 21^{\prime} \mathrm{E}$. Given a ch. at of $46^{\circ} 47^{\prime} \mathrm{N}$ and a ch.long of $20^{\circ} 30^{\prime} E$ to $D$, what is the position of $D$ ?
9. What is the position of the Rhumb Line between 2 points relative to the Great Circle between the same points if the points are:
a. In the Northern hemisphere?
b. In the Southern hemisphere?
10. Any Meridian Line is a:
a. Rhumb Line
b. Semi Great Circle
c. Rhumb Line and a semi Great Circle
11. A Rhumb Line cuts all meridians at the same angle. This gives:
a. The shortest distance between two points.
b. A line which could never be a Great Circle track
c. A line of constant direction

## Answers

1 a. $12^{\circ} 40^{\prime} \mathrm{S}$
b. $\quad 109^{\circ} 55^{\prime} \mathrm{S}$
c. $\quad 39^{\circ} 50^{\prime} \mathrm{N}$
d. $\quad 21^{\circ} 55^{\prime}$
2. 2285 NM is 4232 km using a calculator or 4230 km using the Navigation Computer.
3. a. $49^{\circ} 55^{\prime} \mathrm{W}$
b. $\quad 05^{\circ} 40^{\prime} \mathrm{W}$
c. $\quad 28^{\circ} 50^{\prime} \mathrm{E}$
d. $\quad 56^{\circ} 42^{\prime} \mathrm{W}$ (Not $303^{\circ} 18^{\prime}$ - we want the smaller arc).
4. a. $01^{\circ} 29^{\prime} \mathrm{N} 002^{\circ} 05^{\prime} \mathrm{W}$
b. $\quad 03^{\circ} 36^{\prime} S 001^{\circ} 13^{\prime} \mathrm{E}$
c. $\quad 00^{\circ} 26^{\prime} \mathrm{N} 005^{\circ} 22^{\prime} \mathrm{E}$
d. $\quad 67^{\circ} 24^{\prime} \mathrm{N} 004^{\circ} 44^{\prime} \mathrm{E}$
e. $\quad 00^{\circ} 01^{\prime} \mathrm{N} 014^{\circ} 36^{\prime} \mathrm{E}$
5. a. 90 NM 167 km
b. $\quad 482 \mathrm{NM} 893 \mathrm{~km}$
c. $\quad 3360$ NM 6223 km
d. $\quad 660 \mathrm{NM} 1222 \mathrm{~km}$
e. $\quad 10800 \mathrm{NM} 20000 \mathrm{~km}$
6. a. $360^{\circ}(\mathrm{T})$
b. No
c. Route over the North Pole, initial track True North, then once over the Pole, True South.
7. $70^{\circ} 40^{\prime} \mathrm{N} 023^{\circ} 00^{\prime} \mathrm{W}$
8. $10^{\circ} 00^{\prime} \mathrm{N} 160^{\circ} 09^{\prime} \mathrm{W}$
9. a. Nearer the Equator (South of the Great Circle)
b. Nearer the Equator (North of the Great Circle)
10. C
11. C

## Chapter

3

## Earth Magnetism

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## True Direction

In the chapter on the Form of the Earth, we defined direction with reference to the poles about which the Earth spins (North or South) and the direction in which the Earth spins (East). This is the basis of what is known as True Direction and is what is shown on maps. If you need to fly from, say, Oxford to Leicester, you draw a straight line on the map (assuming that there are no air traffic or other obstacles) between the two airports and measure its direction. This is a True direction. Notice that, at this stage, there has been no reference to magnetic compasses. If there were such a thing as a "compass that works in True Direction", we would not even need to consider Earth magnetism. However, until about 50 years ago, no such thing existed. (They do now - the Inertial Navigation System (INS) finds its direction by establishing the direction of Earth rotation, and so is a genuine "true compass". But these are very expensive and are fitted only to sophisticated aircraft). Most aircraft are still steered by a magnetic compass, and even aircraft with INS will have a standby compass, which will be magnetic.

## Magnetic Direction

It has been known for hundreds, and probably for thousands, of years, that it is possible to hold a reasonably constant direction by making use of Earth magnetism. Because the centre of the Earth is still cooling down, at its centre is a mass of molten liquid. This has the effect of making the Earth act magnetically as though there were a huge bar magnet running through it, aligned fairly closely to the North and South True poles.

The alignment of this magnetic effect is only approximately in line with the True poles and this can lead to marked differences between True and Magnetic direction in some parts of the world. Furthermore, the magnetic pattern is not symmetrical in the $\mathrm{N}-\mathrm{S}$ sense; the North and South magnetic poles are not exactly opposite each other on the Earth. In effect, it is like a bent bar magnet.


Figure 3.1 The Earth's poles

## Magnetic North

Magnetic North is the horizontal direction indicated by a freely suspended magnet influenced only by the Earth's magnetic field. This direction is sometimes referred to as the 'magnetic meridian' at that point.

Magnetic Direction is measured from Magnetic North clockwise through $360^{\circ}$, and is suffixed by the letter ' M ', e.g. $043(\mathrm{M})$ and $270(\mathrm{M})$.

## Variation

Variation is the angular difference between the directions of True and Magnetic North at any point.

Variation is the angle between True and Magnetic North and is measured in degrees East or West from True North.

A line on the surface of the Earth joining points of equal magnetic variation is called an Isogonal.

## Change of Variation According to Observer's Position on the Earth

The amount and the direction of variation depends on the relative geometry of the observer, the True Poles, and the Magnetic Poles.

Consider Figure 3.2. It shows the True North Pole and the Magnetic North Pole. The red line on the globe is the current longitude of the Magnetic North Pole (about $120^{\circ} \mathrm{W}$ ).


Figure 3.2 Idealized model of world variation
To an observer at A, True North is the direction up the meridian at A to the True North Pole. Magnetic North is the direction that his compass needle will point, i.e. towards the Magnetic North Pole, which, from A, is to the right (East) of True North. In other words, Magnetic North is East of True North.

To an observer at B, the direction of Magnetic North will be left of True North because the geometry of the relative positions is different. Magnetic North is West of True North.

Now consider an observer at C. For him, as for the case of A, Magnetic North is East of True North. However, because he is so much further away from both of them, the difference in their directions (i.e. the variation) is a smaller angle.

Similarly, for an observer at D, like the one at B, Magnetic North is West of True North, so variation is West. However, it will be a smaller angle of variation than for the observer at B. Note also the situation for an observer at E. We have started at the True Pole, taken the Great Circle to the Magnetic Pole and then continued it in a Great Circle round the Earth. To an observer at E, the line joining him to the Magnetic North Pole and to the True North Pole will be the same Great Circle. It will be the direction of True North (i.e. straight up the meridian) and it will also be the direction in which his compass needle will point. For him, the value of variation will be zero.

This line is called the Agonic Line. It is the line connecting points of zero variation.

## Situation at the Poles

For an aircraft flying between the North True Pole and the North Magnetic Pole, the variation on that shorter arc of the Great Circle is not zero, it is $180^{\circ}$. Consider the diagram below.


Figure 3.3 True and magnetic north poles - idealized
An aircraft is at position ' A ', somewhere on the line between the North True Pole and the North Magnetic Pole. The meridian connecting him to the True Pole is the direction of True North. However, its compass needle will point at the Magnetic North Pole, in exactly the opposite direction. The variation at this point will be $180^{\circ}$. The zero variation line is shown in yellow and the $180^{\circ}$ variation line is shown in green.

Therefore, the maximum possible value of variation is $180^{\circ}$ and this occurs at both the North and the South Poles.

## The Real Variation Map

Figure 3.3 showed a somewhat idealized situation. As we said earlier, the North and South Magnetic Poles are not actually antipodal (i.e. not directly opposite each other). There is no reason why they should be. Earth magnetism is not really caused by a large bar magnet this is simply a convenient analogy. It is actually caused by the swirling of molten magnetic magma below the surface of the Earth. The effect is more like a bent bar magnet. The National Environmental Research Council (NERC) magnetic map for 1st Jan 2000 positions the North Magnetic Pole at approximately 81 N 110 W whilst the South Magnetic Pole is at 63 S 135 E . The position of the North Magnetic Pole in 2009 was 84N 120W.

The actual situation is shown by the charts at Figure 3.4, Figure 3.5, and Figure 3.6. The disposition of variation is not quite as geometrically neat as the diagram in Figure 3.3, but there definitely are 2 lines of zero variation, one running southwards from the True North Pole and the other running southwards from the Magnetic Pole. One runs down through Europe (the variation near Stuttgart, in Germany, is zero) and the other runs down through the USA.


Figure 3.4 Variation at the North Pole


Figure 3.5 Variation at the South Pole (Charts supplied by the British Geological Survey (NERC) in March 2002)

The agonic line running down through the USA continues through South America (Figure 3.4) and then continues to the True South Pole (Figure 3.5), much as one would expect from the idealized model. However, the agonic line running down out of the North True Pole passes through Stuttgart into Central Africa, then curves upward again back onto the North Polar chart (Figure 3.4) into North Central Asia, then southwards again through Australia to the South Magnetic Pole.

It is important to realize that isogonals are not the actual magnetic lines of flux (which are a natural phenomenon). They are the difference between the alignment of the lines of flux and the local direction of True North at any point.

## Real Variation at the Poles

Figure 3.6 shows the actual variation at the North Poles. The agonic line running up from Europe (yellow) changes to $180^{\circ}$ (black) at the True North Pole. The red isogonals round the True Pole go from $015^{\circ} \mathrm{E}$ to $180^{\circ} \mathrm{E}$ variation whilst the blue ones go from $015^{\circ} \mathrm{W}$ to $180^{\circ} \mathrm{W}$.


Figure 3.6 Variation at the North Pole (Chart supplied by the British Geological Survey (NERC) in March 2002)
Therefore the isogonals converge on the True North Pole.
However, exactly the same thing occurs round the Magnetic North Pole. The agonic line running up from the USA (yellow) changes to $180^{\circ}$ variation (black) between Magnetic and True Poles. Again, the red isogonals round the Magnetic Pole go from zero to $180^{\circ} \mathrm{E}$ variation whilst the blue ones go from zero to $180^{\circ} \mathrm{W}$. Therefore, the isogonals converge on the Magnetic North Pole as well. A study of Figure 3.5 will show the same phenomenon at the South Poles also.

Isogonals converge on both the True and the Magnetic North and South Poles.

## Changes in Variation over Time

The value of variation at any point on the Earth changes over a period of time. For instance, the present (2012) value of variation at Oxford is about $2^{\circ} \mathrm{W}$. However, this appears to be reducing at a rate of approximately one degree every 9 years. In 1960, Oxford's variation was about $8^{\circ} \mathrm{W}$ and, if the present trend continues, the value should fall to zero in about 2040, becoming easterly subsequently. Other changes are taking place in other parts of the world.

The reason for the change is that the position of the Magnetic North Pole (which is to the West of Oxford, but by more than $90^{\circ} \mathrm{W}$ of longitude) is moving westwards round the North True Pole, thereby reducing the variation. The reasons for this movement are not fully understood, but it is evidently associated with movement of the molten magma in the Earth's core.

At one time it used to be thought that the Magnetic North Pole rotated in a near-perfect circle round the True Pole at a rate of one revolution every 960 years. However, this hypothesis was based simply on empirical observation over a period of time and more recent and accurate observation over the last 50 years has shown that, at least over the last few years, the Magnetic Pole has moved northwards as well as westwards. All that can be said is that the Earth's magnetic field is certainly changing, and by observing it over a period of time and extrapolating the change, we can make reasonably accurate forecasts of variation for up to about ten years ahead.

There appear to be at least 3 predictable cycles in the pattern:
Secular. The secular movement is this long-term change described above.
Annual. Superimposed on this long-term change is a sinusoidal change with a period of one year. This is associated with the Earth's orbit round the Sun.

Diurnal. Superimposed on these 2 patterns is a sinusoidal change with a period of one day. This appears to be associated with the daily changes in the height of the ionosphere as the Earth rotates, presenting different areas of the upper atmosphere to the Sun. The variation can change up to about $0.1^{\circ}$ over the course of a day.

In addition, there are unpredictable changes. One is associated with solar activity and one with local anomalies.

Solar Activity. The Sun experiences cycles of sunspot activity which peak every 11 years. Huge solar flares are expelled far out into space. The period is predictable, but whether it affects the Earth is not. If one of these flares is pointed towards Earth, a tongue of intense ionisation curls around the upper atmosphere, causing various effects, the most notable of which is the Aurora Borealis (the Northern Lights). There is also a similar effect round the South Magnetic Pole, called the Aurora Australis.

These are known as 'magnetic storms' and the effects can be very intense. During the most recent of these 11 -year cycles, variation changes of up to $7^{\circ}$ were observed. Sunspot activity and solar flares occur at other times as well and minor magnetic storms can occur outside the 11-year peak of the period.

Local Anomalies. Local magnetic anomalies are caused by magnetic deposits or rock formations that cause the field to be different within a particular area.

In addition, scientific surveys have shown that variation can change slightly with altitude, though few aircraft systems would be sensitive enough for this effect to be noticed.

Accordingly, it is very difficult to know the precise instantaneous value of variation affecting an aircraft to better than about 2 degrees and, even if great care is taken over finding and correcting for it, to better than about half a degree over a period of time. This is why the emergence of systems based on highly accurate gyros, particularly INS, in the 1960s/70s was such a major advance in navigation technology. It was not merely that they calculated present position, important though that was. It was because, for the first time, there was a source of accurate reliable heading.

## Updating Isogonals

Pilots should always fly with the most up-to-date chart for flight safety reasons. Aeronautical information is constantly changing, airways are re-aligned, the positions and frequencies of VORs and DMEs are changed and danger areas alter shape or are moved as civil and defence requirements change within a country. If the chart is republished at frequent intervals, the isogonals should be comparatively recent as well and for most radio navigation charts it is not normally necessary to update the isogonals.

However, the interval between reissues of topographical maps may be considerably longer perhaps every 5 or 10 years. In this case, if the variation has changed significantly, it may be necessary for the pilot to bring the isogonals up to date during the flight planning process. On most maps and charts the year of origin is shown and some indication of the annual change (due to the movement of the magnetic poles) is given. This may be done by a small arrow showing the direction and distance of the annual change of the position of the isogonal as in Figure 3.7,


Figure 3.7
or by a statement giving the annual change in the variation quoted on the isogonal as in Figure 3.8.


Annual change 7' E
Figure 3.8

## Magnetic Dip Angle

Consider the diagram at Figure 3.9, showing the lines of magnetic force round the Earth's surface. Imagine aircraft at positions A, B, and C. The lines of force will make different angles to the horizontal in each location. This angle is known as the Angle of Dip and is shown diagrammatically in Figure 3.10.


Figure 3.9 Lines of force

The Earth's Magnetic field is along the total line of force, shown as T. This can be resolved into a horizontal component H and a vertical component Z .

Vertical Component. The vertical component $Z$ is of no value in determining horizontal direction. In fact, it is undesirable for two reasons.

Firstly, it causes the needle of a direct reading magnetic compass to dip from the horizontal. This is partially corrected by the use of pendulous suspension, but the end result is that the needle still hangs down to some extent, so that the centre of gravity is no longer directly below the centre of suspension, thereby resulting in the well-known problems of turning and acceleration errors.


Figure 3.10 The angle of dip

Secondly, the vertical component of the Earth's field induces vertical soft-iron magnetism in the aircraft, thereby increasing the deviation.

Horizontal Component. The horizontal component is the part which is detected by the compass needle in order to determine magnetic north and is known as the directive force. In the region of the magnetic equator the strength of the directive force H approaches the value of T , while $Z$ approaches zero as does the angle of dip.

Figure 3.11 considers positions A, B, and C on the Earth's surface as shown in Figure 3.9. It becomes apparent that the directive force $H$ decreases as the angle of dip increases, and vice versa.


Figure 3.11 The effect of latitude on the components of dip1
When either of the Earth's magnetic poles is approached, this component approaches zero strength, while the value of $Z$ approaches that of $T$.

Magnetic field strength is measured in units of microteslas. The generally accepted figure at which the horizontal component of the Earth's field becomes too small to be detected by a compass is 6 microteslas. Clearly, in practice, the actual detection threshold will depend on the design of the particular compass being used, but $6 \mu \mathrm{~T}$ is the notional figure normally quoted.

The maximum possible dip angle is $90^{\circ}$ and this occurs overhead the North and South Magnetic Poles.

Figure 3.12 is an illustration of the $6 \mu \mathrm{~T}$ zone round the North Poles. This particular diagram is dated about 1950, but it has been chosen because the type of projection shows a reasonably representative shape near the Poles.


Figure 3.12 The $6 \mu$ T zone in the Arctic - polar projection
Figure 3.13 and Figure 3.14 show more recent surveys of the $6 \mu$ t zone. They have been included because they demonstrate that there is a $6 \mu \mathrm{~T}$ zone around the South Magnetic Pole as well as the North. They are dated 01 Jan 2000. The projection used is a Winkel, which introduces distortions near the True Pole, which has the effect of making the zone round the North Pole appear larger than the one round the South when, in fact, they are approximately the same size and shape.



Figure 3.14 Dip angle

In fact, the relationship between H and dip angle is not quite as simple as suggested by Figure 3.14 because of irregularities in the pattern of the Earth's field and changes with position and time of the total magnetic force T .

The angle of dip at Oxford is about $66^{\circ}$ but the use of pendulous suspension in a simple Direct Reading Compass reduces the dip of the magnetic compass assembly to about $2^{\circ}$. However, this small residual angle is still sufficient to give the well-known turning and acceleration errors.

## Deviation

In addition to the correction for the difference between True and Magnetic North (variation), there is a further correction to be applied. This is for the difference between Magnetic North and the direction that a compass needle points. Why are these not the same?

The reason is that the aircraft itself is full of magnetic influences (metal, electric currents, etc.) and so acts partially as a magnet itself. Therefore, the compass needle is mainly attracted to the Magnetic North Pole, but partially attracted to the magnetism within the aircraft. This aircraft magnetism deflects the compass needle from Magnetic North to a new direction, which we call Compass North. The difference between Magnetic North and Compass North is called Deviation.

Deviation is defined as the angle measured at a point between the direction indicated by a compass needle and the direction of Magnetic North.

It is termed East or West according to whether Compass North lies to the East or West of Magnetic North.

Deviation can also be quoted as plus or minus. For instance a deviation of $-3^{\circ}$ is the same as $3^{\circ} \mathrm{W}$ deviation. Plus is East, minus is West. The reason for this is that deviation is seen as a correction to be applied to the compass in order to establish Magnetic heading.

When deviations are quoted as plus or minus they are to be applied to the compass heading in order to give magnetic heading.

## Application of Variation and Deviation

When Magnetic direction is the same as True direction the variation is nil; otherwise Magnetic North direction may lie either to the West or East of the True North Direction. See Figure 3.15 \& Figure 3.16.


Figure 3.15 Westerly variation

The direction of the aircraft relative to True North is $105^{\circ}$

The direction of the aircraft relative to Magnetic North is $122^{\circ}$.

Magnetic North is to the West of True North of True North.

## VARIATION IS $17^{\circ} \mathrm{W}$



Figure 3.16 Easterly variation

The direction of the aircraft relative to True North is $105^{\circ}$

The direction of the aircraft relative to Magnetic North is $088^{\circ}$.

Magnetic North is to the East of True North.

## VARIATION IS $17^{\circ} \mathrm{E}$

The direction in which the fore and aft axis of the aircraft is pointing is known as the aircraft's heading.

We are thus able to complete the following tables:-

From Figure 3.15

| Heading <br> True | Variation | Heading <br> Magnetic |
| :---: | :---: | :---: |
| $105^{\circ}$ | $17^{\circ} \mathrm{W}$ | $122^{\circ}$ |

From Figure 3.16

| Heading <br> True | Variation | Heading <br> Magnetic |
| :---: | :---: | :---: |
| $105^{\circ}$ | $17^{\circ} \mathrm{E}$ | $088^{\circ}$ |

and from this we deduce the following rule which must be remembered:-

Deviation is the angle between Magnetic North and Compass North measured in degrees East or West from Magnetic North. Figure 3.17 \& Figure 3.18 illustrate this.


Figure 3.17 Westerly deviation

The direction of the aircraft relative to True North is $100^{\circ}$.

The direction of the aircraft relative to Magnetic North is $125^{\circ}$

The direction of the aircraft indicated by the Compass is $135^{\circ}$.


Figure 3.18 Easterly deviation

The direction of the aircraft relative to Magnetic North is $100^{\circ}$

The direction of the aircraft relative to Magnetic North is $125^{\circ}$.

The direction of the aircraft indicated by the Compass is $115^{\circ}$.
and we are able to complete the following tables:-
From Figure 3.17

| Heading <br> True | Variation | Heading <br> Magnetic | Deviation | Heading <br> Compass |
| :---: | :---: | :---: | :---: | :---: |
| $100^{\circ}$ | $25^{\circ} \mathrm{W}$ | $125^{\circ}$ | $10^{\circ} \mathrm{W}$ | $135^{\circ}$ |

From Figure 3.18

| Heading <br> True | Variation | Heading <br> Magnetic | Deviation | Heading <br> Compass |
| :---: | :---: | :---: | :---: | :---: |
| $100^{\circ}$ | $25^{\circ} \mathrm{W}$ | $125^{\circ}$ | $10^{\circ} \mathrm{E}$ | $115^{\circ}$ |

and from this we deduce the following rule which must be remembered:-

| DEVIATION EAST | - | COMPASS LEAST |
| :--- | :--- | :--- |
| DEVIATION WEST | - | COMPASS BEST |

The signs for plus and minus are sometimes used as a prefix instead of the suffixes East or West, in which case you will see that the rule is to apply the deviation (as stated) to the Compass Heading to obtain the Magnetic Heading and conversely apply the deviation (with the sign changed) to Magnetic Heading to obtain Compass Heading.

For example:-

## Deviation $3^{\circ} \mathrm{E}$

Hdg 263(M) deviation $3^{\circ}$ E Deviation East Compass Least Hdg 260(C)
Hdg 260(C) deviation $3^{\circ} \mathrm{E}$ Deviation East Compass Least Hdg 263(M)
Alternatively Deviation $+3^{\circ}$
Hdg 263(M) deviation +3 to convert from Magnetic to Compass.
Change the sign and apply to $\mathrm{Hdg}(\mathrm{M})$;
Hdg 263-3 = Hdg 260(C)
Or Apply deviation to Hdg (C)
Hdg 260(C) +3 to convert from Compass to Magnetic.
Hdg 260+3 = Hdg 263(M)
Another rule, equivalent to the one stated above, is also used:-
From Compass to Magnetic the signs are true ( E is + , $\mathbf{W}$ is -)

## Definitions

Definitions of terms associated with direction on the Earth.

Heading is the direction in which the fore and aft axis of the aircraft is pointing; it may be measured from True, Magnetic, or Compass North.

Variation is the angle measured at a point between True North direction and the direction indicated by a freely suspended compass needle influenced only by the Earth's magnetic field. It is termed East or West according to whether Magnetic North lies to the East or West of True North.

Deviation is the angle measured at a point between the direction indicated by a compass needle and the direction of Magnetic North. It is termed East or West according to whether the Compass North lies to the East or West of Magnetic North.

Isogonals are pecked lines on a map or chart joining places of equal magnetic variation.
Agonic Line is the name given to isogonals joining places of zero variation.
The Angle of Dip is the angle in the vertical plane between the horizontal and the Earth's magnetic field at a point.

Isoclinals are lines on a map or chart joining places of equal magnetic dip.
Aclinic Lines is the name given to isoclinals joining places of zero dip.
Note:- Isoclinals and Aclinic lines do not appear on Navigation charts.

## Questions

1. The sensitivity of a direct reading magnetic compass is:
a. inversely proportional to the horizontal component of the Earth's magnetic field
b. proportional to the horizontal component of the Earth's magnetic field
c. inversely proportional to the vertical component of the Earth's magnetic field
d. inversely proportional to the vertical and horizontal components of the Earth's magnetic field
2. What is the definition of magnetic variation?
a. The angle between the direction indicated by a compass and Magnetic North
b. The angle between True North and Compass North
c. The angle between Magnetic North and True North
d. The angle between Magnetic Heading and Magnetic North
3. At the magnetic equator:
a. Dip is zero
b. Variation is zero
c. Deviation is zero
d. The isogonal is an agonic line
4. Which of these is a correct statement about the Earth's magnetic field?
a. It acts as though there is a large blue magnetic pole in Northern Canada
b. The angle of dip is the angle between the vertical and the total magnetic force
c. It may be temporary, transient, or permanent
d. It has no effect on aircraft deviation
5. Where is a compass most effective?
a. About midway between the Earth's magnetic poles
b. In the region of the magnetic South Pole
c. In the region of the magnetic North Pole
d. On the geographic equator
6. The value of variation:
a. is zero at the magnetic equator
b. has a maximum value of $180^{\circ}$
c. has a maximum value of $45^{\circ} \mathrm{E}$ or $45^{\circ} \mathrm{W}$
d. cannot exceed $90^{\circ}$
7. The agonic line:
a. is midway between the magnetic North and South poles
b. follows the geographic equator
c. is the shorter distance between the respective True and Magnetic North and South poles
d. follows separate paths out of the North polar regions, one currently running through Western Europe and the other through the USA
8. The angle between True North and Magnetic North is known as:
a. deviation
b. variation
c. alignment error
d. dip
9. The value of magnetic variation on a chart changes with time. This is due to:
a. Movement of the magnetic poles, causing an increase
b. Increase in the magnetic field, causing an increase
c. Reduction in the magnetic field, causing a decrease
d. Movement of the magnetic poles, which can cause either an increase or a decrease
10. Isogonal lines converge as follows:
a. At the North Magnetic Pole
b. At the North and South Magnetic and Geographical Poles
c. At the North and South Magnetic Poles
d. At the Magnetic equator
11. What is the maximum possible value of dip angle?
a. $66^{\circ}$
b. $\quad 180^{\circ}$
c. $\quad 90^{\circ}$
d. $45^{\circ}$
12. What is the dip angle at the South Magnetic Pole?
a. $\quad 0^{\circ}$
b. $\quad 90^{\circ}$
c. $\quad 180^{\circ}$
d. $\quad 64^{\circ}$
13. What is a line of equal magnetic variation?
a. An isocline
b. An isogonal
c. An isogriv
d. An isovar
14. If variation is West, then:
a. True North is West of Magnetic North
b. Compass North is West of Magnetic North
c. True North is East of Magnetic North
d. Magnetic North is West of Compass North
15. Fill in the blank spaces in the table below.

| ${ }^{\circ}$ TRUE | VARN | ${ }^{\circ} \mathrm{MAG}$ | DEVN | ${ }^{\circ} \mathrm{COMP}$ |
| :---: | :---: | :---: | :---: | :---: |
| 260 |  | 291 | 3E |  |
|  | 10W |  | 1E | 070 |
|  | 7W | 001 | 2E |  |
|  | 17W |  | 0 | 020 |
| 306 | 10E |  | 1W |  |
| 036 |  | 031 |  | 033 |
|  | 5 E | 025 |  | 023 |
| 359 | 3W |  | 2 E |  |
|  | 23E |  | 2W | 221 |
| 312 |  | 322 |  | 319 |
| 002 | 3W |  | 1W |  |
| 260 |  | 291 | -3 |  |
|  | 5E |  | +1 | 070 |
|  | 3W | 001 | +2 |  |
| 022 | 10W |  | 3W | 035 |

Answers

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| b | c | a | a | a | b | d | b | d | b | c | b |


| 13 | 14 |
| :---: | :---: |
| b | c |

15. 

| ${ }^{\circ}$ TRUE | VARN | ${ }^{\circ} \mathrm{MAG}$ | DEVN | ${ }^{\circ} \mathrm{COMP}$ |
| :---: | :---: | :---: | :---: | :---: |
| 260 | 31W | 291 | 3E | 288 |
| 061 | 10W | 071 | 1E | 070 |
| 354 | 7W | 001 | 2E | 359 |
| 003 | 17W | 020 | 0 | 020 |
| 306 | 10E | 296 | 1W | 297 |
| 036 | 5E | 031 | 2W | 033 |
| 030 | 5E | 025 | 2E | 023 |
| 359 | 3 W | 002 | 2E | 000 |
| 242 | 23E | 219 | 2W | 221 |
| 312 | 10W | 322 | 3E | 319 |
| 002 | 3W | 005 | 1W | 006 |
| 260 | 31W | 291 | -3 | 294 |
| 076 | 5E | 071 | +1 | 070 |
| 358 | 3W | 001 | +2 | 359 |
| 022 | 10W | 032 | 3W | 035 |

## Chapter 4

## The Navigation Computer - Slide Rule Face

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## The Navigation Computer

The Navigation Computer is an analogue computer used to solve navigation and flight planning problems quickly and easily. The EASA regulations allow students to take the analogue computer of their choice into the General Navigation examination and several manufacturers make suitable products. The diagrams in these notes are based on the Pooley CRP-5. If students choose to buy other computers, it will be their own responsibility to adapt the examples to their own instruments.

Navigation Computer calculations form a large part of the General Navigation EASA ATPL examination. Almost $25 \%$ of the exam comprises direct Navigation Computer questions and many of the other questions can often be solved faster with a Navigation Computer than by calculator. Rapid operation of the Navigation Computer is also fundamental to Flight Planning. It is essential that you learn to operate the Navigation Computer quickly and accurately. Those who do not make the effort to master it completely lose marks unnecessarily in exams.

The Navigation Computer has 2 faces:-
A circular slide rule face
A wind face.
This chapter deals with the slide rule face.

## Slide Rule Theory

It is easier to learn how to use the Navigation Computer if you understand the theory behind it rather than simply learning how to do the operations parrot-fashion. The advent of digital calculators over the last 30 years has meant that few students have experience of the oldfashioned engineer's slide rule. The Navigation Computer uses the same slide rule theory except that instead of the scales being presented on a straight ruler they are on 2 circular discs.

We will start this explanation by imagining that we have a problem of addition, not multiplication.

Imagine that you had to add two numbers, but you had never learnt how to do simple addition. One way to solve the problem would be to use two rulers.


Figure 4.1 Two straight linear rulers

Suppose you had to add $3+2$. Put the zero of the top ruler against the ' 3 ' on the bottom, as in Figure 4.2

Now look down from the ' 2 ' on the top ruler to read off the ' 5 ' on the bottom.


Figure 4.2 Adding $3+2$
In addition, note that with just one alignment of the ruler you can also read off the answer to the sum:-


Figure 4.3 Multiple simultaneous additions
$3+$ any other number = ?
Note: You can also subtract. In Figure 4.4, by aligning '4' on the bottom ruler with '1' on the top ruler and then looking back to the zero on the top ruler, you can read off the answer '3'.


Figure 4.4 Subtraction
Now, keeping that at the back of our mind, let us deal with a different problem. Suppose you had to multiply $4 \times 8$, but had never learnt to do simple multiplication. Another way to tackle this problem would be to treat it as $2^{2} \times 2^{3}$.

This is the same as saying $2 \times 2 \times 2 \times 2 \times 2$, which is $2 \times 2 \times 2 \times 2 \times 2$.
So $2^{2} \times 2^{3}=2^{5}$
In other words, to multiply 2 numbers expressed as powers of the same base, you add their indices.

Now, if we put our numbers on a ruler again, but on a logarithmic, not a linear scale, the length of the rulers is scaled in proportion to their indices.


Figure 4.5 Logarithmic v linear scale
We can now use our rulers for multiplying and dividing instead of adding and subtracting. This is much more useful.

For example, in order to multiply by 2 , we align the ' 1 ' on the top scale with the ' 2 ' on the bottom scale.


Figure 4.6 Multiplication
Then, by simply reading off the top scale at any point we can find that number multiplied by 2 on the bottom scale.

This works for any number. Suppose we now want to multiply by 3 . All we do is align the ' 3 ' on the bottom scale with the '1' on the top scale as in Figure 4.7 below. We have now set the scales in the ratio 3:1. We do not need a different scale for each number. The same 2 scales do all numbers.


Figure 4.7 Multiplying by a different number

Now ' 1 ' is aligned with ' 3 ', ' 2 ' is aligned with ' 6 ', ' 3 ' is aligned with ' 9 ', and so on.
Note that it works with the positions of the two scales reversed. We can have our 2 scales like this:


Figure 4.8

Or like this:


Figure 4.9
Note: We can use it for dividing as well as multiplying, in the same way that we could use the linear scale for subtracting as well as adding.

When set up like this, we can divide 6 (top scale) by 2 (bottom scale), and read off the answer '3'.


Figure 4.10 Division

The Navigation Computer turns our straight slide rule into a circular one, as follows:-


Figure 4.11 The circular slide rule

## Multiplication

Before we can start using the Navigation Computer, we need to look at the scale.
From Figure 4.12, you can see that there is no number smaller than 10 or larger than 99.9. Therefore, we need to mentally insert the decimal point; for instance, the number 12 can be used to represent $0.12,12,120,1200$, etc. We put the decimal point in after the Navigation Computer operation.

This means that you have to carry out a rough calculation in your head to know what order of magnitude your solution will be.


Figure 4.12 No number smaller than 10 or greater than 99.9
An example should make this clear.
Suppose you have to multiply 11.5 by 2.54 .
First, the approximate calculation.
This is approximately 10 (11.5) times approximately $2 ½$ (2.54).
The answer will be approximately 25 . It will not be $21 / 2$ or 250 .
Now we know the order of magnitude of the solution, we can use the slide rule.
Against 11.5 on the outer scale, line up 10 (representing 1.0) on the inner scale - Figure 4.13.


Figure 4.13 Line up 11.5 (outer scale) with 1.0 (inner scale)
Now find 2.54 on the inner scale (shown as 25.4). Align the cursor (see Figure 4.14). Read off the answer (29.2) on the outer scale.


Figure 4.14 Multiplication
Is this the correct order of magnitude? Is the answer 2.92, 29.2, or 292?
We know from our rough calculation that the answer is about 25 . So 29.2 is the correct order of magnitude.

## Division

Divide 3050 by 3.28.

Again, the rough calculation. This is approximately 3000 divided by approximately 3. The answer will be just under 1000, i.e. about nine hundred and something.

Find 3050 on the outer scale and align the cursor. Now find 3.28 on the inner scale and align it with the cursor. It should then look as in Figure 4.15 below.


Figure 4.15 Setting up for division

Now find 10 on the inner scale and read off against the outer scale, as in Figure 4.16 below.


Figure 4.16 Reading off the answer-division

The initial answer is 93.0. Is this the right order of magnitude? We know from our rough calculation that it should be about 900, so the correct answer must be 930.

## Rates and Ratios

Let us reinforce the division process by doing another example. Placing it into an aviationrelated situation:-.

The average rate of climb of a particular aircraft is given as 700 feet per minute. How long will it take to climb from 3000 to 20000 feet?

The aircraft is to climb through 17000 feet at 700 feet per minute. Straightforward division of 17000 by 700 will determine the number of minutes for the climb.

What is the rough order of magnitude?

700 feet in one minute so. $\qquad$

7000 feet in ten minutes, and. $\qquad$
14000 feet in twenty minutes; therefore $\qquad$
17000 feet in a few more minutes than twenty $\qquad$
Alternatively, 700 fpm is not too far from 1000 fpm. So the sum is approximately
17000 / 1000, which will be a bit more than 17, i.e. somewhere in the low twenties.
Hold the computer with the 17000 (shown as 17) of the outer scale at the top.
Rotate the disc to bring the 700 (shown as 7) of the inner scale immediately opposite the outer scale 17.

Turn the instrument the shortest way (clockwise in this case) to bring 10 of the inner scale to the top.

Opposite this '10' read off from the outer scale the answer digits 243.

Which leads us to a final answer of 24.3 minutes.
You may prefer a slightly different approach to this type of problem, solving in terms of proportion rather than division. Restating the problem we have:-

Aircraft climbs 700 feet in one minute
It will climb 17000 feet in ' $t$ ' minutes (where ' $t$ ' is the answer we wish to discover)
This can be written as:-

$$
\frac{700}{1}=\frac{17000}{t}
$$

Now if you arrange this equation on your circular slide rule with the numerators 700 and 17000 on the outer scale set respectively above the denominators 1 and ' $t$ ' on the inner scale, the problem is solved.


Figure 4.17700 fpm set up as a ratio
As shown in Figure 4.17, align the 700 (shown as 70 ) of the outer scale exactly over the 1 (shown as 10) on the inner scale. This sets up the left hand side of the equation.


Figure 4.18 The solution as a ratio
Turn the instrument clockwise and look for the right hand side of the equation. Look for 17000 (shown as 17) on the outer scale.

Immediately under it, read off 24.3 from the inner scale.
This is the same answer that we had before. If you solve the problem in this manner, you still have to perform a rough mental calculation, as before, to position the decimal point correctly.

This proportioning, or ratio, technique is important. It is the basis of all distance, speed and time calculations and of fuel consumption calculations. We will return to it in the next lesson.

## Questions

MULTIPLY:

| 1. | $9 \times 8=$ |  | 6. | $275 \times 45=$ |
| :---: | :---: | :---: | :---: | :---: |
| 2. | $11 \times 30=$ | ..... | 7. | $121.5 \times 14=$ |
| 3. | $111 \times 12=$ | .................... | 8. | $12 \times 12 \times 12=$ |
| 4. | $202 \times 2.1=$ | .................... | 9. | $237 \times 7 \times 0.5=$ |
| 5. | $315 \times 0.3=$ | ................. | 10. | $12 \times 15 \times 13=$ |

DIVIDE:

1. $35 \div 7=$ $\qquad$ 6. $\mathbf{2 7 5} \div \mathbf{4 5}=$
2. $210 \div 7=$ $\qquad$ 7. $2705 \div 4.5=$ $\qquad$
3. $72960 \div 12=$ $\qquad$
4. $755 \div 0.8=$
5. $325 \div 5=$ $\qquad$
6. $39 \div 60=$
7. $20.2 \div 22=$

## PROPORTIONING

1. In the Ruritanian currency, there are 73 skroats to a blat. Fill in the following table:

Skroats 73
$\begin{array}{lllllll}\text { Blats } & 1 & 2 & 5 & 7 & 13 & 17\end{array}$
2. Your rate of descent is $\mathbf{2 7 0}$ metres per minute. How many minutes to descend through 2100 metres?
3. How far do you descend in 7 minutes?

## Answers

MULTIPLY

1. $9 \times 8=$
72.0
2. $11 \times 30=330$
3. $111 \times 12=1332$
4. $12 \times 12 \times 12=1728$
5. $202 \times 2.1=424.2$
6. $315 \times 0.3=94.5$
7. $237 \times 7 \times 0.5=830$
8. $12 \times 15 \times 13=2340$

DIVIDE

1. $35 \div 7=$
5.00
2. $\mathbf{2 7 5} \div \mathbf{4 5}=$
6.11
3. $210 \div 7=\quad 30.0$
4. $2705 \div 4.5=601.1$
5. $32 \div 8=$
4.00
6. $72960 \div 12=6080$
7. $325 \div 5=$
65.0
8. $755 \div 0.8=944$
9. $39 \div 60=$
0.65
10. $20.2 \div 22=0.917$

## PROPORTIONING

1. $\begin{array}{llllllll} & \text { Skroats } & 73 & 146 & 365 & 511 & 949 & 1241\end{array}$
$\begin{array}{lllllll}\text { Blats } & 1 & 2 & 5 & 7 & 13 & 17\end{array}$
2. $\quad 7.8$ minutes
3. 1890 metres

## Chapter

## 5

## The Navigation Computer - Distance, Speed, Time and Conversions

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## Introduction

In the last chapter, we saw how it was possible to use a proportioning, or ratio, technique on the Navigation Computer when dealing with problems of rates, i.e. quantities per minute or quantities per hour. The example we used was a rate of climb, which was expressed as 700 feet per minute. By setting ' 700 ' (outer scale) against ' 1 ' (inner scale), we could read off the number of feet gained (outer scale) in 2 minutes, 3 minutes, 4 minutes, etc (inner scale).


Figure 5.1700 fpm set up as a ratio
The red arrow on the inner scale of the Navigation Computer is the indicator of ' 1 ', and is used as the inner scale reference if the answer we want is in the same units. If the red arrow is used to indicate ' 1 minute', then the answer will be in minutes (and decimals of a minute). This is useful if the rate is given in quantity per minute, as, for instance, rate of climb or descent usually is. If the red arrow is used to indicate ' 1 hour', then the answer will be in hours and decimals of an hour. This may not be so useful, because few of us work in decimals of an hour. Usually, even if the rate is given 'per hour' we want the answer in minutes, or hours and minutes.


Figure 5.2 Base '1' and Base '60' Indices
If this is the case - for instance, fuel flow is usually given in pounds (or kilograms) per hour but we usually want the answer in minutes or hours and minutes - then we put the ' 60 ' index (the black triangular arrow) against the hourly rate. Suppose our fuel flow is $2000 \mathrm{~kg} / \mathrm{hour}$. Then by putting 60 minutes (inner scale) instead of one hour against 2000 (outer scale) we are dividing 2000 by 60 . If you now look at the red arrow you are getting the rate per minute ( 33.33 kg ).


Figure 5.32000 kg/hour (black triangle) equals 33.3 kg/min (red arrow)

## Distance, Speed and Time

For distance, speed and time problems the outer scale is always distance or speed and the inner scale is always time.

Example. How far do you go in 48 minutes at 120 knots ground speed?

First, set what you know. Set the ground speed. Bring the '60' black triangle against 120 knots on the outer scale.

Now find 48 minutes on the minutes (inner) scale. Read off the answer - 96 NM on the outer scale.


Figure 5.4 Finding distance from speed and time

Example. In 30 minutes you have covered 90 NM over the ground. What is your ground speed?

Set what you know on the computer. Set 30 minutes (inner scale) against 90 NM (outer scale).

Now read off the ground speed against the ' 60 ' black triangle. The answer is 18.0 .

What order of magnitude? At 18 knots, you would stall. At 1800 knots, you would strip the wings off. Common sense tells you that this must be 180 knots - which is the right answer.


Figure 5.5 Finding ground speed from distance and time

Example. Your ground speed is 220 knots and you have 57 NM to go. How long will it take you?

As before, set what you know. Move the black ' 60 ' triangle against 220 knots.

Now bring the cursor round to 57 NM (outer scale, because it is a distance).

Read off the time on the inner scale. The answer is 15.5 . Are we expecting 15 minutes or 150 minutes? 57 NM is about a quarter of 220, so about a quarter of an hour is obviously the right answer. It is 15.5 minutes.


Figure 5.6 Finding time from distance and speed

## The Yellow Scale

What is the purpose of the yellow scale? It is simply to allow you to convert from minutes to hours and minutes. If we use the ' 60 ' black triangle for an hourly rate, we get the answer in minutes, even when it is in excess of one hour. Often, in aviation, we want the answer in hours and minutes.

Suppose an answer comes to 142.6 minutes. Looking at the yellow ring, you can see that this is between 2 hours and 2 hours 30 min . Therefore it must be 2 hours 22.6 min .

This is the sole purpose of the yellow scale


Figure 5.7 Between 2 h and 2 h 30 min

## The Blue Scale

The blue scale gives the value of ram rise. Ram rise is the difference between Static Air Temperature (SAT) and Total Air Temperature (TAT).

$$
\text { Ram Rise }=(\text { TAT }- \text { SAT })
$$

Static Air Temperature is the temperature of the ambient atmosphere surrounding the aircraft with no inaccuracies caused by the measurement process. This is the temperature we need to calculate True Airspeed (TAS) from Calibrated Airspeed (CAS) and Pressure Altitude, or to calculate TAS from Mach No. However, what we measure with our temperature probe is the TAT, which is affected by compressibility and kinetic or adiabatic heating, and is always a higher temperature than the SAT.

A good approximation for the value of ram rise is:

$$
\text { Ram Rise }=\left(\frac{\text { TAS }}{100}\right)^{2} \text { where TAS is in knots }
$$

Thus, if the TAS is 400 knots, the ram rise is approximately $16^{\circ} \mathrm{C}$, if the TAS is 500 the ram rise is approximately $25^{\circ} \mathrm{C}$, and so on. However, although this approximation is usually good enough for most practical purposes, it tends to be slightly low and is not as accurate as the answer produced by the Navigation Computer, which is the one which will be expected for EASA ATPL examinations.

Suppose your TAS is 400 knots. Align the cursor with 400 knots (upper portion of blue scale). Read off the Ram Rise on the lower portion of the blue scale. It is about $17^{\circ} \mathrm{C}$.

So, if the TAT at 400 knots TAS were, say, $-22^{\circ} \mathrm{C}$, then the SAT would be $-39^{\circ} \mathrm{C}$.


Figure 5.8400 knots TAS gives a Ram Rise of $17^{\circ} \mathrm{C}$

## Fuel Consumption

Fuel consumption problems work exactly like distance/speed/time calculations. We normally get a fuel consumption per hour, use base 60, and read the answer in minutes. If it comes to more than 60 minutes we use the yellow scale to read the answer in hours and minutes.

Example. Your fuel consumption is $2300 \mathrm{~kg} /$ hour. Your time to destination is 1 hour 37 minutes. How much fuel will you use?

Remember - minutes are always on the inner scale.

Set the black triangle against 2300. Align the cursor with 1 hour 37 minutes ( 97 minutes). Read off the answer-37.2. The correct answer is obviously 3720 kg .

Using a calculator gives you the answer 3718.33 kg . This should reassure you that the navigation computer is very accurate - in fact, probably considerably more accurate than your fuel gauges.


Figure 5.9 Fuel consumption

Example Your fuel flow is 130 imperial gallons per hour. You have 380 gallons left above your reserves. What is your Safe Endurance?

Set the black triangle against 130.
Align the cursor with 380 (outer scale).
Read off the answer - 175.5 (inner scale). Use the yellow ring to see that this is 2 hours and 55.5 minutes.


Figure 5.10 Fuel consumption - 2

Example You have used 3700 litres of fuel in the last 1 hour 45 minutes. What is your fuel consumption rate?

Find 3700 on the outer scale and align the cursor.

Bring 1 hour 45 minutes ( 105 minutes) on the minutes scale into alignment with the cursor. Use the yellow scale to convert hours and minutes to minutes if necessary.

Read off the consumption against the black '60' triangle.

The answer is 21.1. A common-sense check shows that this must be $2110 \mathrm{~kg} /$ hour.

The electronic calculator solution is 2114.29.


Figure 5.11 Fuel consumption

## Distance Conversions

The Navigation Computer can also be used for distance, volume and weight conversions. Taking distance first:

Round the edge of the outer scale are various index marks with red labels. These 3 are for distance conversions:


Figure 5.12 Kilometres to statute miles and nautical miles

Example: Convert 148 kilometres to statute miles and to nautical miles.

The km-m-ltr index on the outer scale is at 10 and the naut m index is at 54 (or 5.4). You should remember that the distance from the Equator to the pole is 10000 km and is also ( 90 $\times 60$ ), i.e. 5400 nautical miles. So $10000 \mathrm{~km}=$ 5400 NM and $10 \mathrm{~km}=5.4 \mathrm{NM}$.

Align 148 on the inner scale with the $\mathrm{km}-\mathrm{m}-\mathrm{ltr}$ index.


Figure 5.13148 km against the $k m$ red index

Now move the cursor to the naut $m$ index.

Read off the answer 8.0 (on the inner scale) which must be 80 NM.


Figure 5.14 Align with naut m index

To get the answer in statute miles, now move the cursor to the stat $m$ index and read off 92 .
Always remember to read the answer off the inner scale.
A common mistake is to read these conversion answers off the outer scale and then to wonder why they are always the same - 54 NM and 62.5 stat.m.

Now do one for metres, yards and feet:
Example: Convert 256 metres to yards and feet.

Set on what you know:-
Opposite the $m$ (metres and 10) index on the outer scale, set 256 (which is 25.6 ) on the inner scale.

Then, opposite the yards index on the outer scale read off 28.0 (obviously 280 ).
Against the feet index on the outer scale, read off 84.0 (obviously 840) on the inner scale.
256 metres $=280$ yards $=840$ feet.

## Volume Conversions

You can also do volume conversions.
These are the litres, imperial gallons and U.S. gallons index marks.


Figure 5.15 Volume conversion index marks

Example: Convert 136 litres to imperial and to US gallons.
As always, set what you know on the navigation computer.
Put 136 against the litres index Read off against the imp.gal index


Figure 5.16136 against the litres index


This gives 30 imperial gallons
Figure 5.17 Reading off the imp.gal index


This gives 36 US gallons
Figure 5.18 Reading off the u.s. gal index
You need to remember that there are about $41 / 2$ litres to an imperial gallon and that there are 5 imperial galls to 6 US gallons in order to get the order of magnitude right.

## Volume to Weight

There is also a facility on the Navigation Computer to convert volume in litres or gallons to weight in kilograms or pounds if you know the specific gravity of the liquid (usually fuel). These indices have been spaced so that you can begin with an argument in one set of units and read off the answer in the other units.

A little revision on specific gravity may be useful here. Specific gravity (also known as 'relative density') is the density compared with the density of water. One litre of water weighs ONE kilogram in the metric system and one imperial gallon of water weighs TEN pounds in the imperial system. For example, if a fuel has a specific gravity of 0.80 , then one litre weighs 0.80 kg and one imp.gal weighs 8.0 pounds.

The calorific value of fuel is related to its mass, not its volume. You need a greater volume of fuel at a lower SG to give the same amount of energy as a smaller volume at a higher SG. This energy may be expressed as range in nautical air miles, or possibly as endurance. Thus, 200 litres at .80 SG weighs 160 kg . If fuel at .75 SG is used, then 213 litres will be needed to cover the same distance.

Example: Convert 300 US gallons to:
a) imperial gallons
b) litres
c) pounds weight
d) kilograms
using a Specific Gravity of 0.80 .

As always, set what you know.
Align 300 on the inner scale against the u.s. gal index on the outer scale


Figure 5.19 300 US gallons


Figure 5.20250 Imperial gallons

Now read off against the litres index.
This gives 1135 litres.


Figure 5.211135 litres

So far, so familiar. Now look at these 2 scales.


Figure 5.22 SG scales

Let us do the 'pounds weight' conversion first.
Without moving the inner scale, which is already set up as you require, move the cursor to 0.80 specific gravity on the Sp.G lbs scale. This scale is marked from 70 to 95 , representing SGs of 0.70 to 0.95 .

Read off 20.0 on the inner scale.
What is the order of magnitude? Remember, we are dealing with 250 imp.gal. Remember, the $S G$ is 0.8 . Therefore we are expecting an answer of about 2000 lb.

In this case, the answer is exactly 2000 lb.

Now, the conversion to kilograms.
Again, we do not move the inner scale.
Move the cursor to 0.80 on the Sp.G kgs scale. The scale is marked for 70 to 90 , representing SGs of 0.70 to 0.90 .

Read off 9.08 on the inner scale.

What is the order of magnitude?
There are various ways you can estimate this. Either 1135 litres times .8 is about 900 kg , or 2000 lb is going to be just under 1000 kilograms.

Therefore the answer is 908 kg .


Figure 5.23 Converting to pounds weight


Figure 5.24 Converting to kilograms

## Questions

Complete the tabulations:-

| Ground speed (kt) | Distance (NM) | Time (min) |
| :---: | :---: | :---: |
| 230 | 33 |  |
| 175 | 234 |  |
| 182 | 510 | 77 |
| 145 |  | 51 |
| 260 | 46.5 | 16.5 |
| 200 | 231 | 97 |
|  | 156 | 43 |
| 320 |  | 3.5 |
|  |  |  |
|  |  |  |


| Fuel Flow | Time. | Fuel Used |
| :---: | :---: | :---: |
| 140 litres $/ \mathrm{h}$ | 1 h 20 min |  |
| $2120 \mathrm{~kg} / \mathrm{h}$ |  | 570 kg |
|  | 3 h 25 min | $400 \mathrm{imp} . g \mathrm{l}$ |
| $8.7 \mathrm{US} . g a \mathrm{l} / \mathrm{h}$ | 2 h 15 min |  |
| $9800 \mathrm{lb} / \mathrm{h}$ | 5 h 20 min | 420 kg |
| $165 \mathrm{~kg} / \mathrm{h}$ |  | $1075 \mathrm{lb} / \mathrm{hr}$ |
|  | 1 h 47 min |  |
| $295 \mathrm{US} . g a \mathrm{~h} / \mathrm{h}$ | 4 h 40 min |  |
| $160 \mathrm{litres} / \mathrm{h}$ | 1 h 45 min | $1120 \mathrm{imp} . g \mathrm{l}$ |
| $260 \mathrm{imp} . g a l / \mathrm{h}$ |  |  |

Complete the following conversions:-

| Nautical Mile | Statute Mile | Kilometre |
| :---: | :---: | :---: |
| 88 |  |  |
|  | 27 | 87 |
| 141 | 147 | 310 |
|  |  | 10000 |
| 433 |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


| Metres | Feet |
| :---: | :---: |
| 1000 |  |
|  | 29 |
| 120 |  |
| 356 |  |


| Litres | imp.gal | US.gal | SG | kg | lb |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 |  |  | .78 |  |  |
|  | 420 |  | .81 |  |  |
|  |  | 800 | .84 |  |  |
|  |  |  | .80 | 1000 |  |
|  |  |  | .72 |  | 2000 |

## Answers

| Groundspeed (kt) | Distance (NM) | Time (min) |
| :---: | :---: | :---: |
| 230 | 33 | 8.6 |
| 175 | 234 | 80 |
| 182 | 510 | 168 |
| 145 | 36 | 15 |
| 260 | 334 | 77 |
| 200 | 170 | 51 |
| 169 | 231 | 16.5 |
| 143 | 156 | 97 |
| 218 | 18.5 | 43 |
| 320 |  | 3.5 |


| Fuel Flow | Time | Fuel Used |
| :---: | :---: | :---: |
| 140 litres $/ \mathrm{hr}$ | 1 h 20 min | 187 litres |
| $2120 \mathrm{~kg} / \mathrm{hr}$ | 16 min | 570 kg |
| $117 \mathrm{imp} . g a l$ | 3 h 25 min | $400 \mathrm{imp} . g a l$ |
| $8.7 \mathrm{US} . g a \mathrm{l} / \mathrm{hr}$ | 2 h 15 min | $19.6 \mathrm{US} . g a l$ |
| $9800 \mathrm{lb} / \mathrm{hr}$ | 5 h 20 min | $52 \mathbf{2 6 7 \mathrm { lb }}$ |
| $165 \mathrm{~kg} / \mathrm{hr}$ | 2 h 33 min | 420 kg |
| $603 \mathrm{lb} / \mathrm{hr}$ | 1 h 47 min | $1075 \mathrm{lb} / \mathrm{hr}$ |
| $295 \mathrm{US} . g a \mathrm{l} / \mathrm{hr}$ | 4 h 40 min | $1377 \mathrm{US} . g \mathrm{l}$ |
| $160 \mathrm{litres} / \mathrm{hr}$ | 1 h 45 min | 280 litres |
| $260 \mathrm{imp} . g a l / \mathrm{hr}$ | 4 h 18 min | $1120 \mathrm{imp} . g a l$ |


| Nautical Mile | Statute Mile | Kilometre |
| :---: | :---: | :---: |
| 88 | 101 | 163 |
| 23.4 | 27 | 43.5 |
| 47 | 54 | 87 |
| 141 | 162 | 261.5 |
| 127.7 | 147 | 237 |
| 167 | 192 | 310 |
| 5400 | 6210 | 10000 |
| 433 | 499 | 805 |


| Metres | Feet |
| :---: | :---: |
| 1000 | 3280 |
| 8.8 | 29 |
| 120 | 393 |
| 356 | 1166 |


| Litres | imp.gal | US.gal | SG | kg | lb |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 | 220 | 264 | .78 | 780 | 1716 |
| 1909 | 420 | 504 | .81 | 1546 | 3402 |
| 3031 | 667 | 800 | .84 | 2546 | 5600 |
| 1250 | 275 | 330 | .80 | 1000 | 2200 |
| 1263 | 278 | 333 | .72 | 909 | 2000 |

5

## Chapter

## 6

## The Navigation Computer - TAS and Altitude Conversions

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6

## TAS and Altitude Conversions

The Navigation Computer can also be used for the following functions:

- Calculation of TAS from CAS
- Correction of compressibility error
- Calculation of TAS from Mach Number
- Calculation of True Altitude from Indicated Altitude
- Calculation of Density Altitude from Pressure Altitude


## Airspeed Indicator Errors - Application of Corrections

Figure 6.1 overleaf is a breakdown of the corrections to the airspeed indicator (ASI) and the order in which they are applied.

What you see on the instrument is called Indicated Airspeed (IAS). Instrument Error and Pressure (or Position) Error are then applied to give Calibrated Airspeed (CAS). This is the more modern and generally accepted term, but many documents use the older term Rectified Airspeed (RAS). The corrections are applied by use of a correction card usually stuck beside the instrument.

In practice, we do not have separate cards for the Instrument Error and the Pressure Error. It is simpler to combine both corrections onto a single card. IAS is therefore corrected for both Instrument and Pressure Error by the correction card, to give CAS.

Strictly speaking, CAS is then corrected for compressibility error to give Equivalent Airspeed (EAS), to allow for the fact that the density of air is rarely precisely 1225 grams per cubic metre, which is the value for which the ASI is calibrated. (The ISA atmosphere at Mean Sea Level). In practice, as an airline pilot, you are unlikely to deal with EAS and it is rarely encountered outside scientific and test flying. The ASI is already calibrated to allow for compressibility at ISA at MSL and, under those conditions, no compressibility correction is necessary. Compressibility correction is small at True Airspeeds (TAS) lower than 300 knots and no correction is considered necessary.

You will therefore always calculate the Density Error correction first to give True Airspeed (TAS). If the TAS you find is 300 knots or less, no further correction is necessary. If the TAS is greater than 300 knots, Compressibility Error correction must be applied.

Summary of Corrections to Airspeed Indicator
Indicated Airspeed (IAS)


Figure 6.1 Sequence of ASI error correction

## Calculation of TAS from CAS - Correction of Density Error

To correct CAS to TAS we need to divide by the square root of the relative density. You will recall this formula from Phase 1:

$$
\text { CAS }=\text { TAS } \sqrt{\text { relative density }}
$$

Relative to what? The answer is that it is relative to ISA at Mean Sea Level, i.e. 1225 gram per cubic metre. Look at Figure 6.2 below, which is a table of standard values of the ISA atmosphere.

| Flight Level | ISA Temp <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Pressure <br> (hectopascals) | ISA Density <br> $\left(\mathrm{g} / \mathrm{m}^{3}\right)$ | Number of feet <br> to a <br> hectopascal at this <br> level |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 15 | 1013.25 | 1225 | 27 |
| 50 | 5 | 843 | 1056 | 32 |
| 100 | -5 | 697 | 905 | 37 |
| 150 | -15 | 572 | 771 | 43 |
| 200 | -25 | 466 | 653 | 51 |
| 250 | -35 | 376 | 549 | 61 |

Figure 6.2 Values for the ISA standard atmosphere
Imagine that we are flying at 100 knots CAS at FL200 in an ISA atmosphere. The density at that level is 653 gram per cubic metre. To calculate the relative density:

$$
\text { relative density }=\frac{653}{1225}=.5331
$$

Therefore:

$$
\sqrt{\text { relative density }}=\sqrt{.5331}=.7301
$$

So, with a CAS of 100 knots, TAS $=\frac{100}{.7301}=137$ knots
If we had an instrument which measured the density of the air we are flying through, we could calculate our TAS in this way. But, in fact, there is no flight deck instrument which gives such a reading.

However, you should remember from your Principles of Flight lectures that:

$$
\frac{P}{T \rho}=\text { constant } \quad \text { or } \rho \propto \frac{P}{T}
$$

where $P=$ pressure, $T=$ temperature and $\rho=$ density
So, if we know any two of Pressure, Density and Temperature, we can find the other one.

You should also remember from your Piston Engine lectures that:
The "Combined Gas Law" is a combination of Boyle's law and Charles' Law and represents the relationship between Volume, Pressure and Temperature.

This may be shown as:

$$
\begin{array}{ll}
\frac{P \times V}{T}=K \quad \text { or, alternatively } & P \times V=K \times T \\
\text { Or } \quad \frac{P_{1} \times V_{1}}{T_{1}}=\frac{P_{2} \times V_{2}}{T_{2}} &
\end{array}
$$

We do have an instrument that gives us the Temperature - the air thermometer. We have to correct from Total Air Temperature to Static Air Temperature, but we know how to do that.

We also have an instrument that gives us the atmospheric pressure at our level - the altimeter. As we know, it does not actually measure altitude, but pressure. The altimeter is, in fact, a form of barometer, but the readout is in feet. However, the law relating altitude in the ISA atmosphere to hectopascals of pressure is known, and we can use it.

So, we can use pressure from the altimeter and temperature from the thermometer to find the relative density, and then find TAS from CAS. If you tried to do this longhand with a calculator, it would take quite a long time. However, the Navigation Computer uses precisely this method, and does it quickly and easily.

Let us now return to our previous example:

CAS $=100$ knots FL200
SAT $=-25 \quad$ Density $=653 \mathrm{~g} / \mathrm{m}^{3}$

On the Navigation Computer, we have the 4 windows marked AIRSPEED, COMP CORR, ALTITUDE and DENSITY ALTITUDE.


Figure 6.3 CRP-5 windows

In the AIRSPEED window, align FL200 against $-25^{\circ} \mathrm{C}$.

This has altered the position of the inner scale relative to the outer scale. You have aligned them up in the ratio of the square root of the relative density. If you want to check this, look at the '1' (shown as '10') on the outer scale. Against it, on the inner scale, you will see the ratio .7301.


Figure 6.4 Align FL200 with $-25^{\circ} \mathrm{C}$


Figure 6.5 CAS (inner scale) against TAS (outer scale)

You do not require to know, or understand, the theoretical explanation. All that is required is to be able to use the Navigation Computer practically.

Another example, just using the Navigation Computer method:
You are at 18000 feet pressure altitude and the SAT (corrected outside air temperature) is $-30^{\circ} \mathrm{C}$. CAS (RAS) is 170 knots. What is TAS?

In the AIRSPEED window set pressure altitude ( 18000 feet) against COAT $\left(-30^{\circ} \mathrm{C}\right.$ ).


Figure 6.6 Set 18000 ft against $-30^{\circ} \mathrm{C}$

Find 170 knots CAS on the inner scale.

Move the cursor if you need to.

Now read off the value on the outer scale (220 knots TAS).


Figure 6.7220 knots TAS against 170 knots CAS

## Correction of Compressibility Error

The ASI is calibrated to allow for compressibility at an atmospheric pressure of 1225 grams per cubic metre (ISA at MSL), but at any other density (nearly always), compressibility error must be corrected.

Compressibility correction is insignificant at airspeeds less than 300 knots TAS, but must be applied if the initial calculation gives a TAS of more than 300 knots.

Example: Flight Level 350, COAT $=-47^{\circ} \mathrm{C}, \mathrm{CAS}=280$ knots. What is TAS?

Start by calculating TAS from CAS in the usual way.

In the AIRSPEED window, set FL350 against $-47^{\circ} \mathrm{C}$.


Figure 6.8 FL370 against $-47^{\circ} \mathrm{C}$

Find 280 knots CAS on the inner scale.

Now read off the value on the outer scale (500 knots TAS).


Figure 6.9280 KCAS against 500 KTAS

This is more than 300 knots TAS. So how do we correct for compressibility?
Under the COMP CORR window you will see the equation here:

$$
\frac{\text { TAS }}{100}-3 \text { DIV }
$$

Divide the initial TAS (500 in this case) by 100 and subtract 3 .

$$
\frac{500}{100}-3=2.0
$$

Now, in the COMP CORR window, move the arrow 2 divisions to the left.

From here:


Figure 6.10 The COMP CORR Window

To here:


Figure 6.11 Index moved to the left
This has moved the inner scale to the left, altering the relationship of CAS to TAS, correcting it for compressibility.

Now read off the new value of TAS on the outer scale corresponding to 280 knots CAS on the inner scale.

This gives a revised TAS of 480 knots.


Figure 6.12 TAS corrected for compressibility

## Calculation of TAS from Mach Number

Finding TAS from Mach Number is easier than from CAS because there is only one variable temperature - and no correction for compressibility.

There are 2 methods - a formula, or the Navigation Computer method
The formula is as follows:

$$
\text { TAS }=\text { Mach No } \times 38.95 \sqrt{\mathrm{~T}^{\circ}} \mathrm{K}
$$

0 K is $-273.16^{\circ} \mathrm{C}$, though $-273^{\circ} \mathrm{C}$ is a good enough approximation.

## Example:

```
COAT = -47 }\mp@subsup{}{}{\circ}\textrm{C}\mathrm{ Mach No. = 0.82 What is the TAS?
-47}\mp@subsup{}{}{\circ}\textrm{C}=(273-47)=226\textrm{K
TAS = 0.82 \times 38.95 \times \sqrt{}{226}=480 knots
```

Now we will do the same calculation using the Navigation Computer:-

Move the AIRSPEED window round until the Mach No. index appears.


Figure 6.13 Location of the Mach No. Index

Set the COAT of $-47^{\circ} \mathrm{C}$ against the index arrow.


Figure 6.14 COAT against Index Arrow


Figure 6.15 Mach No. (inner) against TAS (outer)

## Calculation of True Altitude from Indicated Altitude and Temperature

The altimeter is calibrated against an ISA atmosphere, so a particular set of values of temperature is assumed. If the temperatures on the day at the various levels of the atmosphere are different from ISA, the indicated value will not correspond to true altitude. At higher levels this is not a problem since all aircraft experience the same temperature error in the same part of the atmosphere and can still maintain vertical separation from each other. However, when close to the ground it is necessary to know true altitude in order to maintain terrain clearance.

If the other altimeter corrections have been made (position error, instrument error) and the correct QNH is set, the indicated error differs from the true altitude by temperature error. This can be corrected either by use of a formula or with the Navigation Computer.

The formula is as follows:
True Altitude $=$ Indicated Altitude $+\left(\frac{\text { ISA Deviation }}{273} \times\right.$ Indicated Altitude $)$
However, if you treat $1 / 273$ as $1 / 250$, then you can write it as $4 / 1000$. This is the basis of the " $4 \%$ rule" used in Meteorology.

True Altitude $=$ Indicated Altitude $+\left(\right.$ ISA Deviation $\times \frac{4}{1000} \times$ Indicated Altitude)
Example: Indicated Altitude is 20000 feet. SAT is $-35^{\circ} \mathrm{C}$. What is True Altitude?
ISA at FL200 is $-25^{\circ} \mathrm{C}$, so our SAT of $-35^{\circ} \mathrm{C}$ is ISA $-10^{\circ} \mathrm{C}$.
Substituting into the above equation:

$$
\begin{aligned}
& \text { True Altitude }=20000+\left(-10 \times \frac{4}{1000} \times 20000\right) \\
& \text { True Altitude }=20000+(-800)=19200 \text { feet }
\end{aligned}
$$

This corresponds to what you would expect. The column of air is $10^{\circ} \mathrm{C}$ colder than ISA. This means that it is denser air. Therefore, the pressure reduces more rapidly than ISA as you climb through the atmosphere because there are fewer feet to a hectopascal with denser air. So you will pass through the required number of hectopascals needed to give an indication of 20000 feet in less than 20000 feet True Altitude.

However, it is rather easier with the Navigation Computer. We carry out the correction using the ALTITUDE window.


Figure 6.16 The ALTITUDE window

Using the same example, in the ALTITUDE window, align 20000 feet with the SAT of $-35^{\circ} \mathrm{C}$. By doing this, you have set up a relationship between the inner and the outer scales.


Figure 6.17 Setting Ind Alt against SAT

The outer scale now shows the True Altitude corresponding to the Indicated Altitude on the inner scale.

Against 20000 feet (Indicated Altitude) on the inner scale, read 19200 feet (True Altitude) on the outer scale.


Figure 6.18 True Alt (outer scale) against Indicated Alt (inner scale)

In any question which requires you to correct Indicated Altitude to True Altitude, the Navigation Computer should always give an accurate answer and there is no need to use the formula. In addition, you can use the ambient temperature directly and there is no need to calculate ISA deviation. True Altitude questions should, therefore, always be solved by use of the Navigation Computer.

## Calculation of Density Altitude from Pressure Altitude and Temperature

Density Altitude has nothing to do with vertical distance. It is a measure of aircraft performance. Density Altitude is the altitude in the ISA atmosphere which corresponds to the density which you are experiencing in your current ambient atmosphere. It affects aircraft performance and engine power. Most manufacturers quote a declared performance in terms of Density Altitude. Most performance graphs already allow for this correction in the entering arguments to the graph, but if yours does not, you need to be able to calculate it.

As before, there are 2 methods of solving the problem - the formula and the use of the Navigation Computer.

The formula is as follows:

$$
\text { Density Altitude }=\text { Pressure Altitude + (ISA Deviation } \times 120)
$$

(Strictly speaking, the conversion factor should be 118.8, not 120, but 120 is close enough)

## Example:

You are at Nairobi, pressure altitude 5500 feet. SAT is $+35^{\circ} \mathrm{C}$. What is Density Altitude?
ISA at FL5.5 is $+4^{\circ} \mathrm{C}$, so our SAT of $+35^{\circ} \mathrm{C}$ is ISA $+31^{\circ} \mathrm{C}$.
Substituting into the above equation:
Density Altitude $=5500+(+31 \times 120)$
Density Altitude $=5500+(3720)=9220$
Now try the same problem on the Navigation Computer.
This one is not so obvious because you start off with the AIRSPEED window.


Figure 6.19 The AIRSPEED window

Set 5500 feet against $+35^{\circ} \mathrm{C}$.

Read off the answer of 9200 feet.

Unfortunately, the divisions on the Navigation Computer DENSITY ALTITUDE window are rather small and it can sometimes be difficult to separate closely spaced ATPL examination answer options. With Density Altitude calculations, unlike True Altitude, it may sometimes be necessary to use the formula rather than the Navigation Computer to get a sufficiently accurate answer.

Figure 6.21 DENSITY ALT window


Figure 6.22 Reading the answer


Figure 6.20 FL against SAT


## Questions

Complete the following tables:

| CAS | Pressure <br> Altitude | $\operatorname{SAT}\left({ }^{\circ} \mathrm{C}\right)$ | TAS |
| :---: | :---: | :---: | :---: |
| 140 | 10000 | -10 |  |
| 175 | 8000 | +5 |  |
| 220 | 15000 | -22 |  |
| 300 | 30000 | -40 |  |
| 180 | 12500 | -25 |  |
| 165 | 9000 | +7 |  |
| 163 | 11000 | +10 |  |
| 295 | 31000 | -53 |  |
| 160 | 12000 | -10 |  |
| 260 | 35000 | -45 |  |


| Mach No | SAT $\left({ }^{\circ} \mathrm{C}\right)$ | TAS |
| :---: | :---: | :---: |
| .82 | -50 |  |
| .66 |  | 375 |
|  | -38 | 425 |
| .77 | -60 |  |
| .92 |  | 580 |
|  | -47 | 415 |


| Ind Alt | SAT $\left({ }^{\circ} \mathrm{C}\right)$ | True Alt |
| :---: | :---: | :---: |
| 10000 | -20 |  |
| 18000 | -30 |  |
| 6000 | +10 |  |
| 27000 | -50 |  |
| 21500 | -35 |  |
| 8000 | +10 |  |


| Pressure Alt | SAT $\left({ }^{\circ} \mathrm{C}\right)$ | Density Altitude |  |
| :---: | :---: | :---: | :---: |
| 10000 | +15 | Navigation <br> Computer <br> Solution | Formula <br> Solution |
| 6000 | +20 |  |  |
| 27000 | -40 |  |  |
| 23000 | -22 |  |  |
| 25000 | -45 |  |  |
| 29000 | -38 |  |  |

## Answers

| CAS knots | Pressure <br> Altitude | SAT $\left({ }^{\circ} \mathrm{C}\right)$ | TAS |
| :---: | :---: | :---: | :---: |
| 140 | 10000 | -10 | 162 |
| 175 | 8000 | +5 | 200 |
| 220 | 15000 | -22 | 273 |
| 300 | 30000 | -40 | 472 |
| 180 | 12500 | -25 | 212 |
| 165 | 9000 | +7 | 193 |
| 163 | 11000 | 10 | 199 |
| 295 | 31000 | -53 | 460 |
| 160 | 12000 | -10 | 192 |
| 260 | 35000 | -45 | 450 |


| Mach No | SAT $\left({ }^{\circ} \mathrm{C}\right)$ | TAS |
| :---: | :---: | :---: |
| .82 | -50 | 477 |
| .66 | -60 | 375 |
| .71 | -38 | 425 |
| .77 | -60 | 438 |
| .92 | -11 | 580 |
| .71 | -47 | 415 |


| Ind Alt | SAT $\left({ }^{\circ} \mathrm{C}\right)$ | True Alt |
| :---: | :---: | :---: |
| 10000 | -20 | 9450 |
| 18000 | -30 | 17400 |
| 6000 | +10 | 6150 |
| 27000 | -50 | 25600 |
| 21500 | -35 | 20800 |
| 8000 | +10 | 8350 |


| Pressure Alt | SAT $\left({ }^{\circ} \mathrm{C}\right)$ | Density Altitude |  |
| :---: | :---: | :---: | :---: |
|  |  | Navigation <br> Computer <br> Solution | Formula <br> Solution |
| 10000 | +15 | 12500 | 12400 |
| 6000 | +20 | 8200 | 8040 |
| 27000 | -40 | 26300 | 26880 |
| 23000 | -22 | 23600 | 24080 |
| 25000 | -45 | 23100 | 23800 |
| 29000 | -38 | 28900 | 29600 |

## Chapter

7

## The Navigation Computer - Triangle of Velocities

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## The Wind Face of the Navigation Computer

We have completed all aspects of the slide rule face of the Navigation Computer and we now turn our attention to the wind face. However, before we do so we must define some terms, and then discuss the effect of wind on navigation, and introduce the concept of the Triangle of Velocities.

## Some Definitions

Heading is the direction in which the aircraft is pointing. This is not necessarily the direction in which it is tracking over the ground. As we shall shortly see, if there is any crosswind component, there will be a difference between the direction in which the aircraft is pointing and the direction in which it is travelling.

Heading is defined as the direction in which the fore and aft axis of the aircraft is pointing; it may be measured from True, Magnetic, or Compass North.

Track is the direction of the aircraft's path over the ground. It may be measured from True or Magnetic North.

Drift is the difference between heading and track. You always drift from your heading to your track. Thus, if your heading is $090^{\circ}(\mathrm{T})$ and your track is $097^{\circ}(\mathrm{T})$, you are drifting to the right (from heading to track). If your heading is $030^{\circ}(\mathrm{T})$ and your track is $026^{\circ}(\mathrm{T})$, you are drifting to the left. Have a look at the diagram of the Triangle of Velocities at Figure 7.4 of this chapter. It shows drift to the right.

Wind Correction Angle (WCA). Wind Correction Angle is the same value as drift but it is applied in the opposite sense. In the paragraph above, the heading is $090^{\circ}$ and the track is $097^{\circ}$ which means the drift angle is $7^{\circ}$ right or starboard i.e. added to the heading to give track. The WCA in this example would be $-7^{\circ}$ i.e. applied to the track to give heading.

Port and Starboard. Most countries, and certainly the USA, use the terms 'left' and 'right' in aviation. However, the British retain an affection for their maritime tradition and so use the terms 'Port' and 'Starboard' when referring either to the left or right side of the aircraft (looking forwards) or when referring to drift. 'Port' is left (easy to remember - 'Port' has 4 letters and so does 'left'). 'Starboard' is right.

Course is a word that can lead to enormous confusion. Early British terminology used 'course' to mean heading - the direction in which the aircraft is pointing. This practice ceased around 1950, when 'heading' became the accepted UK term. However, many present American publications use the word 'course' to mean track - the direction of the aircraft's path over the ground.

Because of this potential for confusion, we will not use the word 'course'. The unambiguous words 'heading' and 'track' will always be used. However, the word course may sometimes be used in formal examinations to mean 'desired track'.

## The Effect of Wind

Aircraft do not always travel in the direction in which they are pointed. If there is any crosswind, the track will be different from the heading. We will start by explaining this in terms of a boat crossing a river.

Imagine a river 10 NM wide. Imagine that it has a current of 5 knots.


Figure 7.1 River with current
If you stand at Point $A$ and throw a stick into the river, after one hour it will have been carried 5 NM downstream.

Now imagine that the river has been dammed, and has become a lake. There is now no current and the water is still. This time, you are at Point A with a boat with an outboard motor and a speed of 10 knots through the water. If you set off from Point A to cross directly to the opposite bank, one hour later you will be at Point B.


Figure 7.2 Triangle of velocities - boat crossing river
Now combine these two ideas. The dam is removed and the lake has become a river again. We have our 5 knots current back. We set off in our boat from Point A heading towards Point B. However, the effect of the current is to take us downstream, as before, and this time we finish up at Point $\mathrm{C}, 10$ miles across the river and 5 miles downstream. Note that we pointed the boat
(heading) towards B, but have actually travelled (track) to C. If we want to finish up at Point B, we need to aim off to the left towards Point D.

This is a continuous process. If we head towards Point $B$, after, say, 12 minutes ( $1 / 5$ th of an hour), we will be 2 miles across the river and one mile displaced to the right. After 24 minutes, we will be 4 miles across the river and 2 miles downstream, and so on. The boat tracks along the vector $A C$.

Exactly the same process takes place with aircraft travelling through the air. On a day when the air is completely calm (analogous to the lake), the aircraft tracks where it is pointed. We get completely calm days about 6 times a year. However, we still have to be able to navigate on the other 359, so we need to be able to take account of wind, or moving air (analogous to the river).

This term 'wind' tends to suggest something transient and many newcomers to aviation find it hard to imagine that something so unsubstantial could have such an effect on such a dense massive aircraft as, say, a Boeing 747, or a dense fast moving aircraft such as a Tornado. This is why this chapter uses this rather elaborate analogy of the river. It is easy to imagine a boat or a twig being carried downstream by a 10 mile expanse of river moving at 5 knots. However, wind is not simply little puffy transient eddies of air. Wind is the continuous movement of an air mass.

An air mass is a large homogenous body of air travelling over the ground - we normally do not regard it as an air mass unless it is at least, say, 60 NM by 60 NM in size. This whole air mass travels over the ground, just like the river. Your aircraft is supported by this broad mass of air in just the same way as the boat or the twig is supported by the river and it therefore travels in it just as the twig travels in the river.

## The Triangle of Velocities

Accordingly, we have to take account of this movement of air when we navigate and this problem is solved by constructing a vector triangle of velocities. The components are always the same:

The Air Vector: The Air Vector consists of Heading and True Airspeed (TAS). TAS is the true (not indicated) speed of the aircraft through the air, and it is the speed that the aircraft would travel over the ground if there were no wind. The Air Vector is always drawn with one direction arrow.

The Wind Vector: The Wind Vector consists of Wind Direction and Wind Speed. Wind direction is always given in terms of the direction that the wind has come from, not where it is blowing to. Thus a wind vector like this:


Figure 7.3 Wind vector
indicates a wind direction of $330^{\circ}(\mathrm{T})$, not $150^{\circ}(\mathrm{T})$. (This does actually correspond to the way that we normally discuss wind direction in everyday speech. A South wind is warm because it has blown up from Morocco, a West wind is warm and wet because it has blown across the Atlantic and over Ireland, etc.). The Wind Vector is always drawn with 3 direction arrows.

The Ground Vector. The Ground Vector is drawn by joining up the Air Vector and the Wind Vector. The resultant vector is Track and Ground Speed. The Ground Vector is always drawn with 2 direction arrows.

Figure 7.4 is a diagram of the Triangle of Velocities. In practice, pilots do not normally draw it out to scale on graph paper, but solve it using an analogue navigation computer. However, as we are just about to find out, the Navigation Computer is merely a device for quickly producing a scale drawing and is actually drawing the Triangle of Velocities for you.


Figure 7.4 Triangle of velocities

## A Practical Example

Take the following situation:
You are in a Warrior PA 28.
TAS is 100 knots.
Heading of $000^{\circ}(\mathrm{T})$.
The forecast W/V is 240/30.
What will be your track and ground speed?

Start by drawing out your Triangle of Velocities.
On a piece of paper (graph paper, ideally), draw in the Air Vector. It will have a direction of $000^{\circ}(\mathrm{T})$ and a vector length equivalent to 100 knots (say, 100 mm ).


Figure 7.5 The air vector

Now draw in the Wind Vector. It was 240/30.
Remember, it is from $240^{\circ}$. It will actually point in the direction $060^{\circ}(\mathrm{T})$.
Whatever units you used in proportion to 100 knots TAS ( 100 mm ), draw the length of the Wind Vector in the same units ( 30 mm ).


Figure 7.6 The air vector and the wind vector

Now join the start of the Air Vector to the end of the Wind Vector to establish the resultant - the Ground Vector:-


Figure 7.7 The complete triangle of velocities

When you do that on your piece of graph paper, if you measure the track with a protractor you will find that it is $012^{\circ}$ and if you measure the length of the ground vector you will find a vector length equivalent to 118 knots.

What this is telling you is that if you fly a heading of $000^{\circ}(\mathrm{T})$ at a TAS of 100 knots in a wind of 240/30, your path over the ground will actually be a track of $012^{\circ}(\mathrm{T})$ at a ground speed of 118 knots.

It would be a bit long-winded if every time you wanted to fly, you had to start doing little scale drawings on graph paper. There has to be a quicker way. There is. It is called the Navigation Computer - but using the Wind Face this time.

## Solving the Same Problem on the Navigation Computer

Same situation:
You are in a Warrior PA 28.
TAS is 100 knots.
Heading of $000^{\circ}(\mathrm{T})$.
The forecast W/V is 240/30.
What will be your track and ground speed?

Start off by putting the W/V on the Navigation Computer.

Rotate the wind face to bring 240 up to the index (the 12 o'clock position).

Put any convenient datum to measure from (say 100 knots) under the blue circle in the centre of the wind face. (Alternatively, you can use the 'graph paper' square grid at the bottom of the slide).

Now plot downwards (in the 6 o'clock direction).

Mark off 30 knots on the wind face. We recommend that you use a soft pencil. You can use a felt-tip pen PROVIDED that it is waterbased (non-permanent) ink (pens with grey barrels). DO NOT use spirit-based (permanent) pens (with black barrels).


Figure 7.8 Plotting the W/V vector

That is the Wind Vector plotted. Now for the Air Vector.

Rotate the wind disc to put the Heading $\left(000^{\circ}\right)$ against the true heading index (12 o'clock position)

Move the slide to put your TAS (100 knots) under the blue circle in the centre.

It should now look like this.


Figure 7.9 Navigation computer showing the drift and ground speed

Now look where the wind mark is positioned. It is about 12 degrees right of the centre line. This means that you have 12 degrees of starboard drift. Your heading is $000^{\circ}$, so your track is $012^{\circ}$.

The ground speed is obtained by reading it off against the radial arcs. In the above example, it is 118 knots.

These are the same answers that we got by scale drawing.
All that the Navigation Computer has done is to produce an analogue model of our scale drawing.


Figure 7.10 The triangle of velocities on the face of the navigation computer
The centre line is the Air Vector, pointing $000^{\circ}$, with a length of 100 knots TAS.
It is joined to the Wind Vector, which you have plotted downwind yourself.
The resultant vector, which connects the start of the Air Vector to the end of the Wind Vector, is the Ground Vector. It tracks up the $12^{\circ}$ starboard drift line to give Track direction and ground speed vector length.

Now try the examples on the next page:

## Questions

To calculate track and ground speed:

1. Set wind direction under true heading index.
2. Mark in wind dot on centre line of computer BELOW centre dot at a distance equal to wind speed scale to scale.
3. Set true heading at true heading index.
4. Set TAS under centre dot.

THE COMPUTER IS NOW SET UP CORRECTLY WITH HEADING AT THE TRUE HEADING INDEX. DO NOT DISTURB THE SETTINGS.
5. Read off drift under wind dot.
6. Read off track on circular scale beneath drift on drift scale.
7. Read off ground speed beneath wind dot.

| HDG ${ }^{( }{ }^{(T)}$ | W/V | TRACK MADE GOOD ${ }^{\circ}(\mathrm{T})$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 273 | 230/40 |  | 150 |  |
| 181 | 150/30 |  | 90 |  |
| 054 | 350/28 |  | 88 |  |
| 084 | 255/55 |  | 210 |  |
| 141 | 280/35 |  | 190 |  |
| 274 | 030/40 |  | 150 |  |
| 213 | 060/35 |  | 150 |  |
| 029 | 090/40 |  | 170 |  |
| 305 | 350/16 |  | 100 |  |
| 187 | 270/60 |  | 110 |  |
| 310 | 045/45 |  | 320 |  |

## Questions

| HDG ${ }^{\circ}(\mathrm{T})$ | W/V | TRACK MADE GOOD ${ }^{\circ}(\mathrm{T})$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 138 | 125/50 |  | 390 |  |
| 253 | 180/70 |  | 415 |  |
| 131 | 210/40 |  | 485 |  |
| 100 | $315 / 55$ |  | 480 |  |
| 270 | 315/55 |  | 110 |  |
| 200 | 125/50 |  | 210 |  |
| 180 | 210/40 |  | 135 |  |
| 140 | 180/70 |  | 170 |  |
| 093 | 045/45 |  | 210 |  |
| 103 | 060/35 |  | 240 |  |
| 237 | 270/60 |  | 205 |  |
| 343 | 030/40 |  | 350 |  |
| 030 | 350/16 |  | 390 |  |
| 315 | 280/35 |  | 375 |  |
| 145 | 090/40 |  | 420 |  |
| 290 | 255/55 |  | 490 |  |
| 191 | 150/30 |  | 480 |  |
| 257 | 350/28 |  | 475 |  |
| 157 | 230/140 |  | 460 |  |

Answers

| $\operatorname{HDG}{ }^{(T)}$ | W/V | TRACK MADE GOOD ${ }^{\circ}(\mathrm{T})$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 273 | 230/40 | 286 | 150 | 124 |
| 181 | 150/30 | 194 | 90 | 66 |
| 054 | 350/28 | 073 | 88 | 80 |
| 084 | 255/55 | 082 | 210 | 264 |
| 141 | 280/35 | 135 | 190 | 217 |
| 274 | 030/40 | 262 | 150 | 171 |
| 213 | 060/35 | 218 | 150 | 181 |
| 029 | 090/40 | 016 | 170 | 155 |
| 305 | 350/16 | 297 | 100 | 89 |
| 187 | 270/60 | 157 | 110 | 119 |
| 310 | 045/45 | 302 | 320 | 327 |


| HDG ${ }^{\circ}(\mathrm{T})$ | W/V | TRACK MADE GOOD ${ }^{\circ} \mathrm{T}$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 138 | 125/50 | 140 | 390 | 340 |
| 253 | 180/70 | 263 | 415 | 400 |
| 131 | 210/40 | 126 | 485 | 480 |
| 100 | 315/55 | 104 | 480 | 525 |
| 270 | 315/55 | 242 | 110 | 81 |
| 200 | 125/50 | 214 | 210 | 204 |
| 180 | 210/40 | 168 | 135 | 102 |
| 140 | 180/70 | 119 | 170 | 125 |
| 093 | 045/45 | 103 | 210 | 182 |
| 103 | 060/35 | 109 | 240 | 215 |
| 237 | 270/60 | 225 | 205 | 158 |
| 343 | 030/40 | 338 | 350 | 322 |
| 030 | 350/16 | 032 | 390 | 379 |
| 315 | 280/35 | 318 | 375 | 346 |
| 145 | 090/40 | 150 | 420 | 400 |
| 290 | 255/55 | 294 | 490 | 445 |
| 191 | 150/30 | 193 | 480 | 459 |
| 257 | 350/28 | 254 | 475 | 480 |
| 157 | 230/140 | 139 | 460 | 440 |

## The Navigation Computer - Calculation of Heading and Wind Finding

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## Uses of the Wind Face

In the previous chapter we showed how, given heading, TAS and a forecast wind, we could use the wind face of the Navigation Computer to calculate the resulting track and ground speed. In fact, there are several operations that can be carried out using the wind face. Three of the most common are tabulated below:

| To Find | From | When Used |
| :---: | :---: | :--- |
| Track <br> Ground speed | Heading <br> TAS <br> W/V | If held on a mandatory heading |
| Heading <br> Ground speed | Track <br> TAS <br> W/V | Flight planning, or re-calculating <br> a heading in flight (new track or <br> new wind). |
| W/V | Heading <br> TAS <br> Track <br> Ground speed | Wind finding (the found wind <br> will be more recent and more <br> local than a forecast W/V) |

## Application of Finding Track

Finding track, when given heading, as in the last chapter is the simplest operation, though not the one most generally used, and you may have thought that the reason we show it is because it is the simplest one to use to explain the Triangle of Velocities, but has no practical operation. This is not true; its practical use can occur as follows:

Suppose that you are on some pre-planned track, for instance, on airway R123 between Brookman's Park and Clacton. You have used your Navigation Computer to calculate the desired heading to maintain the track (we have not covered how you do this yet, but that follows shortly) and you still have the forecast W/V marked up on your Navigation Computer. Then, because of a conflict of tracks with other traffic, ATC gives you a mandatory radar heading of $030^{\circ} \mathrm{T}$, taking you temporarily off the airway at time 1003. You make a note of your VOR/DME position at the time.


Figure 8.1 Mandatory radar heading off the airway
After a while, the conflict is resolved and 1009 at ATC inform you that "You are cleared own navigation to Clacton". At just that moment, the VOR unlocks. You have no area navigation equipment. How are you going to find the required track to Clacton?

Answer: You use your Navigation Computer. You should have the forecast wind (let us say $315 / 15$ ) marked up on the wind face from when you were flight planning before the flight. Put your TAS (say, 160 knots) under the centre circle, set the heading of $030^{\circ}$ and read off the drift and ground speed, exactly as in the last lesson. This gives $5^{\circ}$ starboard drift, making a track of $035^{\circ}(\mathrm{T})$, and a ground speed of 157 knots.

From the position you noted at 1003, draw a track line of $035^{\circ}(\mathrm{T})$ on your chart. You have gone 6 minutes (one-tenth of an hour) at 157 knots ground speed, so that is 15.7 nautical miles. That is your best estimate of position (known as a "dead reckoning" or "DR" position and shown by the following symbol: ©).


Figure 8.2 Calculating the DR position
From the DR position draw a straight line to Clacton. That is your new desired track.
The purpose of this last example is to make the point that there are occasions where you need to be able to calculate track from a given heading, not the other (more common) way round. You will be tested on all 3 Navigation Computer operations in the ATPL examination and you need to be completely familiar with all 3 and to be able to distinguish the use of one from the other.

## Solving for Heading and Ground Speed

Chapter 7 showed how, if you are given the heading, the TAS, and the W/V, you can find your track and ground speed. The first few pages of this chapter showed the practical use of this and how you can use this when held on a fixed heading.

However, the most common requirement in navigation and planning is finding the heading to fly. In planning a trip, you will join up your turning points on a map using a ruler and then measure the track of the straight line. This means that you already know your track. What you need to know is the heading to fly, because the wind will probably cause some drift. It is the reverse of the problem we solved earlier.

So let us see how you use the Navigation Computer, given track, to find the heading to fly.

Take this example:
You are in an aircraft with a planned TAS of 146 knots. The forecast $W / V$ is $315 / 20$.
You need to fly a track of $040^{\circ}(\mathrm{T})$.
What is the heading to fly, and what will your ground speed be?
Start off by plotting the wind on in the usual way. It is $315 / 20$.


Figure 8.3 Plotting a wind of 315/20
Now put the TAS (146 knots) under the blue circle and rotate the central disc round to put the desired track ( $040^{\circ}$ ) against the 12 o'clock index. $^{\prime}$.

This will show a drift of 8 degrees starboard, like this:


Figure 8.4 Heading $040^{\circ}$, TAS 146 , drift $8^{\circ}$ starboard
What this is telling you is that if you fly a HEADING of $040^{\circ}$, you will experience $8^{\circ}$ starboard drift and fly a TRACK of $048^{\circ}$.

You want to fly a TRACK of $040^{\circ}$. So what heading do you need?
If a heading of $040^{\circ}$ gives you $8^{\circ}$ starboard drift, try aiming off $8^{\circ}$ to the left and seeing if the drift changes. Try a heading of $032^{\circ}$.

Rotate the disc to put $032^{\circ}$ under the 12 o'clock index and check to see whether the drift $^{\prime}$ has changed. Sometimes it does, sometimes it does not. It depends on the geometry of your particular triangle of velocities.


Figure 8.5 Adjusting the heading to give the desired track
The drift is still $8^{\circ}$ starboard. This means that we have solved the problem. With a heading of $032^{\circ}$ and $8^{\circ}$ starboard drift, we get a track of $040^{\circ}$ - which is what we wanted.

Now there is just one final thing to do - establish what the ground speed will be.


Figure 8.6 Indication of ground speed
You can now see that the ground speed will be 143 knots.

## Changing Drift

Whether the drift changes from your initial attempt at a heading ( $040^{\circ}$ in our last example) to your final heading which gives you the right answer ( $032^{\circ}$ in our last example) depends on the geometry of your triangle of velocities. In the last example, the drift did not change. It was $8^{\circ}$ starboard in both cases.

An example of a heading change which gives no change of drift is shown below.


Figure 8.7 Heading change but no drift change
However, in the example below (Figure 8.8), the heading change, with the same wind, gives a change of drift. The drift in the diagram on the right is obviously smaller. It just depends on how big the wind vector is compared to the air vector, how large the heading change is, and the initial value of the angle between wind and heading.


Figure 8.8 Heading change which causes drift change

## Balancing the Drift

Take the situation:

TAS = 76 knots

Desired Track $=230^{\circ}$

Forecast $W / V=110 / 40$

Start in the usual way. Plot the W/V of 110/40.


Figure 8.9 Plotting a W/V of 110/40


Put the blue centre circle onto the TAS point of 76 knots.

Now bring the desired track of $230^{\circ}$ up to the top.

Now observe that the drift under the wind point is $20^{\circ}$ starboard and rotate the disc $20^{\circ}$ clockwise to bring the desired track of $230^{\circ}$ against $20^{\circ}$ on the DRIFT STBD scale.

Look at the wind point. The drift has increased to $25^{\circ}$ starboard so rotate the disc $5^{\circ}$ clockwise to bring $230^{\circ}$ against $25^{\circ}$ on the DRIFT STBD scale. Check the drift again. It is now $27^{\circ} \mathrm{S}$.

Rotate the disc a further $2^{\circ}$ clockwise so that track $230^{\circ}$ is against $27^{\circ}$ on the DRIFT STBD scale.
Check the wind point again. The drift is still $27^{\circ}$ S. Check that 76 is still exactly under the centre dot (the blue circle).

Read off true heading $203^{\circ}$ against the TRUE HEADING index and ground speed 87 knots from under the wind point.


Figure 8.11 Balancing the drift

## Questions

To calculate heading and ground speed (Note that this is the one which is normally required):

1. Mark in wind dot.
2. Set true track at true heading index.
3. Set TAS under centre dot.

THE COMPUTER IS NOW SET UP INCORRECTLY, TRACK IS SET AT THE HEADING INDEX. THE COMPUTER NEEDS RE-SETTING.
4. Read off drift indicated beneath the wind dot.
5. Move the track reading on the circular scale to coincide with the drift read off at 4 above on the drift scale.
6. Repeat 4 and 5 above until drift indicated under wind dot agrees with drift indicated on the drift scale.

THE COMPUTER IS NOW CORRECTLY SET UP. TRUE HEADING IS AT THE TRUE HEADING INDEX AND TAS IS BENEATH THE CENTRE DOT. THE DRIFT HAS BEEN BALANCED.
7. Read off true heading at the true heading index.
8. Read off ground speed under the wind dot.

| HDG $^{\circ}(\mathbf{T})$ | W/V | TRACK <br> REQUIRED <br> ${ }^{\circ}(T)$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
|  | $040 / 40$ | 155 | 140 |  |
|  | $280 / 27$ | 226 | 94 |  |
|  | $320 / 14$ | 198 | 136 |  |
|  | $190 / 52$ | 284 | 260 |  |
|  | $310 / 105$ | 262 | 544 |  |
|  | $300 / 60$ | 355 | 572 |  |
|  | $270 / 75$ | 113 | 520 |  |
|  | $240 / 80$ | 301 | 455 |  |
|  | $210 / 55$ | 313 | 545 |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |


| $\mathrm{HDG}^{\circ}(\mathrm{T})$ | W/V | TRACK REQUIRED ${ }^{\circ}(\mathrm{T})$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
|  | 335/75 | 220 | 553 |  |
|  | 040/20 | 005 | 110 |  |
|  | 125/35 | 045 | 196 |  |
|  | 170/23 | 113 | 210 |  |
|  | 145/20 | 021 | 80 |  |
|  | 170/30 | 287 | 220 |  |
|  | 300/30 | 200 | 137 |  |
|  | 180/40 | 130 | 190 |  |
|  | 280/24 | 116 | 199 |  |
|  | 262/90 | 234 | 605 |  |
|  | 210/80 | 071 | 620 |  |
|  | 190/80 | 339 | 523 |  |
|  | 305/30 | 038 | 585 |  |
|  | 295/60 | 355 | 515 |  |
|  | 340/85 | 091 | 588 |  |
|  | 235/66 | 032 | 485 |  |
|  | 055/35 | 197 | 176 |  |
|  | 255/15 | 018 | 180 |  |
|  | 090/15 | 147 | 74 |  |
|  | 020/25 | 140 | 99 |  |

## Answers

| HDG $^{\circ}(\mathrm{T})$ | W/V | TRACK <br> REQUIRED <br> ${ }^{\circ}(T)$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 140 | $040 / 40$ | 155 | 140 | 152 |
| 239 | $280 / 27$ | 226 | 94 | 75 |
| 203 | $320 / 14$ | 198 | 136 | 142 |
| 272 | $190 / 52$ | 284 | 260 | 258 |
| 138 | $270 / 83$ | 132 | 544 | 600 |
| 270 | $310 / 105$ | 262 | 572 | 496 |
| 350 | $300 / 60$ | 355 | 620 | 585 |
| 116 | $270 / 75$ | 113 | 570 | 637 |
| 292 | $240 / 80$ | 301 | 455 | 413 |
| 307 | $210 / 55$ | 313 | 545 | 555 |


| $\mathrm{HDG}^{\circ}(\mathrm{T})$ | W/V | TRACK REQUIRED ${ }^{\circ}$ (T) | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 227 | 335/75 | 220 | 553 | 582 |
| 011 | O40/20 | 005 | 110 | 92 |
| 055 | 125/35 | 045 | 196 | 187 |
| 118 | 170/23 | 113 | 210 | 198 |
| 033 | 145/20 | 021 | 80 | 89 |
| 280 | 170/30 | 287 | 220 | 232 |
| 212 | 300/30 | 200 | 137 | 140 |
| 139 | 180/40 | 130 | 190 | 161 |
| 118 | 280/24 | 116 | 199 | 221 |
| 238 | 262/90 | 234 | 605 | 526 |
| 076 | 210/80 | 071 | 620 | 675 |
| 334 | 190/80 | 339 | 523 | 592 |
| 035 | 305/30 | 038 | 585 | 585 |
| 349 | 295/60 | 355 | 515 | 480 |
| 083 | 340/85 | 091 | 588 | 610 |
| 029 | 235/66 | 032 | 485 | 545 |
| 190 | 055/35 | 197 | 176 | 202 |
| 014 | 255/15 | 018 | 180 | 188 |
| 137 | 090/15 | 147 | 74 | 65 |
| 127 | 020/25 | 140 | 99 | 108 |

## Practice Flight Plan

We now bring together those Navigation Computer facilities on both the Slide Rule face and the Wind face which are needed to complete a flight plan.

In all of the following exercises, work across the rows, row by row. Do not do them column by column. Doing it by rows makes you think about each individual operation and what it is that you are trying to achieve. Doing it by columns just turns it into a no-thought mechanical exercise.

## FLIGHT PLAN 1

|  | ALT <br> FT | TEMP <br> ${ }^{\circ} \mathrm{C}$ | CAS <br> KT | TAS <br> KT | TRACK <br> ${ }^{\circ}(\mathrm{T})$ | W/V | HDG <br> ${ }^{\circ}(\mathrm{T})$ | VAR | HDG <br> ${ }^{\circ}(\mathrm{M})$ | DEV | HDG <br> ${ }^{\circ}(\mathrm{C})$ | GS <br> KT | DIST <br> NM | TIME <br> MIN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2000 | -10 | 160 |  | 100 | $330 / 17$ |  | $5^{\circ} \mathrm{W}$ |  | $1^{\circ} \mathrm{W}$ |  |  | 50 |  |
| 2 | 4700 | -2 | 170 |  | 227 | $300 / 10$ |  | $3^{\circ} \mathrm{E}$ |  | $2^{\circ} \mathrm{E}$ |  |  | 112 |  |
| 3 | 4000 | -10 | 175 |  | 134 | $262 / 17$ |  | $3^{\circ} \mathrm{W}$ |  | $1^{\circ} \mathrm{E}$ |  |  | 30 |  |
| 4 | 2000 | +2 | 170 |  | 157 | $110 / 12$ |  | $6^{\circ} \mathrm{W}$ |  | $2^{\circ} \mathrm{W}$ |  |  | 38.5 |  |
| 5 | 6000 | -3 | 154 |  | 100 | $320 / 18$ |  | $4^{\circ} \mathrm{E}$ |  | $1^{\circ} \mathrm{W}$ |  |  | 60 |  |
| 6 | 5000 | -15 | 150 |  | 050 | $110 / 45$ |  | $8^{\circ} \mathrm{W}$ |  | $2^{\circ} \mathrm{E}$ |  |  | 13 |  |
| 7 | 10000 | -8 | 173 |  | 320 | $181 / 27$ |  | $3^{\circ} \mathrm{E}$ |  | -3 |  |  | 44 |  |


|  | ALT <br> FT | TEMP <br> ${ }^{\circ} \mathrm{C}$ | CAS <br> KT | TAS <br> KT | TRACK <br> ${ }^{\circ}(\mathrm{T})$ | W/V | HDG <br> ${ }^{\circ}(\mathrm{T})$ | VAR | HDG <br> ${ }^{\circ}(\mathrm{M})$ | DEV | HDG <br> ${ }^{\circ}(\mathrm{C})$ | GS <br> KT | DIST <br> NM | TIME <br> MIN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5000 | +5 | 170 |  | 070 | $230 / 25$ |  | $5^{\circ} \mathrm{W}$ |  | $1^{\circ} \mathrm{E}$ |  |  | 86 |  |
| 2 | 6500 | -2 | 170 |  | 234 | $210 / 30$ |  | $3^{\circ} \mathrm{W}$ |  | $2^{\circ} \mathrm{E}$ |  |  | 71 |  |
| 3 | 6000 | +2 | 174 |  | 240 | $210 / 22$ |  | $4^{\circ} \mathrm{W}$ |  | $1^{\circ} \mathrm{E}$ |  |  | 57 |  |
| 4 | 7500 | +2 | 175 |  | 166 | $110 / 30$ |  | $6^{\circ} \mathrm{W}$ |  | $3^{\circ} \mathrm{E}$ |  |  | 89 |  |
| 5 | 8000 | -2 | 174 |  | 190 | $245 / 36$ |  | $4^{\circ} \mathrm{E}$ |  | $1^{\circ} \mathrm{W}$ |  |  | 60 |  |
| 6 | 9000 | -5 | 180 |  | 050 | $110 / 45$ |  | $11^{\circ} \mathrm{W}$ |  | $2^{\circ} \mathrm{E}$ |  |  | 13 |  |
| 7 | 12000 | -11 | 184 |  | 324 | $190 / 47$ |  | $3^{\circ} \mathrm{E}$ |  | +2 |  |  | 50 |  |

FLIGHT PLAN 3

| Tr | W/V | Hdg <br> T | Var | Hdg <br> M | Dev | Hdg <br> C | CAS | PA/ <br> Temp | TAS | GS | Dist | Time |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 315 | $045 / 33$ |  | 7 W |  | 2 E |  | 190 | 3000 <br> +15 |  |  | 88 |  |
| 324 | $335 / 21$ |  | 6 E |  | 1 W |  | 205 | 9000 <br> -10 |  |  | 105 |  |
| 071 | $030 / 55$ |  | 11 W |  | 1 W |  | 142 | 14000 <br> +5 |  |  | 80 |  |
| 300 | $190 / 25$ |  | 8 E |  | 3 E |  | 132 | 4000 <br> -10 |  |  | 332 |  |
| 019 | $300 / 28$ |  | 9 W |  | 2 E |  | 130 | 3500 <br> -17 |  |  | 110 |  |
| 342 | $130 / 40$ |  | 3 E |  | 1 W |  | 160 | 7500 <br> 0 |  |  | 92 |  |
| 262 | $010 / 38$ |  | 34 E |  | 2 W |  | 180 | 9500 <br> 0 |  |  | 145 |  |
| 021 | $265 / 32$ |  | 16 W |  | 4 W |  | 159 | 7000 <br> +15 |  |  | 112 |  |
| 033 | $310 / 50$ |  | 7 W |  | 2 E |  | 202 | 22500 <br> -30 |  |  | 46 |  |
| 162 | $210 / 20$ |  | 14 W |  | 3 E |  | 194 | 28000 <br> -55 |  |  | 1312 |  |
| 228 | $280 / 20$ |  | 10 W |  | 2 E |  | 148 | 2000 <br> +4 |  |  | 84 |  |
| 351 | $060 / 30$ |  | 12 W |  | 2 W |  | 123 | 3000 <br> +16 |  |  | 95 |  |
| 043 | $123 / 46$ |  | 9 E |  | 1 W |  | 166 | 5000 <br> -5 |  |  | 62 |  |
| 032 | $120 / 30$ |  | 4 E |  | 1 E |  | 123 | 2500 <br> 0 |  |  | 49 |  |
| 162 | $140 / 40$ |  | 9 W | 1 W |  | 96 | 3500 <br> +8 |  |  | 130 |  |  |
| 343 | $220 / 30$ |  | 11 E | 1 W |  | 111 | 4000 <br> -5 |  |  | 82 |  |  |

Answers
FLIGHT PLAN 1

|  | ALT <br> FT | TEMP <br> ${ }^{\circ} \mathrm{C}$ | CAS <br> KT | TAS <br> KT | TRACK <br> ${ }^{\circ}(\mathrm{T})$ | W/V | HDG <br> ${ }^{\circ}(\mathrm{T})$ | VAR | HDG <br> ${ }^{\circ}(\mathrm{M})$ | DEV | HDG <br> ${ }^{\circ}(\mathrm{C})$ | GS <br> KT | DIST <br> NM | TIME <br> MIN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2000 | -10 | 160 | 159 | 100 | $330 / 17$ | 095 | $5^{\circ} \mathrm{W}$ | 100 | $1^{\circ} \mathrm{W}$ | 101 | 169 | 50 | 18 |
| 2 | 4700 | -2 | 170 | 180 | 227 | $300 / 10$ | 230 | $3^{\circ} \mathrm{E}$ | 227 | $2^{\circ} \mathrm{E}$ | 225 | 177 | 112 | 38 |
| 3 | 4000 | -10 | 175 | 180 | 134 | $262 / 17$ | 138 | $3^{\circ} \mathrm{W}$ | 141 | $1^{\circ} \mathrm{E}$ | 140 | 190 | 30 | 9.5 |
| 4 | 2000 | +2 | 170 | 172 | 157 | $110 / 12$ | 154 | $6^{\circ} \mathrm{W}$ | 160 | $2^{\circ} \mathrm{W}$ | 162 | 165 | 38.5 | 14 |
| 5 | 6000 | -3 | 154 | 167 | 100 | $320 / 18$ | 096 | $4^{\circ} \mathrm{E}$ | 092 | $1^{\circ} \mathrm{W}$ | 093 | 180 | 60 | 20 |
| 6 | 5000 | -15 | 150 | 155 | 050 | $110 / 45$ | 064 | $8^{\circ} \mathrm{W}$ | 072 | $2^{\circ} \mathrm{E}$ | 070 | 127 | 13 | 6 |
| 7 | 10000 | -8 | 173 | 200 | 320 | $181 / 27$ | 315 | $3^{\circ} \mathrm{E}$ | 312 | -3 | 315 | 219 | 44 | 12 |

## FLIGHT PLAN 2

|  | ALT <br> FT | TEMP <br> ${ }^{\circ} \mathrm{C}$ | CAS <br> KT | TAS <br> KT | TRACK <br> ${ }^{\circ}(\mathrm{T})$ | W/V | HDG <br> ${ }^{\circ}(\mathrm{T})$ | VAR | HDG <br> ${ }^{\circ}(\mathrm{M})$ | DEV | HDG <br> ${ }^{\circ}(\mathrm{C})$ | GS <br> KT | DIST <br> NM | TIME <br> MIN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5000 | +5 | 170 | 183 | 070 | $230 / 25$ | 073 | $5^{\circ} \mathrm{W}$ | 078 | $1^{\circ} \mathrm{E}$ | 077 | 206 | 86 | 25 |
| 2 | 6500 | -2 | 170 | 186 | 234 | $210 / 30$ | 230 | $3^{\circ} \mathrm{W}$ | 233 | $2^{\circ} \mathrm{E}$ | 231 | 158 | 71 | 27 |
| 3 | 6000 | +2 | 174 | 190 | 240 | $210 / 22$ | 237 | $4^{\circ} \mathrm{W}$ | 241 | $1^{\circ} \mathrm{E}$ | 240 | 171 | 57 | 20 |
| 4 | 7500 | +2 | 175 | 197 | 166 | $110 / 30$ | 159 | $6^{\circ} \mathrm{W}$ | 165 | $3^{\circ} \mathrm{E}$ | 162 | 179 | 89 | 30 |
| 5 | 8000 | -2 | 174 | 196 | 190 | $245 / 36$ | 196 | $4^{\circ} \mathrm{E}$ | 192 | $1^{\circ} \mathrm{W}$ | 193 | 180 | 60 | 20 |
| 6 | 9000 | -5 | 180 | 206 | 050 | $110 / 45$ | 061 | $11^{\circ} \mathrm{W}$ | 072 | $2^{\circ} \mathrm{E}$ | 070 | 180 | 13 | 4 |
| 7 | 12000 | -11 | 184 | 220 | 324 | $190 / 47$ | 315 | $3^{\circ} \mathrm{E}$ | 312 | +2 | 310 | 250 | 50 | 12 |

## FLIGHT PLAN 3

| Tr | $\mathrm{W} / \mathrm{V}$ | Hdg <br> T | Var | Hdg <br> M | Dev | Hdg <br> C | CAS | PA/ <br> Temp | TAS | GS | Dist | Time |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 315 | $045 / 33$ | 325 | 7 W | 332 | 2 E | 330 | 190 | 3000 <br> +15 | 200 | 197 | 88 | 27 |
| 324 | $335 / 21$ | 325 | 6 E | 319 | 1 W | 320 | 205 | 9000 <br> -10 | 230 | 210 | 105 | 30 |
| 071 | $030 / 55$ | 059 | 11 W | 070 | 1 W | 071 | 142 | 14000 <br> +5 | 182 | 137 | 80 | 35 |
| 300 | $190 / 25$ | 290 | 8 E | 282 | 3 E | 279 | 132 | 4000 <br> -10 | 135 | 141 | 332 | 141 |
| 019 | $300 / 28$ | 007 | 9 W | 016 | 2 E | 014 | 130 | 3500 <br> -17 | 132 | 122 | 110 | 54 |
| 342 | $130 / 40$ | 349 | 3 E | 346 | 1 W | 347 | 160 | 7500 <br> 0 | 179 | 211 | 92 | 26 |
| 262 | $010 / 38$ | 272 | 34 E | 238 | 2 W | 240 | 180 | 9500 <br> 0 | 208 | 217 | 145 | 40 |
| 021 | $265 / 32$ | 012 | 16 W | 028 | 4 W | 032 | 159 | 7000 <br> +15 | 180 | 192 | 112 | 35 |
| 033 | $310 / 50$ | 023 | 7 W | 030 | 2 E | 028 | 202 | 22500 <br> -30 | 284 | 274 | 46 | 10 |
| 162 | $210 / 20$ | 165 | 14 W | 179 | 3 E | 176 | 194 | 28000 <br> -55 | 291 | 279 | 1312 | 281 |
| 228 | $280 / 20$ | 234 | 10 W | 244 | 2 E | 242 | 148 | 2000 <br> +4 | 150 | 137 | 84 | 37 |
| 351 | $060 / 30$ | 003 | 12 W | 015 | 2 W | 017 | 123 | 3000 <br> +16 | 130 | 116 | 95 | 49 |
| 043 | $123 / 46$ | 058 | 9 E | 049 | 1 W | 050 | 166 | 5000 <br> -5 | 175 | 161 | 62 | 23 |
| 032 | $120 / 30$ | 046 | 4 E | 042 | 1 E | 041 | 123 | 2500 <br> 0 | 124 | 120 | 49 | 25 |
| 162 | $140 / 40$ | 153 | 9 W | 162 | 1 W | 163 | 96 | 3500 <br> +8 | 101 | 63 | 130 | 124 |
| 343 | $220 / 30$ | 330 | 11 E | 319 | 1 W | 320 | 111 | 4000 <br> -5 | 115 | 129 | 82 | 38 |

## Wind Finding

A pilot needs to know what wind is affecting his aircraft in flight at his particular level. This knowledge is available from the meteorological forecast, but the problem with forecasts is that they cover a large area and their period of validity extends over several hours. They can, therefore, only be a general guide for your area. If you want to find out exactly what wind is affecting you in your local area at the present time, the Navigation Computer can be used to find the wind.

Take the following situation:
You are heading $060^{\circ}(\mathrm{T})$ at a TAS on 140 knots, in order to fly some planned track.


Figure 8.12 Heading $060^{\circ}(\mathrm{T})$
Once airborne, you get a fix at B, say 20 minutes after passing over a known landmark $A$.
You measure the track angle from A to B on your chart and find that it is $065^{\circ}(T)$.
You also measure the distance $A$ to $B$ on your chart and find that it is 40 nautical miles.
If you have flown 40 NM in 20 minutes, your ground speed must be 120 knots.
You now know the following:
Heading $\quad=060^{\circ}(\mathrm{T})$
TAS $=140$ knots
Track $=065^{\circ}(\mathrm{T})$
Ground speed $=120$ knots
Your triangle of velocities looks like this (Figure 8.13):


Figure 8.13 The Complete Triangle of Velocities
What is the wind velocity?
On your Navigation Computer, put the heading $060^{\circ}$ against the heading index and the TAS of 140 knots against the blue circle.

The track is $065^{\circ}$, which means you have $5^{\circ}$ starboard drift. Make a mark where the $5^{\circ} \mathrm{S}$ line crosses 120 knots ground speed.

This is the wind point.


Figure 8.14 Plotting the Wind Point

Your triangle of velocities looks like this:

Now we need to measure the wind.


Figure 8.15 The Triangle of Velocities


Figure 8.16 Wind Direction and Wind Speed

However, we need to measure it a bit more accurately than this.
This is the only reason we rotate the wind vector - to measure it more accurately.
Rotate the wind cross to the 6 o'clock position. You can now read the direction accurately from the 12 o'clock index.

You can also read the length of the wind vector accurately.

So the W/V is, in this example, $034^{\circ} / 23$ knots.

This is the wind that has affected you over the last 20 minutes. It will be far more recent and far more local than any area met forecast.


Figure 8.17 Reading the Wind Velocity Accurately

## Questions

To calculate the wind velocity:

1. Set true heading at the heading index.
2. Set TAS under the centre dot.
3. Read off drift on drift scale opposite track on circular scale.
4. Mark in wind dot at intersection of ground speed and drift line.
5. Rotate face until wind dot lies on centre line BELOW centre dot.
6. Read off wind direction at true heading index and wind speed to scale.

| $\mathrm{HDG}^{\circ}(\mathrm{T})$ | W/V | TRACK MADE GOOD ${ }^{\circ}(\mathrm{T})$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 209 |  | 219 | 150 | 134 |
| 270 |  | 266 | 180 | 202 |
| 223 |  | 224 | 206 | 246 |
| 069 |  | 079 | 138 | 124 |
| 299 |  | 293 | 125 | 120 |
| 167 |  | 162 | 202 | 190 |
| 213 |  | 231 | 85 | 80 |
| 014 |  | 005 | 200 | 218 |
| 330 |  | 348 | 90 | 90 |
| 179 |  | 168 | 196 | 226 |
| 098 |  | 100 | 310 | 338 |
| 091 |  | 080 | 440 | 435 |
| 050 |  | 055 | 398 | 415 |
| 351 |  | 340 | 393 | 405 |
| 288 |  | 292 | 434 | 462 |
| 276 |  | 281 | 335 | 316 |
| 280 |  | 275 | 430 | 435 |
| 291 |  | 292 | 330 | 360 |
| 097 |  | 095 | 343 | 322 |


| $\left.\mathrm{HDG}{ }^{( } \mathrm{T}\right)$ | W/V | TRACK MADE GOOD ${ }^{\circ}(\mathrm{T})$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 056 |  | 055 | 287 | 320 |
| 108 |  | 109 | 480 | 530 |
| 176 |  | 171 | 428 | 438 |
| 270 |  | 265 | 335 | 308 |
| 248 |  | 243 | 406 | 332 |
| 247 |  | 252 | 380 | 352 |
| 200 |  | 202 | 435 | 403 |
| 198 |  | 202 | 340 | 325 |
| 275 |  | 275 | 435 | 395 |
| 274 |  | 275 | 290 | 270 |
| 321 |  | 326 | 338 | 280 |
| 045 |  | 061 | 285 | 305 |
| 196 |  | 205 | 210 | 245 |
| 311 |  | 295 | 482 | 435 |
| 011 |  | 358 | 198 | 211 |
| 355 |  | 008 | 248 | 270 |
| 111 |  | 115 | 120 | 114 |
| 228 |  | 245 | 460 | 505 |
| 175 |  | 168 | 121 | 114 |

## Answers

| $\mathrm{HDG}{ }^{(T)}$ | W/V | TRACK MADE GOOD ${ }^{\circ}(\mathrm{T})$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 209 | 155/30 | 219 | 150 | 134 |
| 270 | 055/25 | 266 | 180 | 202 |
| 223 | 050/40 | 224 | 206 | 246 |
| 069 | 014/26 | 079 | 138 | 124 |
| 299 | 002/14 | 293 | 125 | 120 |
| 167 | 217/22 | 162 | 202 | 190 |
| 213 | 144/26 | 231 | 85 | 80 |
| 014 | 129/38 | 005 | 200 | 218 |
| 330 | 250/28 | 348 | 90 | 90 |
| 179 | 300/50 | 168 | 196 | 226 |
| 098 | 300/30 | 100 | 310 | 338 |
| 091 | 172/80 | 080 | 440 | 435 |
| 050 | 298/40 | 055 | 398 | 415 |
| 351 | 085/75 | 340 | 393 | 405 |
| 288 | 155/41 | 292 | 434 | 462 |
| 276 | 224/35 | 281 | 335 | 316 |
| 280 | 014/40 | 275 | 430 | 435 |
| 291 | 125/30 | 292 | 330 | 360 |
| 097 | 124/25 | 095 | 343 | 322 |


| HDG ${ }^{( }$( ${ }^{\text {( }}$ | W/V | TRACK MADE GOOD ${ }^{\circ}(\mathrm{T})$ | TAS | GS |
| :---: | :---: | :---: | :---: | :---: |
| 056 | 227/32 | 055 | 287 | 320 |
| 108 | 298/50 | 109 | 480 | 530 |
| 176 | 277/40 | 171 | 428 | 438 |
| 270 | 312/40 | 265 | 335 | 308 |
| 248 | 270/80 | 243 | 406 | 332 |
| 247 | 200/41 | 252 | 380 | 352 |
| 200 | 172/32 | 202 | 435 | 403 |
| 198 | 143/25 | 202 | 340 | 325 |
| 275 | 275/40 | 275 | 435 | 395 |
| 274 | 245/22 | 275 | 290 | 270 |
| 321 | 298/60 | 326 | 338 | 280 |
| 045 | 308/85 | 061 | 285 | 305 |
| 196 | 065/50 | 205 | 210 | 245 |
| 311 | 011/135 | 295 | 482 | 435 |
| 011 | 110/50 | 358 | 198 | 211 |
| 355 | 262/62 | 008 | 248 | 270 |
| 111 | 060/10 | 115 | 120 | 114 |
| 228 | 130/148 | 245 | 460 | 505 |
| 175 | 233/16 | 168 | 121 | 114 |

## Chapter <br> 9

## The Navigation Computer - Multi-drift Winds and Wind Components

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## Further Uses of the Wind Face

There are 2 other applications of the wind face which can be of use to pilots. One is finding winds from measuring drift only, not ground speed. This is called "multi-drift winds". The other is the use of the Navigation Computer to establish head/tailwind and crosswind components. The technique could well be of practical use to a pilot operating in a basic environment without sophisticated navigation aids.

## Multi-drift Winds

Experienced pilots can often assess their drift with considerable accuracy just by estimation of the angle between their heading and the angle that a particular object on the ground makes as it appears to track underneath their aircraft at low altitude. Bush pilots and maritime patrol pilots, in particular, tend to develop this skill with experience. However, it is very difficult to assess ground speed quickly, except by timing the rate of DME distance change over a short period, and this only works when tracking directly towards or away from the DME, which may not always be the case. The alternative is using a stopwatch over a short precalculated distance, but this requires detailed preflight preparation and is not usually practicable.

However, it is possible to find a W/V by use of drift only, though it requires the pilot to fly on 2 , or preferably, 3 headings which cut at a large angle to each other. This technique is known as the "multi-drift wind".

The best multi-drift winds come from a 3-heading check, with the headings at $60^{\circ}$ to each other. We will assume that a pilot is flying an initial heading of $030^{\circ}$ at a TAS of 120 knots and decides that he needs an update of the W/V. He therefore assesses his drift on his present heading, then turns $60^{\circ}$ left and makes another drift assessment, then turns $120^{\circ}$ right for a third drift assessment. He then returns to his original heading. He notes the drift in each case and comes up with the following summary:-

| Heading | Drift |
| :---: | :---: |
| $030^{\circ}$ | $10^{\circ} \mathrm{P}$ |
| $330^{\circ}$ | $5^{\circ} \mathrm{P}$ |
| $090^{\circ}$ | $5^{\circ} \mathrm{P}$ |

On the wind face of the Navigation Computer, rotate the heading to $030^{\circ}$ and ensure that the TAS of 120 knots is under the centre dot. Then, using a straight-edge, draw a line along the whole length of the $10^{\circ} \mathrm{P}$ drift line.


Figure 9.1 Draw in the $10^{\circ} \mathrm{P}$ drift line
Now rotate the heading to the second reading of $330^{\circ}$. Ensuring that you still have 120 knots TAS under the centre dot, plot in the drift on this heading. In our example, it is $5^{\circ} P$. There is no need this time to draw the line along the whole wind face. You just need to have a small crosscut - say, 1 cm either side, as follows.


Figure 9.2 Draw in the second drift line
Now rotate the heading to the third reading of $090^{\circ}$. Ensuring that you still have 120 knots TAS under the centre dot, plot in the drift on this heading. In our example, it is $5^{\circ} \mathrm{P}$ again.

If the drifts have been assessed correctly, the third line should lie over or very near the other two, like this:-


Figure 9.3 Plotting the final line
If the 3 lines do not coincide, a triangle will be formed by the intersection. This triangle is known as the "cocked hat". The size of the triangle is an indication of the accuracy of the drift assessment. If they coincide, it is probable that all the drift assessments were correct. A small cocked hat means that the probable error is small. With a larger cocked hat, probably one or more of the drifts is in error to some extent.

However, assuming no cocked hat or only a small one (in which case you take the centre), you have established the W/V by taking the intersection of the drift lines. The 3 lines intersect at the wind point. Rotate it round to the 6 o'clock position to measure the wind speed and direction. In this example it is 115/21.


Figure 9.4 Read off the W/V

## Two-drift Winds

The 3-drift wind would only be necessary for particular applications (for example, bush flying and maritime flying, as previously discussed). However, it would be possible to use 2-drift winds when flying on airways in the UK in an aircraft which is not equipped with more than airways minimum navaids (i.e. no RNAV, GPS, INS or FMS). In this case, the 2 headings need to be at nearly right angles to each other, and certainly in excess of $70^{\circ}$. Consider the situation below:


Figure 9.5 An airways route from Brecon to Ottringham via Wallasey
The planned track from Brecon to Wallasey is $008^{\circ}(\mathrm{M})$ along airway A 25 , then from Wallasey to Ottringham on a track of $085^{\circ}(\mathrm{M})$ along airway B1. The pilot homes into Wallasey by applying his planned drift to the VOR track. If his drift is wrong, the VOR radial will change. He gets back onto the correct centre line, then modifies his drift depending on how the radial changed. After a short period of trial and error he will establish the correct drift to hold the radial.

He then arrives at Wallasey and turns onto the planned heading to hold the outbound track along B1. Again, if the planned drift was not correct, there will another short series of trials and corrections until he finds the heading that holds the VOR radial consistently. He now has drifts for 2 headings and can use the Navigation Computer to find the wind point from the cross-cut of the drifts on the appropriate headings. The only problem with a 2-drift wind is that there is no confirmatory third cut to give some confidence in the other two, but it should be possible to assess the drift reasonably accurately when flying along a VOR centre line.

## Questions

1 TAS = 180 knots

| Heading | Drift |
| :---: | :---: |
| 130 | $6^{\circ} \mathrm{P}$ |
| 070 | $4^{\circ} \mathrm{P}$ |
| 190 | $2^{\circ} \mathrm{P}$ |
| $\mathbf{W} / \mathbf{V}$ |  |

2 TAS = 150 knots

| Heading | Drift |
| :---: | :---: |
| 045 | $3^{\circ} \mathrm{S}$ |
| 345 | $6^{\circ} \mathrm{P}$ |
| 105 | $8^{\circ} \mathrm{S}$ |
| $\mathbf{W} / \mathbf{V}$ |  |

3 TAS = 120 knots

| Heading | Drift |
| :---: | :---: |
| 270 | $2^{\circ} \mathrm{S}$ |
| 210 | $12^{\circ} \mathrm{S}$ |
| 330 | $9^{1} 1^{\circ} \mathrm{P}$ |
| $\mathbf{W} / \mathbf{V}$ |  |

4 TAS = 165 knots

| Heading | Drift |
| :---: | :---: |
| 100 | $2^{\circ} \mathrm{P}$ |
| 040 | $14^{\circ} \mathrm{P}$ |
| 160 | $12^{\circ} \mathrm{S}$ |
| $\mathbf{W} / \mathbf{V}$ |  |

## Answers

1 TAS = 180 knots

| Heading | Drift |
| :---: | :---: |
| 130 | $6^{\circ} \mathrm{P}$ |
| 070 | $4^{\circ} \mathrm{P}$ |
| 190 | $2^{\circ} \mathrm{P}$ |
| $\mathbf{W} / \mathbf{V}$ | $205 / 18$ |

2 TAS = 150 knots

| Heading | Drift |
| :---: | :---: |
| 045 | $3^{\circ} \mathrm{S}$ |
| 345 | $6^{\circ} \mathrm{P}$ |
| 105 | $8^{\circ} \mathrm{S}$ |
| $\mathbf{W} / \mathbf{V}$ | $026 / 22$ |

3 TAS = 120 knots

| Heading | Drift |
| :---: | :---: |
| 270 | $2^{\circ} \mathrm{S}$ |
| 210 | $12^{\circ} \mathrm{S}$ |
| 330 | $9^{1} 1^{\circ} \mathrm{P}$ |
| $\mathbf{W} / \mathbf{V}$ | $098 / 30$ |

4 TAS = 165 knots

| Heading | Drift |
| :---: | :---: |
| 100 | $2^{\circ} \mathrm{P}$ |
| 040 | $14^{\circ} \mathrm{P}$ |
| 160 | $12^{\circ} \mathrm{S}$ |
| $\mathbf{W} / \mathrm{V}$ | $107 / 40$ |

## Wind Components

The resolution of wind into head or tailwind and crosswind components is of importance both in en route flying (particularly as part of the Flight Planning syllabus) and in take-off and landing. There are various techniques of rapidly calculating crosswind components using mental arithmetic and rule of thumb methods, but these are not precise enough for General Navigation ATPL exam questions, which have to be resolved by use of the Navigation Computer in order to ensure the required level of accuracy.

The wind rarely blows directly along the line of an en route track or along the centre line of a runway. There is almost always an angle between the track or the runway direction and the direction from which the wind is blowing.


Figure 9.6 Wind direction and runway relationship
If the wind direction and the runway direction were the same, the full force of the wind would be directed along the runway and there would be no effect across it.

However, in the above example where the wind is blowing at an 'angle-off' of $30^{\circ}$ from the right, only part of the force is directed along the runway. The other part is directed across the runway from right to left. The wind velocity of $300^{\circ}(\mathrm{M}) / 30$ knots is resolved into 2 components, one acting along the runway, called the headwind component, and the other acting across the runway, called the crosswind component.

It would be possible to solve this by taking the sine and cosine of the wind angle of $30^{\circ}$. The headwind component will be 30 knots $\times \cos 30^{\circ}$, i.e. 26 knots, whilst the crosswind component Computer.


Figure 9.7 Wind components

## Requirement for Wind Component Calculations

Pilots need to be able to calculate both headwind and crosswind components. Taking headwind first:

Threshold speed is CAS (or IAS, for all practical purposes). This corresponds to a TAS. However, stopping on a runway depends on the value of $1 / 2 m v^{2}$. The ' $v$ ' is ground speed. A headwind will reduce the ground speed on touchdown, requiring less brake energy to stop in a given runway length. For a heavy aircraft on a short runway, there may be a minimum headwind requirement, below which it will not be possible to stop with an adequate safety margin.

Crosswind can be even more critical. This is because:
With a crosswind, the pilot needs to line up on the approach using the extended centre line of the runway as a reference, then point the nose into wind to lay off for the drift on finals. At the moment that the undercarriage is about to touch down, it is necessary to apply rudder to re-align the aircraft with the runway. This is usually called "kicking off the drift", although it should be a controlled application of rudder, not a "kick", and applying the correct amount requires a certain amount of skill and judgement. The greater the crosswind, the more difficult this problem becomes.

Even if the pilot judges this perfectly and the aircraft is precisely lined up with the runway at the moment the tyres make contact, there will still be a sideways velocity. Tyres are mainly stressed to take fore-and-aft loads, not sideways ones, and it is possible to burst tyres on crosswind landings.

Finally, once the pilot has got the aircraft down, he may have trouble holding it on the runway, especially if the aircraft has a high tail fin. As the speed slows down, the rudder has less control authority and may not be able to counteract the crosswind component acting on the rest of the fin. The aircraft may run off the runway.

For these reasons, there will be a maximum crosswind component specified for the type of aircraft. Companies may also apply their own rules, quite apart from the aircraft limitation. For instance, an aircraft may be cleared to land in up to 25 knots crosswind, but a company might have rules that the first officer may only land with a crosswind of up to 15 knots. Above that, it has to be the captain's landing.

## Wind Directions - True or Magnetic?

Any FORECAST wind is given in true direction. Thus any written meteorological information (TAF, METAR, etc) will be in true. Forecast winds are used for planning navigation as well as landing and navigation tracks are normally initially measured in true.

Any information used to help the pilot line the aircraft up on the runway will be given in magnetic, because runway directions are always magnetic. Thus a wind from an ATC controller will be magnetic and the ATIS always quotes magnetic wind direction.

To summarize:-

| MAGNETIC | - | ATC and ATIS |
| :--- | :--- | :--- |
| TRUE | - | VOLMET, TAF and METAR |

## Calculation of Wind Components

## Navigation Computer

Take the following example:-

| R/W | 27 |
| :--- | :--- |
| ATIS W/V | $310 / 30$ |

Firstly, using the 'graph paper' part of the Navigation Computer scale at the bottom of the slide, plot the wind velocity in the normal way.


Figure 9.8 Plot W/V of 310/30
Now rotate the wind disc to put the runway QDM of 270 at the 12 o'clock index.


Figure 9.9 Rotate to runway QDM (270)

Now read vertically 'down' the $y$-axis. This gives you the headwind component. In this example it is 23 knots headwind.

Read across from the centre line (the $x$-axis). This gives you the crosswind component. In this example it is 19 knots crosswind (from right to left).


Figure 9.10 Reading off headwind and crosswind

## Maximum Wind Strength for a Given Crosswind Component

Sometimes the wind is blowing from a known given direction but it is gusting. A gust is defined as an increase in wind speed lasting less than one minute. Gusts will only be forecast or reported if 10 kt or more above the mean wind speed. You may wish to try to land between gusts. In this case you continue the approach down to the point at which you must commit to a landing or go around. For a visual approach this will be far lower than an instrument decision height. It may be 50 feet or lower, depending on the handling of your aircraft. At the last moment you request a spot readout of wind from the tower. If the wind strength from the actual direction gives a crosswind component of your crosswind limit or less, you can land. Otherwise, you must go around.

The moment of visual committal decision is no time to start calculating crosswind limits. You should have thought about it beforehand. Before you start the approach, you calculate what wind strength from the forecast direction corresponds to your crosswind limit on the runway in use. The calculations are made at the top of descent to allow an instant decision at the visual committal point.

Take the following case:

| TAF W/V | $180 / 10 \mathrm{G50}$ |
| :--- | :--- |
| Variation | 15 W |
| R/W | 23 |
| Crosswind limit | 25 knots |

This type of calculation requires a different technique. In this case, you do not have a single value of wind, so you cannot put it on your Navigation Computer.

Instead you put the runway and the crosswind on your computer, as follows:
Put the R/W QDM (230) up at the top.
Then draw in 25 knot cross limit parallel lines either side of the centre line.


Figure 9.11 Mark in crosswind limits

Now rotate the wind disc to the direction that the wind is coming from.

Do not forget to apply variation, if necessary.
In this example, the wind comes from a TAF, which is true direction. So the magnetic direction will be $195^{\circ}(\mathrm{M})$.

Now note where the applicable crosswind limit line crosses the centre line.

In this case, it is at 41 knots.


Figure 9.12 Rotate to wind direction

What this is telling you is that when the wind from $195^{\circ}(\mathrm{M})$ reaches 41 knots, it will give you 25 knots crosswind on R/W 23. Provided that the wind is less than 41 knots from $195^{\circ}(\mathrm{M})$, you may land.

## Headwind and Crosswind Limits

On occasions you can be limited by both crosswind AND headwind. You might have to land a heavy aircraft on to a single-runway airport and that runway might be short. In this case, you may be on your Landing Distance Available (LDA) performance limits and may need, say, at least 10 knots down the runway to land. At the same time, there is a crosswind.

Take this example:-

| R/W | 35 |
| :--- | :--- |
| ATC W/V | $030 / 10 \mathrm{G40}$ |
| Min Headwind | 10 knots |
| Crosswind Limit | 20 knots |

As before, draw in the 20 knot crosswind parallel markers. But this time, in addition, draw in a 10 knot headwind limit line.


Figure 9.13 Parallel crosswind limit lines and a headwind limit line

Now rotate to the direction that the wind will be coming from - in this case, $030^{\circ}(\mathrm{M})$ :


Figure 9.14 Rotate and read off centre line intersections

Note where the lines intersect the centre line - at 11 knots and 31 knots.
What this is telling you is that, when landing on R/W 35 , if the wind from $030^{\circ}(\mathrm{M})$ is less than 11 knots, you have less than 10 knots headwind down the runway. You cannot land. If the wind is any greater than 31 knots from $030^{\circ}(\mathrm{M})$, you have more than 20 knots of crosswind component. You cannot land.

If the wind is between 11 and 31 knots from $030^{\circ}(\mathrm{M})$, you have 10 knots or more of headwind component AND you have 25 knots or less of crosswind component. You can land.

## Tailwind Components

In general, you should avoid landing with a tailwind. If you are given a runway with a tailwind component, ask ATC if you can use the other end. It is usually poor airmanship to accept a tailwind and you will not have a leg to stand on afterwards if you run off the end - even if your brakes fail!

However, you may occasionally have questions that involve tailwind components, so you should know how to deal with them.

The first thing to notice is that you cannot directly plot the horizontal line representing a tailwind component in the normal way because it is above the horizontal scale line. You are in the curved part of the wind slide.

Let us take a worked example.

| R/W | 09 |
| :--- | :--- |
| Var | 10 E |
| ATIS W/V | $320 / 10 \mathrm{G} 30$ |

Your maximum tailwind component for this particular landing is 10 knots. What is the maximum wind speed from the given direction that you can accept?

ATIS is magnetic direction, so the value of variation is unnecessary information. Ignore it. (Watch out for this trap).

Bring the R/W direction (09) round to the top.
Now move the speed slide to bring 10 knots under the centre dot.

Draw in the tailwind component limit line.


Figure 9.15 Drawing in the tailwind limit line
then do not forget to move the speed slide back to put zero under the centre DOT!


Figure 9.16 Position of a tailwind limit line

Now rotate round to the wind direction.
Read off the value of the intersection.
In this example, it is 16 knots.


Figure 9.17 Rotate to wind direction

If you find all that too complicated, do not forget that you can always treat it as a headwind problem on the reciprocal runway!

Try treating it as the same type of problem as shown on pages 176 and 177, with the same wind of 320/10G30 but landing on R/W 27. You get the same answer - 16 knots.

## Allowable Wind Angle for a Given Wind Speed

Another problem sometimes arises which is quickly solved on the computer. It takes the form of determining between what directions a wind of given speed can blow without exceeding a stated crosswind component.

For example, it is required to find the directions between which a wind of 50 kt may blow without creating a crosswind component of more than 30 kt on a runway of direction $250^{\circ}(\mathrm{M})$.

1. With the runway direction set against the true index, mark a cross on the centre vertical line to represent the wind speed of 50 kt .
2. Turn the plastic disc clockwise until the cross indicates a crosswind of 30 kt. Note the direction which now appears against the true index ( $214^{\circ}(\mathrm{M})$ ).
3. Now turn the plastic disc anticlockwise and repeat the procedure, reading off the direction which appears against the true index $\left(286^{\circ}(\mathrm{M})\right)$.
4. The same crosswind component will be present if the wind direction is between $034^{\circ}(\mathrm{M})$ $-106^{\circ}(\mathrm{M})$ - the reciprocals of the directions found in 2) and 3) above, but a tailwind component will be present on this runway.

## Questions

1 An airfield has runways $18 / 36$ and $07 / 25$, all of equal length. The TAF W/V is $310 / 25$. Variation is $20^{\circ} \mathrm{E}$. Which will be the best R/W for take-off and landing?
a. $\quad 18$
b. $\quad 36$
c. 07
d. 25

2 In the Question 1 situation, what will be the head and crosswind components on this runway?
Head Cross
$\begin{array}{lll}\text { a. } & 24 & 17\end{array}$
b. $13 \quad 19$
c $19 \quad 16$
d. 17

3 An aircraft requires 10 knots headwind component for take-off. The crosswind limit for the aircraft is 20 knots. The only runway available is 25 . The ATIS W/V is 290/10G40. What are the minimum and maximum wind speeds for take-off?

Min Max
$\begin{array}{lll}\text { a. } & 17 & 31\end{array}$
b. $13 \quad 17$
c. $13 \quad 31$
d. 17 26

4 On a particular take-off, you can accept up to 10 knots tailwind. The runway QDM is 047 , the variation is $17^{\circ} \mathrm{E}$ and the ATIS gives the wind direction as 210 . What is the maximum wind strength you can accept?
a. 18 knots
b. $\quad 11$ knots
c. 8 knots
d. 4 knots

Answers

| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| $d$ | $c$ | $c$ | $b$ |

## Chapter <br> 10

## The 1 in 60 Rule

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## The 1 in 60 Rule

When you are flying, one or both hands will be on the control yoke. The other may have to be on the throttle some of the time. Either way, this does not leave you many spare hands to use to measure an angle on your map with a protractor.

Over the years, therefore, pilots have had to find other ways to calculate angles quickly and easily. One of these is called the " 1 in 60 rule".

Imagine that you have a line on a piece of paper exactly 60 mm long:
$\square$
Figure 10.1

Now raise a perpendicular at one end, exactly 10 mm high, and join them with a hypotenuse:


Figure 10.2

This will create an angle, $\mathbf{z}$, in the above diagram. If the adjacent is 60 cm long and the opposite is 10 cm long, then the angle $z$ will be $10^{\circ}$.

Similarly, if the adjacent is 60 cm long and the opposite is 5 cm high, $z$ will be $5^{\circ}$. If the opposite is 8 cm high, $z$ will be $8^{\circ}$, and so on - up to a maximum of about 20 degrees, when the theory starts to break down.

## Measuring Angles in Flight

Suppose you are flying along a planned track of $100^{\circ}(\mathrm{T})$ and you have flown 60 miles since your last fix, which was on track. You now pass over a clearly recognizable ground feature and you note from your map that it is 4 miles right of track. Using the 1 in 60 rule, you can establish that your actual track flown (i.e. the Track Made Good, or TMG) was 4 degrees right of your planned track and you have actually flown a TMG of $104^{\circ}(\mathrm{T})$. You are now in a position to calculate some sort of correction - and you have not had to use a protractor to establish the angle. We will discuss how you make the appropriate correction in the next chapter.

## The Geometry of the 1 in 60 Rule

To use the 1 in 60 rule as a practical pilot, you do not really need to know why it works - as long as you just accept that it does. However, for those who are interested, the explanation is as follows:

Imagine a circle of 1 metre radius.
What is the formula to calculate the circumference?

Circumference $=2 \pi r$
Taking $\pi$ as 3.142 , which is close enough for our purposes, then the
circumference $=2 \times 3.142 \times 1=6.284$ metres.


Figure 10.3

Now imagine that we go one metre round the circumference - not the short cut across the chord, but along the circumference, like this:

One metre at the circumference will subtend an angle $\theta$ at the centre. We need to calculate the value of $\theta$. Those of you who have done some engineering or applied maths will know that $\theta$ has a value of 1 radian. However, for those who have not, we can calculate its value from first principles.

Starting at any point on the circumference and going all the way round once gives an angle of $360^{\circ}$ and a distance of 6.284 metres. Going just one metre along the circumference gives an angle $\theta$. These are in proportion, so we can write the following equation:

$$
\frac{360}{6.284}=\frac{\theta}{1}
$$



Figure 10.4

Therefore $\theta=57.3^{\circ}$

It is a fundamental principle of geometry that provided we keep the ratios the same, we can scale a geometrical diagram up or down and the angles will not change. For instance, if we take a circle of 2 metres radius then, as long as we also take a 2 metre length of the circumference, $\theta$ will still be $57.3^{\circ}$. Similarly, $\theta$ will remain at $57.3^{\circ}$ if we take a circle of 10 metres radius, as long as we also take 10 metres of circumference.

Therefore, if we take a circle with a radius of 57.3 metres and consider a length of the circumference of 57.3 metres, we now have a situation where 57.3 degrees corresponds to 57.3 metres - a perfect one to one ratio. However, this only occurs with a radius of 57.3 metres.

Strictly, then, this should be the " 1 in 57.3 rule". But 57.3 is a difficult number to use in mental arithmetic when flying, so we approximate it to 60 , which introduces only about $5 \%$ error, which is perfectly acceptable for our purposes.

## Another Explanation

Another way of looking at it is as follows:


Figure 10.5
Let us write a series of values for $z$ between 1 and 20 in the above diagram and then compare the angles with their tangents and see if we can see any relationship.

| $z$ | 1 | 2 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tan \mathbf{z}$ | .017 | .035 | .087 | .176 | .268 | .364 |

Nothing leaps out of the page at us. If there is a relationship, it is not a clearly obvious one. Looking at the diagram above, what this tangent relationship is saying is, if the adjacent side is 1 metre, then the length of the opposite side is the number of metres in the "tan $z^{\prime \prime}$ row, i.e. 0.017 metres for $1^{\circ}, 0.035$ metres for $2^{\circ}$, and so on.

Let us now expand the adjacent side to 60 metres in length. This will mean multiplying the opposites by 60 as well.

Adding a third row to the above table:-

| $\mathbf{z}$ | 1 | 2 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tan \mathbf{z}$ | .017 | .035 | .087 | .176 | .268 | .364 |
| $60 \tan \mathbf{z}$ | 1.02 | 2.1 | 5.22 | 10.56 | 16.08 | 21.84 |

Suddenly, there seems to be a very close correlation between the angle $z$ and $60 \tan z$. Now a one degree track difference gives 1.02 NM off track in 60 NM along track. A ten degree track difference gives 10.56 NM off track in 60 NM along track. Even at 15 and 20 degrees, the relationship is pretty close. But you should not really need to use the 1 in 60 rule for angles much above 10 degrees - you should never get that far off track.

There are 2 reasons why the above rule does not give a perfect one-to-one correlation as the angles get bigger, whilst the first explanation, using the circle, gave an exact relationship - 57.3 degrees gave exactly 57.3 metres. The first reason is that we intentionally introduced a $5 \%$ error to make the arithmetic easier - we use a 1 in 60 rule in the air, not a 1 in 57.3 rule. Knock $5 \%$ off all the above figures in the " 60 tan z " row, and the correlation becomes very close.

The second reason is that the perfect correlation comes from using the arc of a circle. The above explanation considers a right-angled triangle. The tangent of an angle is not a linear relationship, but up to about 20 degrees it is very close.

However, as previously stated, you do not need to know why the 1 in 60 rule works as long as you accept that it does and you can apply it in the air.

## Expanding or Contracting the Triangle



Figure 10.6
In the above diagram, the adjacent side represents 60 nautical miles distance gone along track. The pilot fixes himself 8 miles left of track. He therefore has a track angle error of $8^{\circ}$ left.

However, fixes do not always come conveniently at 60 mile intervals. In any event, 60 NM at Warrior speeds corresponds to 30-40 minutes, depending on the ground speed, which is too long an interval between fixes. We therefore need to be able to use the 1 in 60 rule when the along track distance is a figure other than 60 .

The problem is a simple one - a matter of similar triangles. Suppose we have only gone 30 miles along-track. For the same track angle error, $8^{\circ}$ left, we will only be 4 miles left of track.


Figure 10.7
We simply take the ratio of 60 NM to our along-track distance and multiply our cross-track distance by this ratio. This sounds complicated, but is not when you try a few practical examples. Four miles in 30 is obviously the same as 8 in 60 , so the angle is $8^{\circ}$. Ten miles in 120 is obviously the same as 5 in 60 , so the angle is $5^{\circ}$, and so on.

If the numbers really do not lend themselves to simple mental proportioning, there is a formula:-
$z=\frac{\text { distance off }}{\text { distance gone }} \times 60$
For instance:- You find yourself 6 miles right of track after 40 miles along track.
In the above formula:
$z=\frac{6}{40} \times 60$
$=9^{\circ}$

## Questions

1 You are flying from A to B. You find that your position is 60 NM outbound from A and 7 NM left of the required track. What is your track error angle?

2 You are flying from C to D. You find that your position is 120 NM outbound from C and 8 NM right of the required track. What is your track error angle?

3 You are flying from E to F. You find that your position is 90 NM outbound from E and 6 NM right of the required track. What is your track error angle?

4 You are flying from G to H . You find that your position is 30 NM outbound from G and 4 NM left of the required track. What is your track error angle?

5 You are flying from J to K, which is a required track of $045^{\circ}(\mathrm{T})$. You find that your position is 80 NM outbound from J and 4 NM left of the required track. What is your track made good?

6 You are flying from $L$ to $M$, which is a required track of $220^{\circ}(T)$. You find that your position is 45 NM outbound from $L$ and 3 NM right of the required track. What is your track made good?

7 You are flying from $N$ to $P$, which is a required track of $315^{\circ}(T)$. You find that your position is 40 NM outbound from N and 6 NM left of the required track. What is your track made good?

8 A surveyor places himself 660 metres from a mast and measures an elevation angle of $4^{\circ}$ to the top of the mast. What is the height of the mast?

9 You are flying an instrument approach to an airfield and the required glide slope angle is $3.00^{\circ}$. What height should you be passing when you are exactly 2 nautical miles from the touchdown point? (NOTE: For this question, assume 1 NM = 6000 feet).

10 You are flying an instrument approach to an airfield and the required glide slope angle is $2.5^{\circ}$. You are correctly on the glide slope and you are passing a height of 1000 feet QFE. What is your range from the touchdown point? (Assume 1 NM = 6000 feet).

11 You are flying from $Q$ to $R$, which is a required track of $125^{\circ}(T)$. You find that your position is 40 NM from $R$ and 2 NM left of the required track. What track must you fly to arrive overhead $R$ ?

12 You are flying from $S$ to $T$, which is a required track of $272^{\circ}(\mathrm{T})$. You find that your position is 50 NM from T and 5 NM right of the required track. What track must you fly to arrive overhead $T$ ?

Answers

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $7^{\circ} \mathrm{L}$ | $4^{\circ} \mathrm{R}$ | $4^{\circ} \mathrm{R}$ | $8^{\circ} \mathrm{L}$ | $042^{\circ}(\mathrm{T})$ | $224^{\circ}(\mathrm{T})$ | $306^{\circ}(\mathrm{T})$ | 44 m | 600 ft | 4 NM | $128^{\circ}(\mathrm{T})$ | $266^{\circ}(\mathrm{T})$ |

## Chapter 11

## Navigation Using the 1 in 60 Rule

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## Returning to Track

If you find that you have strayed from track, it may be for a variety of reasons. You could have failed to fly the flight plan heading accurately enough, or possibly made too shallow a turn from the last track when over the initial turning point. However, for en route navigation, it is assumed that the most likely reason is that the wind has changed.

It is also assumed that, for small changes, a change of track of, say, 10 degrees, is achieved by a change of heading of exactly 10 degrees. In fact, as you should recall from the chapter on the Navigation Computer and the Triangle of Velocities, sometime as you alter heading, the drift changes. However, these rapid navigation techniques assume you will alter your track by the same amount that you alter heading.

There are a few points to remind ourselves about tracking errors.


Figure 11.1 Planned track and TMG
The track you intended to fly is called the "planned track". The track you actually make over the ground (presumably because the actual wind is different from the forecast) is called the "track made good" (or TMG). The difference between them is called "Track Error Angle".

Track Error Angle is NOT drift. This is a common mistake amongst beginners to this subject. Drift is the angle between Heading and Track (either planned track or TMG). Adding to Figure 11.1 to draw in the heading should make this clear.

Suppose that, with the forecast wind on the day, you needed to fly a true heading of $100^{\circ}(\mathrm{T})$ to give you a planned track of $090^{\circ}(\mathrm{T})$. This gives the following diagram:-


Figure 11.2 Heading, expected drift, actual drift, and track error angle
The difference between the planned track and the TMG (shown as green in Figure 11.2) is the track error angle. The difference between the heading and the planned track (shown in red) is the expected drift. The difference between the heading and the TMG is the actual drift experienced (shown in blue).

It is true that Track Error Angle is the difference between the expected drift and the actual drift, but Track Error Angle is not drift itself. This point is important.

There are 3 basic techniques for getting back to track:

- Double Track Angle Error
- Track Error Angle and Closing Angle
- Combined Track Error Angle and Closing Angle Single Calculation.

The last one is simply a variation on the second one, and is a quicker method of achieving the same answer.

From these 3 basic techniques, we come up with the Oxford preferred technique, which is called New Track Reference.

## Double Track Angle Error Method

Consider the following situation. You are planning to fly a particular track (say $090^{\circ}(\mathrm{T})$ ) and you have done your flight plan and calculated a heading to fly. You fly that heading (accurately, we assume) but after 30 miles along track you get a pinpoint (a visual fix) which puts you 4 miles left of track.


Figure 11.3 4 NM left of track after 30 NM along track
Firstly, calculate the Track Error Angle. Four NM off in 30 NM along is $8^{\circ}$.
If we do nothing about it, we will continue to diverge from track at the same rate. The following situation will develop:-


Figure 11.4 Continuing to diverge from track
The first thing to do is to stop this trend. We are diverging by $8^{\circ}$ to the left, so turn $8^{\circ}$ right.


Figure 11.5 Paralleling track

Just turning $8^{\circ}$ right has not solved the navigation problem, though. It has merely prevented it from getting any worse. We are now paralleling track, 4 NM to the left of it, but not getting back to it.

It is not good enough to just turn a little bit more - say, another $10^{\circ}$ right - and then map-read our way back to track. Firstly, there might not be anything significant to see to tell us when we are back on track and, secondly, we do not want a navigation technique that requires continuous attention from the pilot - i.e. continuous map-reading, in this case. As a singlepilot, you have many other things to do - fly the aeroplane, monitor changing weather, think about fuel, make radio calls as appropriate, carry out engine and electrics checks periodically and, most importantly, look out to avoid other aircraft. We want some method that allows us to minimize the amount of time spent actually navigating. We therefore want to make a measured and controlled turn in a way that will indicate to us when we are back on track without having to map-read.

What we do, therefore, is turn another $8^{\circ}$ right, making a total change of $16^{\circ}$ right. Now consider Figure 11.6 below:


Figure 11.6 Isosceles triangle
By turning through exactly double the track error angle, i.e. $16^{\circ}$ right, we are creating an isosceles triangle. It is symmetrical about the cross-track error line (marked ' 4 ' in the above diagram), i.e. we are converging back to track at a closing angle of $8^{\circ}$. This means that it takes the same distance to get back on track as it took to get off track. Therefore the times will be the same (there may be a tiny difference because the ground speeds are not absolutely identical, but it will only be a matter of seconds - certainly less than a minute).

So, if I start off on track at 1000 hours and find myself off track at 1020, and if I turn by double the Track Angle Error, I will be back on track at 1040. No continuous map-reading is required; all I have to do is look at my watch.


Figure 11.7 Back on track at 1040 hours
However, the story does not end here. If we do nothing about it once we re-cross our planned track, having turned $16^{\circ}$ right at our pinpoint, we will now diverge to the right of track, like this:-


Figure 11.8 Diverging to right of track
So now we turn back $8^{\circ}$ left, once we get back to track. And, unless the wind changes again, we should now stay on track.


Figure 11.9 Maintaining track by turning back again
Notice that we have now finished up on a heading $8^{\circ}$ right of our initial heading. We turned $16^{\circ}$ right, then $8^{\circ}$ left, making a total of $8^{\circ}$ right. However, notice that our initial heading took us $8^{\circ}$ to the left of track - this is why we had the initial track error. Having turned a total of $8^{\circ}$ right, once back on track, we are now on the heading which, if we had followed initially, would never have taken us off track in the first place. So we continue with this for the rest of the leg (or until the wind changes again).

## Track Error Angle and Closing Angle

Sometimes we may wish, once have found ourselves off track, to head directly for the destination (or next turning point), rather than merely regain track. Also, if we are more than half-way along track at the first pinpoint, the double track error angle method will not get us back on track before the next turning point. In these cases we use a combination of the Track Error Angle and the Closing Angle.

Consider the following situation. Your total track distance is 78 NM. As before, after 30 NM along track, you get a pinpoint 4 NM left of track. As before, your track error angle is $8^{\circ}$ to the left.


Figure 11.10 Track Error Angle and Closing Angle
As before, turning $8^{\circ}$ to the right will only parallel track. We now need to calculate the closing angle in order to proceed to the turning point.

This is not too difficult. If we have gone 30 NM along a 78 NM track, there must still be a further 48 NM remaining. We are 4 NM off track in 48 to go, so that is 1 in 12 - which is 5 in 60 . Therefore our closing angle is $5^{\circ}$.

So, we turn $8^{\circ}$ right to parallel track and a further $5^{\circ}$ right to converge, making a total turn of $13^{\circ}$ required.

Therefore, to summarize, the required amount of turn is the sum of Track Error Angle and Closing Angle.

## Combined Track Error Angle and Closing Angle Single Calculation

There is an arithmetical method of solving the above calculation which gives exactly the same answer, but does not require you to calculate Track Error Angle. Only the Closing Angle needs to be established and it is then multiplied by a factor dependent upon the proportion of your distance along track to the total leg distance. This can be quicker.

Consider exactly the same problem. We have a track 78 miles long and after 30 miles along track, we are 4 miles left of track. This time, do not bother to work out the Track Error Angle. Just consider the Closing Angle.


Figure 11.11 Closing Angle only
As before, if you are 30 NM along a 78 NM leg, you must have 48 NM to go. (In practice, you will not even have to work this out, because we normally mark our maps up with distance-togo, not distance gone). As before, 4 NM off track in 48 to go is 1 in 12 - which is 5 in 60 - so our closing angle is $5^{\circ}$.

Take the proportion of your distance along track to the total leg length and invert it. In this case,

$$
\frac{30}{78} \text { becomes } \frac{78}{30}
$$

Now multiply the closing angle by the inverted fraction and, hey presto, you have the required angle to turn in one calculation.

$$
5^{\circ} \times \frac{78}{30}=13^{\circ}
$$

This is not an approximation. You can show by a geometrical proof that this method gives precisely the same answer as separately calculating the Track Error Angle and Closing Angle. In this example, we deliberately chose numbers that were not exactly half or a third or a quarter to show that the arithmetic works even for numbers like 13. In practice, most of the time, all you will do is estimate that you are approximately, say, one-third of the way along your leg, so you need to alter by 3 times your Closing Angle.

## Using These Methods Practically

The material which follows is not examined as part of the EASA ATPL exam. However, it may be used as part of your flying training.

In practice, you do not have to carry out 1-in-60 calculations in order to work out your Track Error Angle and Closing Angle. Instead, what you should do is, in the preflight preparation stage, mark up the track on your map with 5 degree and 10 degree guide lines, as shown in Figure 11.12.


Figure $11.125^{\circ}$ and $10^{\circ}$ guidelines
Now, with the guidelines, you can rapidly assess the Track Error Angle and Closing Angle of any visual fix you get on that leg. You now just have to add the Track Error Angle to the Closing Angle to know the total angle to turn to head for the destination turning point.

Practically speaking, this results in too cluttered a chart. The recommended teaching is to draw just one guideline at each end of track, at $10^{\circ}$, as in Figure 11.13 below:


Figure 11.13 One $10^{\circ}$ guideline each end of track
Similarly, you do not have to use a ruler in the air in order to assess your distances along and across track. Mark up your map with distance-to-go using 10 NM markers. Do not use distance gone - that is of no interest to you. What you need to know is how many miles remain, without having to carry out subtraction in your head. Mark up the map as follows:


Figure 11.14 Distance-to-go markers
Figure 11.14 shows a track of length 44 NM and any time you get a visual fix you can instantly see how far you have to go. You also have a useful 10 NM scale to help you assess distances off track, if you need to.

The recommended technique is a combination of these 2 methods. Mark up the Closing Angle guidelines and the distance-to-go markers. Then, if you get a fix a quarter of the way along track, alter heading by 4 times the Closing Angle. If you get a fix a third of the way along track, alter heading by 3 times the Closing Angle. If you get a fix half-way along track, alter heading by 2 times the Closing Angle, and so on.

## New Track Reference

New Track Reference is the preferred method. Start off as in the previous paragraph.
Firstly, It is necessary to hold a steady heading until the aircraft's position is definitely pinpointed - rather than making rash panicky alterations.

Once the aircraft's position is established, calculate the required heading alteration as follows:

1. Estimate, as a fraction, the proportion of the total leg distance covered.
2. Invert this fraction.
3. Multiply the inverted fraction by the closing angle.


Figure 11.15 Initial heading correction
In Figure 11.15, you pinpoint yourself at C, to the left of your desired track from A to B. The closing angle is $5^{\circ}$ and you are about $1 / 4$ of the way down your track. So:

1. $\quad$ Proportion of leg travelled $=1 / 4$
2. Invert this fraction, giving 4/1
3. Multiply this by the closing angle of $5^{\circ}$.

This gives $20^{\circ}$. You alter heading $20^{\circ}$ (starboard, of course) for destination B.
We hope that this alteration will allow the aircraft to follow a new track from $C$ to $B$.
Now, let us assume that the wind has changed subsequently, and we later fix our position at D, to the port of our 'new' track from $C$ to $B$.


Figure 11.16 Further pinpoint and track correction
We apply the system again, this time with respect to our 'new' track C-B. Suppose the $10^{\circ}$ lines show that angle DBC, which is the closing angle with respect to our 'new' desired track $C-B$ is $2^{\circ}$. Suppose that our new pinpoint $D$ is about halfway between $C$ and $B$. Then our new alteration (to starboard) will be: $2_{1} \times 2^{\circ}=4^{\circ}$.

The only point to bear in mind is that the closing angle must be calculated from the 'new' track, which will not be drawn on your chart. If you use your $10^{\circ}$ guidelines, you must bear in mind that they were drawn with respect to the original track, and mentally allow for the first closing angle. Similarly, you must estimate the proportion you have travelled of the 'new' distance CB, not the old distance $A B$.

## Questions

1 If an aircraft is $3^{\circ}$ off required track at a range of 120 NM , how far in nautical miles (NM) is the aircraft off required track?

2 If an aircraft is 2 miles off required track at a range of 40 NM , what is the angle off track (track error)?

3 An aircraft leaves A to fly to B, 95 NM distance. Having flown 35 NM, the aircraft position is found from a 'pinpoint' (a geographical point over which the aircraft has flown); the pinpoint is 7 NM right of track.
a. What is the track error?
b. What alteration of heading is required to fly direct to $B$ ?
c. What is the drift? (Trap question).

4 An aircraft is flying from Oxford to Cambridge, planned track $074^{\circ}(\mathrm{M})$, distance 70 NM, heading $065^{\circ}(\mathrm{M})$. Having flown 30 NM, the pilot 'pinpoints' the aircraft position overhead Cranfield, 4 NM left of planned track.
a. What is the track error overhead Cranfield?
b. What is the Track Made Good (TMG) from Oxford?
c. What was the expected drift?
d. What has the actual drift been?
e. What alteration of heading should be made over Cranfield to fly direct to Cambridge?
f. What is the new heading to be flown from overhead Cranfield?

5 An aircraft is flying from Norwich to Oxford, planned track $250^{\circ}(\mathrm{M})$, distance 96 NM, heading $260^{\circ}(\mathrm{M})$, ground speed 180 kt. The aircraft departs Norwich at 1000 hrs. At 1012, the aircraft is overhead Ely, 3 NM right of planned track. Use the 1:60 to estimate the following.
a. What was the planned drift?
b. What is the track error at 1012 hrs ?
c. What TMG has been flown between 1000 hrs and 1012 hrs .
d. What has the actual drift been between 1000 hrs and 1012 hrs ?
e. What alteration of heading should be made to track directly to Oxford?
f. What heading is required to fly directly to Oxford?
g. What alteration of heading should be made to regain track at 1024 hrs ?
h. What heading should be flown between 1012 and 1024 hrs to regain track at 1024 hrs?
i. Given the situation in g. and $h$. above, what heading change should be made at 1024 hrs and what heading should be flown from 1024 hrs onwards?
j. Estimate the ETA at Oxford.

## Answers

1. 6 NM
2. $3^{\circ}$
3. a. $12^{\circ}$ right
b. $\quad 19^{\circ}$ left ( $\mathrm{TE}=12^{\circ}, \mathrm{CA}=7^{\circ}$ )
c. Cannot calculate as heading not given. To calculate drift, you must know heading. Drift is defined as the angle between heading and "track".
4. a. $8^{\circ}$ left
b. $\quad 066^{\circ}(\mathrm{M})$
c. $\quad 9^{\circ} \mathrm{S}$
d. $\quad 1^{\circ} \mathrm{S}$
e. $\quad 14^{\circ}$ right $\left(T E=8^{\circ}, C A=6^{\circ}\right)$
f. $\quad 079^{\circ}(\mathrm{M})$
5. a. $10^{\circ} \mathrm{P}$ (Hdg 260M, planned track 250 M )
b. $\quad 5^{\circ}$ right (aircraft has travelled 12 minutes at $180 \mathrm{kt}=36 \mathrm{NM} .3 \mathrm{NM}$ in $36 \mathrm{NM}=5^{\circ}$ error).
c. $\quad \mathrm{TMG}=255^{\circ}(\mathrm{M})$
d. $\quad 5^{\circ}$ port (left) (Heading 260 M, TMG 255 M )
e. $\quad 8^{\circ}$ left $\left(\mathrm{TE}=5^{\circ}, \mathrm{CA}=3^{\circ}\right)$
f. Heading $252^{\circ}(\mathrm{M})$
g. $\quad 10^{\circ}$ left $(2 \times \mathrm{TE}=2 \times 5=10)$
h. Heading $250^{\circ}(\mathrm{M})$
i. Heading $255^{\circ}(\mathrm{M})$ (original planned heading corrected for track error).
j. ETA $1032 \mathrm{hrs}(1000-1012=36 \mathrm{NM} 1012-1024=36 \mathrm{NM})$
(Leaves 24 miles which takes $\frac{24}{36} \times 12=8 \mathrm{~min}$ )

## Chapter <br> 12

## Other Applications of the 1 in 60 Rule

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## Other Applications of the 1 in 60 Rule

The other main applications which may be examined are:

- Calculation of height on a glide slope.
- Calculation of rate of descent to maintain a glide slope.
- Calculation of change of speed on a glide slope.
- Calculation of distance off required track using radio aids such as VOR/DME.


## Height on a Glide Slope

To find a Rule of Thumb for calculating height on a glide slope, we can use the track error technique but view the situation in side elevation rather than plan. For convenience, the glide slope angle is referred to as $Z^{\circ}$. You need only to understand how to apply the formula, not how it is derived.


Figure 12.1
In Figure 12.1, suppose the range is 1 nautical mile. This is 6080 feet, but an acceptable approximation for the 1 in 60 rule is to call it 6000 feet. This introduces only about $1 \%$ error.

If $Z=1^{\circ}$
When range $=60$ feet, the height is 1 foot.
When range $=600$ feet, the height is 10 feet.
When range $=6000$ feet, the height is 100 feet.
If $Z=2^{\circ}$
When range $=60$ feet, the height is 2 feet.
When range $=600$ feet, the height is 20 feet.
When range $=6000$ feet, the height is 200 feet.
This gives the rule:
To make good a glide slope of $Z^{\circ}$, your rate of descent should be $(100 \times Z)$ ft per nautical mile:
$2.5^{\circ}$ glide slope $3^{\circ}$ glide slope $3.5^{\circ}$ glide slope
$5.5^{\circ}$ glide slope

250 ft per nautical mile 300 ft per nautical mile 350 ft per nautical mile 550 ft per nautical mile.

## Example 1

On a $\mathbf{3}^{\circ}$ glide slope at $4 \mathbf{N M}$ from touchdown.
Height (ft) $=3 \times 100 \times 4$
$=1200 \mathrm{ft}$

## Example 2

On a $5.5^{\circ}$ glide slope at 3 miles from touchdown.

```
Height (ft) = 5.5 \ 100 * 3
```

    \(=1650 \mathrm{ft}\)
    Note: $3^{\circ}$ glide slopes are the most common. $2.5^{\circ}$ glide slopes are often found at military airfields operating high speed jets. Glide slopes greater than $3^{\circ}$ are normally found when airfields are located near high terrain or high buildings. London City has an extremely steep glide slope of $5.5^{\circ}$.

In all problems, glide slopes will be given

## Rate of Descent (ROD)

The rate of descent (ROD) required to maintain a glide slope at a given speed can be estimated using the 1:60 rule.

The simple problem is to calculate the height of the aircraft when it is one minute from touchdown. After all, the aircraft has to lose this height in one minute. The range at one minute is given by ground speed divided by 60 .


Figure 12.2

So, if your ground speed is 60 knots, your range is one mile. On a $3^{\circ}$ glide slope, this means that your height is 300 feet. So your rate of descent needs to be 300 feet per minute.

If your ground speed is 120 knots, your range is two miles. On a $3^{\circ}$ glide slope, this means that your height is 600 feet. So your rate of descent needs to be 600 feet per minute.

ROD (in feet per minute) $=5 \times$ Ground speed (in knots) $\quad-\quad$ for a $\mathbf{3}^{\circ}$ glide slope only
If the glide slope is some angle other than $3^{\circ}$, the easiest way is to solve for a $3^{\circ}$ glide slope and then factor the answer by actual glide slope $\div 3^{\circ}$.

For example:
What rate of descent do you require to maintain a $4^{\circ}$ glide slope at a ground speed of 100 knots?

For a $3^{\circ}$ glide slope it would be $5 \times 100-500$ feet per minute.
For a $4^{\circ}$ glide slope, it would be $500 \times 4 / 3$, which is 667 feet per minute -670 for practical purposes.

## Change of Speed on a Glide Slope

Students may need also to calculate the effect on ROD of changing speed. Firstly, apply the following rules:

To maintain a glide slope:

## Decrease ground speed, decrease ROD

Increase ground speed, increase ROD
To calculate the amount of the change:
Change in ROD $=5 \times$ change in speed ( $3^{\circ}$ glide slopes only)
Note: the previous caveats regarding glide slopes other than $3^{\circ}$ still apply

## Example 1

You are on an ILS approach to London Heathrow using a $3^{\circ}$ glide slope. If your ground speed is 140 kt , what Rate of Descent (ROD) is required to maintain the glide slope?

For this problem, involving a $3^{\circ}$ glide slope, you may use the $5 \times$ ground speed estimate.
$R O D=5 \times 140=700 \mathrm{ft} / \mathrm{min}$.

## Example 2

You are approaching London City Airport on a glide slope of $5.5^{\circ}$ at a ground speed of 120 kt . What rate of descent is required to maintain the glide slope?

ROD $=5 \times 120 \times 5.5 / 3=1100$ feet per minute

## Example 3

Approaching London Heathrow ( $3^{\circ}$ glide slope), an aircraft decreases speed from 140 kt to 120 kt . What change in the ROD must be made to maintain the glide slope?

## Decrease speed = Decrease ROD

Again, for a $3^{\circ}$ glide slope only, the $5 \times$ rule can be used.
$\begin{aligned} \text { Change in ROD } & =\quad 5 \times \text { change in ground speed } \quad\left(3^{\circ} \text { glide slopes only) }\right. \\ & =5 \times 20 \\ & =100 \mathrm{ft} / \mathrm{min}\end{aligned}$
Decrease ROD by $100 \mathrm{ft} / \mathrm{min}$ (this links with Example 1 and would result in a new ROD of 600 $\mathrm{ft} / \mathrm{min}$ ).

## Example 4

Approaching London City airport (glide slope $5.5^{\circ}$ ), you reduce ground speed from 120 kt to 110 kt . What change should you make to your ROD to maintain the glide slope?

Decrease speed = Decrease ROD
For a $3^{\circ}$ glide slope.
Change in ROD $=5 \times$ ground speed change $=5 \times 10=50 \mathrm{fpm}$
For a $5.5^{\circ}$ glide slope.
Change in $R O D=50 \times 5.5 / 3=92 \mathrm{ft} / \mathrm{min}$

## VOR/DME Problems

A typical examination example:
You are flying along an airway to VOR/DME 'Q'; the airway QDM is $271^{\circ}(\mathrm{M})$. Your Radio Magnetic Indicator indicates your QDM to Q as $266^{\circ}(\mathrm{M})$, range 48 NM. How far are you off the airway centre line and to which side?


Figure 12.3 VOR crosstrack problem

Again, we can use the Track Error formula.

$$
\begin{aligned}
\text { Track Error } & =\frac{\text { Distance off } \times 60}{\text { Distance gone }} \text { and modifying the formula } \\
\text { Angle off (TE) } & =\frac{\text { Distance off }(\mathrm{DO}) \times 60}{\text { DME range }(\mathrm{DG})} \\
5^{\circ} & =\frac{\text { DO } \times 60}{48} \\
\text { DO } & =\frac{5 \times 48}{60} \quad 4 \mathrm{NM} \text { to the right of centre line. }
\end{aligned}
$$

Note: you are off track but still within the airway because most airways are 10 NM wide, i.e. 5 NM either side of centre line.

## Finding Range from Change of VOR Bearing

It is possible to use the 1 in 60 rule to work out range from a VOR using the change in VOR bearing.

Suppose an aircraft is tracking $090^{\circ}(\mathrm{M})$ at 180 knots G/S and at 1100 hrs the QDM to the VOR is $002^{\circ}$ (or the QDR from it is $182^{\circ}$ ). Five minutes later the QDM to the VOR has changed to $357^{\circ}$ (or the QDR from it is $177^{\circ}$ ). What is the range $R$ from the VOR to the aircraft (at the closest point of approach)?


Figure 12.4 Range from flying abeam a VOR
The closest point of approach occurs as the aircraft is perpendicular to track (i.e. at a relative bearing of $270^{\circ}$, or when the QDM is $000^{\circ}$ ). You can treat this as 2 separate right-angled triangles of $2^{\circ}$ and $3^{\circ}$ at the top, but it is easier to do both sums at once and treat it as a near-right-angled triangle with a $5^{\circ}$ angle at the top.

The aircraft has a G/S of 180 knots. Five minutes flight gives a ground distance of 15 NM between the first bearing and the second. Therefore an angle of $5^{\circ}$ subtends a distance of 15 NM along track.

For each 60 NM of range $R$, the angle will subtend 5 NM of range. The range must therefore be equal to 3 times $60=180 \mathrm{NM}$.

Alternatively, use the formula:

$$
\begin{aligned}
& \mathrm{Z} \\
& =\frac{\text { opposite (15 NM along track) }}{\text { Range } \mathrm{R}} \times 60 \\
& 5=\frac{15}{\mathrm{R}} \times 60
\end{aligned}
$$

re-arranging:

$$
R=\frac{15}{5} \times 60=180 \mathrm{NM}
$$

## Questions

1. An aircraft is flying due South. At 1000 hrs , point $P$ bears $267^{\circ}(\mathrm{T})$ from the aircraft. At 1006 hrs, point $P$ bears $275^{\circ}(T)$ from the aircraft. If the aircraft has a ground speed of 120 kt, estimate the range of the aircraft from point $P$.
2. You are approaching Innsbruck, Austria on a glide slope of $3.5^{\circ}$. What height (QFE) should you be at 2 miles range?
3. You are approaching Rota, Spain on runway 28 which has a glide slope of $2.6^{\circ}$. At what height should you be at 4 miles range?
4. You are approaching Paris/Charles de Gaulle on a glide slope of $3^{\circ}$. At what height (QFE) should you be at a range of 2 NM?
5. Using the detail in questions 2-4, what rates of descent (ROD) are required to maintain the glide slopes in:
a. question 2 if ground speed is $\mathbf{1 2 0} \mathbf{k t}$
b. question 3 if ground speed is 180 kt
c. question 4 if ground speed is 150 kt.
6. On the approach to London Heathrow runway 27, glide slope $3^{\circ}$, you reduce speed from 150 kt to 120 kt. What change should you make to your ROD to maintain glide slope?
7. You are flying an aircraft into Gioia Del Colle, Italy, on a glide slope of $2.5^{\circ}$. Having had a hydraulic failure, you cannot select any flaps. You approach at 220 kt TAS and have a headwind component of 10 kt.
a. What rate of descent do you need to maintain the glide slope?

On the approach, you regain some hydraulic power and can select 'mid' flaps position. This enables you to reduce your approach speed to 190 kt TAS.
b. What change in the ROD is required to continue to maintain the glide slope?
c. What is your new ROD?
8. You are flying an airway with a centre line QDM of $137^{\circ}(\mathrm{M})$ towards VOR/DME ' $\mathrm{A}^{\prime}$. Your RMI reads $141^{\circ}(\mathrm{M}) / D M E 90$ NM.
a. Are you left or right of centre line?
b. What is your distance off the airway centre line?
c. Are you in trouble with ATC? (Airways normally extend 5 NM either side of the centre line).

## Answers

1. 90 NM
2. $700 \mathrm{ft}(2 \times 350)$
3. $\quad 1040 \mathrm{ft}(4 \times 260)$
4. $600 \mathrm{ft}(2 \times 300)$
5. a. $\quad 700 \mathrm{ft} / \mathrm{min}$
b. $\quad 780 \mathrm{ft} / \mathrm{min}$
c. $\quad 750 \mathrm{ft} / \mathrm{min}$
6. Decrease ROD by $150 \mathrm{ft} / \mathrm{min}$
7. This is the same basic question as used in Qs 5 \& 6 .

For ROD, you need ground speed which is $220 \mathrm{kt}-10 \mathrm{kt}$ headwind $=210 \mathrm{kt}$. For change in ROD, you only need the change in TAS.
a. $\quad 875 \mathrm{ft} / \mathrm{min}$
b. Decrease ROD by $125 \mathrm{ft} / \mathrm{min}$
c. $\quad 750 \mathrm{ft} / \mathrm{min}$
8. a. left
b. $\quad 6$ NM
c. Usually, yes. (Airways are generally only 5 NM wide from centre line in most countries, but rules may vary).

## Chapter <br> 13

## Topographical Maps and Map Reading

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## Introduction

Modern aircraft navigation systems tend to be very reliable and accurate. However, they do fail occasionally or suffer degraded accuracy. Moreover, many light aircraft are not equipped with sophisticated navigation systems. There is, therefore, a need for pilots to develop and maintain visual navigation skills using the basic aids of map/compass/stopwatch/eye and brain. It is essential that pilots can interpret the conventional symbols (the legend) of all the aeronautical charts they use. A set of legend decodes is included at the end of this chapter.

Aeronautical charts show 3 basic characteristics:

- Relief (Terrain)
- Other Features
- Scale


## Relief (Terrain)

If an aircraft is to be navigated safely in poor weather, great care must be taken to avoid high ground, either by flying round it or over it at a safe altitude. However, terrain features can make excellent visual references in good weather.

Pilots should therefore understand how these features are depicted.
Information concerning the location and height of high ground is given on maps in various ways.

Contours. Lines drawn joining all places which are at the same elevation above mean sea level are called contours and the intervals at which they are drawn can be read from the chart. Always check whether the contours are depicted in feet or metres. Contours which are close together indicate steeply sloping ground whereas widely spaced contours indicate gently sloping ground.

Layer Tinting (Colour Gradient). A different colour is used for ground in different elevation bands. For example, land between mean sea level and 250 feet may be coloured white, between 250 feet and 500 feet light yellow, and so on. A key to colour code is given in the margin of the map.

It is important to realize that both contours and layer tinting (which is effectively contours with the spaces between them coloured in) are based on elevation, i.e. vertical distance above mean sea level.

Spot Elevations. The elevations of prominent peaks are shown in figures. For easy recognition the highest spot height is printed on a white rectangle which has a black perimeter, and its position is given in the margin. Others are simply a dot, for the exact position, with the elevation printed alongside.

Maximum Elevation Figure (MEF) or Minimum Safe Altitude (MSA). Most Governmentproduced topographical charts (for instance, the CAA charts and the US Government TPC and ONC series) show Maximum Elevation Figures (MEF). Many commercially-produced charts (such as Jeppesen's) show Minimum Safe Altitude (MSA) - which may also be known as Grid Minimum Off-Route Altitude (Grid MORA).

It is important that you check which is shown on the chart you are using. The difference may, quite literally, be a matter of life and death.

Whether MEF or MSA, the basic calculation of the dominant obstacle is the same. The chart is divided up into convenient-sized rectangles of latitude and longitude for the scale of the chart. On the CAA and TPC 1:500 000 charts the rectangles are half a degree of latitude and longitude, on the ONC every whole degree, on some of the Jeppesen small-scale charts every 5 degrees, and so on. Within the rectangle, the highest natural feature (mountain, hill, ridge) is noted. The highest man-made obstruction (TV transmitter mast, crane, tall building, etc) is also noted. Every structure of a height AGL of 300 feet (UK and US) or 100 metres ( 328 feet - continental Europe) or more is required by law in the appropriate country to be notified to the national aviation authority (CAA, FAA, etc). Therefore there may be unknown structures of less than 300 feet (or 100 metres). In order to take account of these possible unknowns, the following rules are applied:

- For a made-made obstruction, the elevation is accurately known, so no further allowance is necessary.
- For a natural feature, someone may have built an obstruction of up to 300 feet ( 100 metres) on top of a hill quite legally without informing the CAA. Therefore a safety factor of 300 feet ( 100 metres) is added to the elevation of natural features.

The higher of the two is taken and then rounded up to the next whole hundred feet, i.e. 968 feet becomes 1000 feet.

Consider an example. Within a particular rectangle there is a TV mast of elevation 1432' and a hill of $1268^{\prime}$. Add $300^{\prime}$ to the hill; it becomes $1568^{\prime}$. Round this up to $\mathbf{1 6 0 0}^{\prime}$.

For a chart displaying Maximum Elevation Features (MEF), the figures 16 would appear in the rectangle.

If the chart shows Minimum Safe Altitude (MSA) - also known as 'Safety Altitude', a further safety factor is added. If the rounded-up figure is 5000 feet or more, 2000 feet is added.

If it is less than 5000 feet, then 1000 feet is added. In the example shown above, the MSA would become 26 .

Note that CAA charts, TPCs and ONCs all show MEFs, not Safety Altitudes. THE MEF IS NOT A SAFETY ALTITUDE.

## Other Features

- Water Whether sea, lake or river, the colour used is blue.
- Woods Shown in green.
- Roads Shown as red lines.
- Railways Shown as black lines.

An exercise to consolidate knowledge of chart symbols is given at the end of this Chapter.
In addition to understanding chart symbols, the pilot will need to measure distances on the chart. In many cases, the use of an appropriate navigation ruler will suffice but the pilot should be sure in this case that the chart being used is constant scale. In any event, the pilot should know the various ways that chart scale is depicted.

## Scale

Scale is the relationship between the length of a line drawn between two positions on a chart and the distance on the Earth between the same points.

Three methods of expressing scale are common:
Statement in words. This is self-explanatory. The statement "One inch to ten nautical miles" means that a line one inch long on the chart represents a line ten nautical miles long on the Earth.

Representative Fraction. This is a statement in words put into mathematical form and is calculated from the equation:

Scale $=\frac{\text { Chart Length }}{\text { Earth Distance }}$
By convention, the chart length is always 1 and the dimensions above and below the line are the same and thus cancel each other to make the fraction dimensionless, i.e. representative. Thus on the ICAO 1:500 000 topo, 1 cm on the chart represents 500000 cm on the Earth and 1 inch on the chart represents 500000 inches on the Earth.
"One inch to ten nautical miles" becomes a representative fraction of 1:729 600
or

$$
\frac{1}{729600}
$$

Graduated Scale Line. Graduated scale lines are normally depicted at the bottom of the chart as in Figure 13.1 and may be in nautical miles, statute miles or kilometres. Since one degree of latitude is equal to 60 nautical miles, the provision of a latitude scale in effect provides a graduated scale line as is shown in Figure 13.2.


Figure 13.1


Figure 13.2 Latitude scale.

## Map Reading

Map reading can be defined as visualizing the physical features of the ground as represented on the map by symbols and forming a mental picture of the ground by relating the features one to another. This is known as reading map-to-ground. This is the basic technique used when confident that you are near track and near on-time. There is a reverse process used when less certain of position called reading ground-to-map which involves seeing an arrangement of features on the ground and being able to find those features on the map and thus locate oneself.

Visual navigation using map reading is a skill which needs practice and which will develop with experience. A number of factors can make the task easier and the most important of these factors are thorough flight planning and preflight route study. It is also important in flight to try to remain on track and time and, if off track, to regain track as soon as possible. But do not concentrate too much on navigation alone. It is equally important to keep a good look-out to reduce collision risks, to be aware of any potential weather problems, and to manage the aircraft systems.

## Visual Checkpoints

In the planning stage, the pilot will study the route and select visual checkpoints normally occurring at about 5 to 10 minute intervals. A visual checkpoint will be a significant ground feature which can be overflown or used as a reference to fly an appropriate distance away from. Ideally, the checkpoint should be recognizable from some distance away to allow for timely alterations of heading. Visual checkpoints do not need to be exactly on track; a good feature close to track is better than a poor feature on track.

## Selecting Visual Checkpoints

The ideal visual checkpoint will be recognizable from some distance away and have the following features:

Large. A large feature will be recognizable from some distance but may be too large to use for precise navigation. A good principle is to use part of a large feature as the checkpoint, e.g. if using Birmingham as a checkpoint, you will see Birmingham from some distance but choose a large motorway junction as your accurate checkpoint.

Unique. Be sure that the checkpoint chosen could not be mistaken for a similar checkpoint in the same area. For example, disused airfields make good checkpoints but in East Anglia there are so many disused airfields (from WWV2 USAF operations) that it is easy to make mistakes.

Vertical Extent. Features which have significant vertical extent, e.g. masts, mountains, should always be identified for safety reasons, but they do make good visual checkpoints particularly because they can be identified, in good visibility, at some distance. Thus, if you can see a radio mast 20 miles away and you know you need to fly 1 mile to the right of the mast and you are flying at 120 kt , you have solved the navigation problem for the next 10 minutes and can spend your time checking aircraft systems, keeping a good lookout and enjoying your flight. But the value of features with good vertical extent alone may diminish if you fly higher - at 5000 ft AGL, you may not easily see a 400 ft high mast.

Contrast. A visual checkpoint should contrast in colour or texture with its surroundings. Contrast changes with the seasons and with geographic location ,i.e. the contrast between features may be entirely different in winter and summer or in polar or desert regions. Remember also that you may have to map-read at night when the rules of contrast are significantly different. Then, unless there is a good level of moonlight, the checkpoints must be lit. Finally, serious flight safety risks occur in meteorological/geographic conditions of poor visual contrast leading to the possibility of visual illusions.

It is rare for a visual checkpoint to have all the attributes mentioned above and your selection of checkpoints for a flight will probably be a compromise. Much will depend on the altitude you intend to fly, the seasonal/meteorological conditions and the nature of the terrain over which you will be navigating. In the next section, the relative merits of various types of visual checkpoint are discussed but the section cannot provide a comprehensive guide - you, the pilot, must weigh your own selections against the ideal criteria.

## Some Typical Visual Checkpoints.

Coastlines. Coastlines generally provide excellent daytime visual checkpoints and, given reasonable moonlight, can also be used at night. Coastlines are rarely uniform for any great distance and precise navigation can normally be achieved by reference to estuaries, headlands, ports or lighthouses.

Water Features. Lakes, large rivers and canals normally show up well even taking into account seasonal variations such as floods and droughts.

Water features tend to occur in the lowest ground in a particular area and, when flying at low level, small water features may be hidden from view by surrounding high ground. Small rivers are often tree-lined which can help by providing contrast but can hinder the identification of a particular feature of the river.

Mountains/Hills. Mountains and hills can make excellent visual navigation features but flight in hilly/mountainous terrain must only be undertaken after due consideration has been given to safety aspects. In general, terrain features with good vertical extent are more useful for navigation at low altitudes (and also more dangerous). Isolated hills provide the best features whereas terrain which undulates fairly uniformly is difficult to use unless additional features are available. Valleys tend to be good visual navigation features as they can normally be identified from their orientation and the presence of other cultural features, e.g. roads, railways and towns, offer the opportunity of many precise checkpoints.

Towns/Villages. Never overfly built-up areas below 1000 ft AGL or below a height from which you could glide clear of the built-up area in the event of an engine failure. That said, towns and villages are useful as general indicators of position and may be used to locate more precise visual checkpoints. Towns are useful features at night because they are normally well-lit but the same is not true of villages.

Roads. Motorways (and major roads) can provide excellent check features by day and night. Even if there is no discernible checkpoint available on the feature, the feature itself is a line feature and can be used for a track check or a timing/ground speed check. Generally, precise checkpoints can be found at regular intervals along these features, e.g. crossroads, motorway junctions and service stations.

Minor roads make less reliable check features but can still be significant in areas of open country and barren mountainous terrain. A good general guideline is, the bigger the road, the better for navigation.

Railways. When flying near large towns or industrial areas it can be difficult to identify particular railway lines but in more open areas, railway lines are excellent visual line features and can provide precise checkpoints at junctions, stations or where there are conspicuous bridges. Railway lines are not normally visible at night.

Woods. The usefulness of woods as visual check features is variable. In general, the rule remains 'the bigger, the better'. Large woods often have distinctive shapes if flying above 1000 ft AGL, but their shapes are often difficult to recognize from lower levels particularly in hilly terrain. Also, woods change shape due to felling and planting programmes and this particularly applies to coniferous plantations (fir forests). Finally, woods are susceptible to seasonal contrast changes. Snow-covered trees do not contrast with snow-covered fields (but on the other hand, snow melts first on the trees and when this happens, shape and contrast are enhanced). Use woods with care.

Other Features. Many other features can provide excellent precise visual checkpoints: power stations or isolated factories can often be seen at great range and often emit steam or smoke which aids identification and also facilitates estimation of surface wind velocity. If you make use of this, remember that the wind veers and increases with height above the surface. Also remember that power stations are often located along rivers and there may be several power stations along a particular stretch of river. Airfields stand out well by day from heights above 1000 ft AGL but always be aware whether the airfield is active or not. Airfield identification beacons are readily visible at night.

Quarries stand out particularly well but check NOTAMS for blasting activity.
Monuments, follies and chalk cuttings are precise features but rarely lit.
Lighthouses are excellent features by day or night but they are not unique unless identified in relation to coastal features by day or by their correct light transmission characteristics by night.

## Some Visual Navigation Hazards

Some meteorological or geographic circumstances when a flight is being flown or navigated visually can lead to increased flight safety risks through the possible occurrence of visual illusions. These circumstances are dealt with in more detail in the Human Performance syllabus but are summarized here below:

In less developed areas such as deserts, jungles, tundra or snow fields usable features may be widely separated or even non-existent. Another problem could be that topographical maps of these areas may be incomplete.

In areas of snow and ice from horizon to horizon and where the sky consists of a uniform layer of clouds so that no shadows are cast, the horizon disappears causing Earth and sky to blend. In this situation and in 'white-outs' caused by blowing snow, distance and height above ground are virtually impossible to assess visually.

In conditions of poor contrast and particularly over snow covered ground, it is possible to focus on high ground well ahead of the aircraft and to fly the aircraft to cross that ground with the appropriate clearance. The poor contrast may mask intervening high ground over which your
clearance may be considerably less than planned. This situation is known as 'the hill in front of the hill' and is particularly dangerous when flying at lower altitudes.

Flying over a smooth water surface, snow or a smooth desert surface makes it extremely difficult to judge height. The difficulty can be exacerbated in the overland situation if there are no features with vertical extent, e.g. trees. In a desert or tundra landscape, it is easy to see bushes and scrub trees and imagine they are full-size trees when they may actually be only a few metres high.

At night when flying over dark areas of sea or land where there are scattered pinpoints of light (ships, oil rigs, small settlements or isolated buildings) it is possible to become disorientated and to confuse the scattered lights below with the scattered stars above. This situation may be exacerbated after or during aircraft manoeuvres.

## Navigation Techniques

Pilot navigation techniques and chart preparation requirements appropriate to visual navigation (including the 1:60 rule) are discussed in relevant flight exercise briefings.

A disciplined approach is always the most successful. When setting heading from a start point or when altering heading at a turning point, always check that you are flying the correct heading and that the compass is synchronized. Always check that you have the appropriate altimeter setting and are flying at a safe altitude. Always check that you have started/restarted your stopwatch, or you know how much you are running early/late on planned time.

Even the best navigators become occasionally uncertain of position. It is very tempting in this situation to carry on regardless. DON'T! Start immediately to try to resolve any uncertainties using all aids available. This is a critical phase of your flight and it is very common for pilots in this situation to over-concentrate on navigation and to forget to check fuel/aircraft systems. Check your fuel/aircraft systems and then follow a 'lost' procedure. If you are still uncertain when you are due at a turning point, turn on time (ETA), and obey the rules below.

## Lost Procedure

- Check compass/ASI to ensure you are flying planned headings/speeds.
- If meteorological/airspace restrictions do not apply, climb to increase your visual/radio horizon.
- Fix your position using all aids available (VOR/DME, VDF, radar fix, or visual fix).
- If restricted to visual (map reading) methods, calculate a DR position. (DR = 'Deduced Reckoning', commonly known as 'Dead Reckoning')
- The DR position should be deduced using the position/time of the last known accurate fix and the track/ground speed/time flown since that fix.
- Then, draw a 'circle of uncertainty' around the DR position. (The circle is normally drawn with a radius of $10 \%$ of the distance flown since the last fix). Remember that the DR position continues to move along track and the 'circle of uncertainty' gets larger with time). Now map-read from ground-to-map. Try to identify a large visual feature and locate the aircraft on the map with reference to this feature. On most occasions the aircraft position should lie within the circle of uncertainty.
- If still in difficulty, consider turning towards a good visual line feature outside the circle of uncertainty, e.g. coastline, motorway, main railway. On reaching this line feature, turn and follow it until you find a definite visual checkpoint from which proper navigation can be resumed.

Be careful that you do not fly towards high ground in poor weather below safety altitude or that you do not fly into controlled airspace without clearance. Always keep a good lookout, monitor your fuel and aircraft systems and remember that you can use the Emergency Fixing Service on 121.5 MHz (PAN call ).

## Chart Symbols




 Topographical Maps and Map Reading

| SYMBOLS FOR AERODROME/HELIPORT CHARTS |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 136 | Hard surface runway |  |  | 144 | Pierced steel plank or Steel mesh runway |  | ¢\#\#\#\#\#\#\# |
| 137 | Unpaved runway |  | $\square$ | 145 | Point light |  | $\bullet$ |
|  |  |  | $\bigcirc$ |  |  |  |
| 138 | Stopway | SWY |  |  | 146 | Obstacle light |  | 米 |
| 139 | Taxiways and parking areas |  | $\sqrt{\square}$ | 147 | Landing direction indicator (lighted) |  | T |
| 140 |  |  | (H) | 148 | Landing direction indicator (unlighted) |  | T |
|  | Helicopter alighting area on an aerodrome |  |  |  |  |  |  |
| 141 | Aerodrome reference point | ARP | ¢ | 149 | Stop bar |  | ... |
|  |  |  |  | 150 | Taxi holding position | Pattern A | \#n |
| 142 | VOR check-point |  | * |  |  |  |  |
| 143 | Runway visual range (RVR) observation site |  | $\checkmark$ |  | Note.- For | , Volume I | graph 5.2.9 |

SYMBOLS FOR AERODROME OBSTACLE CHARTS - TYPE A, B AND C

|  |  | Plan | Profile |
| :---: | :---: | :---: | :---: |
| 151 | Tree or shrub | * | Identification number |
| 152 | Pole, tower, spire, antenna, etc. | - |  |
| 153 | Building or large structure | ■ |  |
| 154 | Railroad | $\pm+$ |  |
| 155 | Transmission line or overhead cable | -T-T- |  |


|  |  |  | Plan | Profile |
| :---: | :---: | :---: | :---: | :---: |
| 156 | Terrain penetrating obstacle plane |  |  |  |
| 157 | Escarpment |  | แ!!!"! |  |
| 158 | Stopway | SWY |  |  |
| 159 | Clearway | CWY |  |  |

## Questions

1. You are flying a VFR route and have become uncertain of your position. Which is the best course of action?
a. set heading towards a line feature - coastline, river or motorway
b. turn round and fly your flight plan tracks in reverse until you see something you recognised before
c. fly a series of ever-expanding circles from your present position till you find your next checkpoint
d. turn round and fly your flight plan in reverse back to base
2. Please refer to Appendix A (Page 233). What is the symbol for an unlighted obstacle?
a. $\quad 9$
b. $\quad 10$
c. $\quad 12$
d. $\quad 15$
3. Using the Jeppesen $\mathrm{E}(\mathrm{LO}) 1$, position 5211 N 00931 W , which of the following denotes all the symbols?
a. military airport, ILS, NDB
b. civil airport, VOR, ILS
c. military airport, VOR, ILS
d. civil airport, ILS, NDB
4. Using the Jeppesen $\mathrm{E}(\mathrm{LO}) 1$ chart, what are the symbols at Galway Carnmore (5318.1N 00856.5W)?
a. VOR, NDB, DME, compulsory reporting point
b. civil airport, NDB, DME, non-compulsory reporting point
c. civil airport, VOR, DME, non-compulsory reporting point
d. VOR, NDB, DME, non-compulsory reporting point

For the following questions please refer to Appendix A.
5. Which of the following is the symbol for an exceptionally high (over 1000 feet AGL) lighted obstruction?
a. 6
b. $\quad 9$
c. $\quad 10$
d. 15
6. What symbol is used to show a VORTAC on a map/chart?
a. $\quad 5$
b. 7
c. $\quad 13$
d. $\quad 14$
7. Which is the symbol for a VOR?
a. 4
b. 5
c. $\quad 13$
d. 14
8. What does symbol 3 represent?
a. lit obstacle
b. lighthouse
c. VRP
d. aeronautical ground light
9. An aircraft flying VFR using visual navigation crosses two parallel roads at right angles to the track. For what purpose could that information be used?
a. Tracking check
b. Ground speed check
c. Heading check
d. Deviation check
10. Which of the following would be most useful as a visual checkpoint when planning a flight?
a. A small copse
b. A large wood
c. A large motorway junction
d. A bend in a river

Appendix A to Questions


Answers

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a | d | d | b | c | d | c | d | b | c |

## Chapter <br> 14

## Convergency and Conversion Angle

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## Introduction

Meridians are great semi-circles joining the poles. From this it follows that meridians converge as they run towards the nearer pole, and diverge as they run away from the nearer pole.

Figure 14.1 demonstrates this. Starting at the North Pole, it can be seen that the two meridians ' $x$ ' and ' $y$ ' diverge until they cross the Equator, where they are parallel to each other. As they continue into the Southern Hemisphere they begin to converge, finally meeting at the South Pole.


Figure 14.1 Diverging, parallel and converging meridians

It is reasonably easy to see that two meridians, say, 60 degrees of longitude apart, must be parallel to each other at the Equator.


Figure 14.2 Parallel at the Equator

At the Pole, however, they will converge at an angle exactly equal to their difference in longitude - in this case, $60^{\circ}$. There are $360^{\circ}$ of longitude $-180^{\circ} \mathrm{E}$ and $180^{\circ} \mathrm{W}$, and a full revolution looking down vertically on the pole is also $360^{\circ}$. Therefore the angle at the pole is the difference in longitude.


Figure 14.3 Maximum convergence at Pole
It also follows that at some intermediate latitude, there will be an intermediate amount of convergence. The angle of inclination of the 2 meridians will be more than zero (the value at the Equator), but less than $60^{\circ}$ (the value at the Pole).


Figure 14.4 Intermediate convergence

## Convergency

This angle is known as Convergency or as Earth Convergence. Convergency is defined as the angle of inclination between two selected meridians measured at a given latitude.


Figure 14.5 Converging and diverging meridians
Figure 14.5 shows our 3-dimensional problem as a 2-D drawing. However, you should have been able to see from the 3-D drawings of the spherical Earth why the angle at the Poles is the change in longitude.

In Figure 14.5 above:

| Position | Latitude | Change of <br> Longitude | Convergency | $\frac{\text { Convergency }}{\text { Ch.long }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Equator | $0^{\circ}$ | $60^{\circ}$ | $0^{\circ}$ | 0 |
| Pole | $90^{\circ}$ | $60^{\circ}$ | $60^{\circ}$ | 1.0 |

At the Equator, where the latitude is $0^{\circ}$, (convergency/ch.long) is zero. At the Poles, where the latitude is $90^{\circ}$, (convergency/ch.long) is a factor of one

Therefore the relationship between convergency and latitude is a function of the sine of the latitude.

This gives us the formula:

```
Convergency = Change in Longitude }\times\mathrm{ Sine Latitude
```



Figure 14.6 Convergency

In Figure 14.6 the latitude chosen is $30^{\circ} \mathrm{N}$, and tangents have been drawn to each meridian at this latitude. The angle between the slope of the two tangents is convergency.

In this example, the change of longitude is $40^{\circ} . \quad\left(\sin 30^{\circ}=0.5\right)$
Therefore:

$$
\begin{aligned}
\text { convergency } & =40^{\circ} \times 0.5 \\
& =20^{\circ}
\end{aligned}
$$

We now have a formula for calculating convergency between any 2 meridians at the same latitude. However, it may not be immediately obvious why a pilot should wish to know what the convergency between 2 meridians at any particular latitude is.

## Convergency and the Great Circle Track

At any point on the Earth, True North is defined with reference to the direction of the local meridian. If you are at a different meridian in flight from the point at which you started, your local direction of True North has changed. This is the reason why a rhumb line is a continuously curved line on the Earth's surface. If it did not constantly curve, it could not cut all the meridians (which are at different angles to each other) at the same angle.

If you are flying a great circle track (all automatic computing systems, such as Inertial Navigation Systems, Global Positioning Systems, and Flight Management Computers do, in fact, compute steering signals along great circle tracks), then your great circle track will alter over a period of time with the respect to True North by exactly the amount of convergency. The track direction is defined by reference to the meridians, and if the direction of the meridians changes, then the track direction changes.

The change in great circle track direction is the angle of inclination of the meridians at the 2 points where the track is measured.

Figure 14.7 shows a great circle cutting two meridians, X and Y . Tangents to the meridians have been drawn at the points where the great circle cuts them, and convergency is shown in the same way as before.

The great circle direction at meridian $X$ is represented by the angle ' $a$ ' and the great circle direction at meridian $Y$ by the angle ' $b$ '. The difference between the two is ' $b$ '-' $a$ ' = convergency.


Figure 14.7 Convergency as the change in great circle direction
In this case, however, the meridians are not crossed by the great circle at the same latitude. Indeed, there are only 2 mean track directions (due East or due West) where they will be at the same latitude. On any other mean track, by definition, there will be some element of north or south latitude change between the meridians.

In this case, we cannot use the simple convergency formula because there is no one term for 'sine latitude'. The calculation requires the sine of the mean latitude between the two points. However, determining the mean latitude is a complex process as it will be closer to the nearer pole than the mid-latitude (with 2 exceptions). Hence, although the formula requires mean latitude, for simplicity we use mid-latitude. The difference between the sine of mid-latitude and the sine of mean latitude is so small that it is virtually insignificant unless large change of longitude is involved (which is unlikely). To summarize, convergency is the angle of inclination between two selected meridians measured at a given latitude and is equal to the difference between the great circle directions measured at each meridian. Its value may be calculated from the formula:-

## Convergency $=$ Change in Longitude $\times$ Sine Mean Latitude

## Practical Examples

Let us examine how this works in practice (and also learn how to answer typical exam questions) by looking at a worked example.

## Question 1

The initial great circle track from $A(4000 N 00200 \mathrm{~W})$ to $B(5000 \mathrm{~N} 01000 \mathrm{E})$ is $060^{\circ}(\mathrm{T})$. What is the initial great circle track from $B$ to $A$ ?

Start by drawing a diagram. It is only a sketch, to get the situation into your mind - not a scale drawing. You are in the Northern hemisphere. So draw a couple of meridians converging northwards, like this:


Figure 14.8 Meridians converging northwards
The initial great circle track from one of them is $060^{\circ}(T)$. So make it the left-hand one, because if you draw it from the right hand one, the bearing goes off the diagram, like this:


Figure 14.9 Incorrect diagram - so choose the other one instead


Figure 14.10 Meridians converging northwards

This means that the left-hand intersection must be $A$ and the right-hand one must be $B$. (The question says "The initial great circle track from A is 060(T).")

Check it out by reference to the latitude and longitude.
If we have it right:


Figure 14.11 Checking the diagram for sense
With an initial track of $060^{\circ}(\mathrm{T})$ at $A, B$ should be north of $A$ (it is: $B$ is at $50 \mathrm{~N}, A$ is at 40 N ) and $B$ should be east of $A$ (it is: $B$ is at $010 \mathrm{E}, A$ is at 002 W ).

Now that we have the diagram right, we can calculate the convergency:
Convergency $=$ Change in Longitude $\times$ Sine Mean Latitude
The change in longitude is $12^{\circ}$ (from 002 W to 010 E ).
The mean latitude is 45 N (halfway between 40 N and 50 N ). Sine $45^{\circ}$ is .7071 .
Convergency $=$ Change in Longitude $\times$ Sine Mean Latitude

$$
=\quad 12 \times .7071
$$

Convergency $=8.5$

Now, there are 2 ways to solve this problem, depending on which definition of convergency you prefer.

If you use the definition that:
convergency $=$ change in great circle track between 2 meridians, then the great circle track has changed 8.5 degrees between $A$ and $B$.

It left A on a track of $060^{\circ}$, so using the rules given by the D-I-I-D diagram in chapter 2 :
Travelling eastwards in the Northern hemisphere, the track will increase. The track angle will be $068.5^{\circ}$ as it passes through $B$. To find the track from $B$ to $A$, as it passes through $B$, take the reciprocal, $248.5^{\circ}$

If you prefer the definition that:
convergency $=$ angle of inclination between 2 meridians, then start like this:


Figure 14.12 Meridians converging northwards

Parallel the meridian at A over to B :
Angle M is $8.5^{\circ}$ (the inclination of the meridians). Angle N is $060^{\circ}$ (corresponding angles between meridians in classical geometry).

Therefore the total of $\mathrm{M}+\mathrm{N}$ is $068.5^{\circ}$.


Figure 14.13 Parallel the meridian

The meridian at $B$ defines True North at that point. So the track angle of the continuation of the line $A B$ is $068.5^{\circ}$ at $B$.

Therefore the great circle bearing of A from B, measured at B, is $248.5^{\circ}$
Now try the following:

## Question 2

The initial great circle track from C (3600N 01500E) to D (latitude 4200 N$)$ is $300^{\circ}(\mathrm{T})$ and the final great circle track at D is $295^{\circ}(\mathrm{T})$.
a. What is the longitude of $D$ ?
b. What is the approximate great circle track direction at longitude 01100E?

## Question 3

The initial great circle track from $\mathrm{H}(4000 \mathrm{~S} 17000 \mathrm{~W})$ to $\mathrm{G}(4500 \mathrm{~S} 17400 \mathrm{E})$ is $250^{\circ}(\mathrm{T})$. What is the initial great circle track from G to H ?

## Answers

## Answer to Question 2

a. As the track has changed from $300^{\circ}(\mathrm{T})$ to $295^{\circ}(\mathrm{T})$, the convergence must be $5^{\circ}$.

The question tells us that C is at $36^{\circ} \mathrm{N}$ and D is at $42^{\circ} \mathrm{N}$, so the mean latitude must be $39^{\circ} \mathrm{N}$.
Now, substituting in to the equation:
convergency $=$ ch.long $\times$ sin mid lat
$5^{\circ}=$ ch.long $\times \sin 39^{\circ}$
Change in longitude $=\frac{5^{\circ}}{\sin 39}=8^{\circ}$


Figure 14.14 The basic diagram
If the change in longitude is $8^{\circ}$ and $C$ is at longitude $015^{\circ} E$ and $D$ is west of $C$, then the longitude of $D$ is 00700 E .
b. Longitude 01100E is halfway between 01500E and 00700E. Therefore the great circle track will be halfway between $300^{\circ}(\mathrm{T})$ and $295^{\circ}(\mathrm{T})$, i.e. $297.5^{\circ}(\mathrm{T})$


Figure 14.15 Mid-longitude

## Answer to Question 3

Southern hemisphere this time. Draw in an initial great circle track of $250^{\circ}(\mathrm{T})$. It must be from the right-hand side of the diagram if it is going to cut the other one. Therefore the right-hand one must be H and the left-hand one must be G .


Figure 14.16 Meridians converging southwards
Just to be 100\% certain we have it right, check the latitudes and longitudes. The question tells us that H is $40 \mathrm{~S}, \mathrm{G}$ is 45 S . So G should be south of H on our diagram; it is. H is 170 W and G is 174E. As we are crossing the Greenwich anti-meridian (180E/W), left and right are the right way round on this diagram - confusing, but correct. Check the explanation in chapter 1 if clarification is required.

```
convergency = ch.long }\times\mathrm{ sin mid lat
    =16
```

Therefore, as before - either say that the convergency is the change in great circle track angle, so the track angle at $G$ is $260.8^{\circ}(\mathrm{T})$ going westwards, therefore the reciprocal at $G$ is $080.8^{\circ}(\mathrm{T})$.

Or, say that convergency is the angle of inclination of the meridians. Draw in a parallel construction line, as follows:

The reciprocal track from the parallel construction line from G to H is $070^{\circ}$. The convergency between the parallel construction line and the meridian at $G$ is $10.8^{\circ}$. Therefore the track angle from True North (i.e. the meridian) at G back to H is $080.8^{\circ}$.


Figure 14.17 Drawing in a parallel

## Conversion Angle

Consider the following situation:
(In this example, the drawing has been done as a Mercator-type projection, with the meridians shown as parallel lines, even though they are in fact convergent on the Earth. This is to make it clear that the Rhumb Line track does not change track angle but the great circle does.


Figure 14.18 Rhumb line and great circle

In Figure 14.18, we have 2 meridians 40 degrees of longitude apart. Now consider both the great circle and the Rhumb Lines running from A (30N 020W) to B (30N 020E).

The Rhumb Line (shown in red) is the parallel of latitude of 30 N and therefore has a constant track direction of $090^{\circ}(\mathrm{T})$ at any point along it.

However, the great circle changes direction. The diagram shows that it leaves A on a track angle of less than $090^{\circ}(\mathrm{T})$ and crosses B on a track angle of more than $090^{\circ}(\mathrm{T})$.

We can calculate the amount that it changes direction, as follows:-

```
convergency = ch.long }\times\mathrm{ sine mid lat
    = 40' change of longitude }\times\mathrm{ sine 30
    = 20
```

So, whatever direction it left A on, its track angle will be $20^{\circ}$ greater at B . Now, the diagram is completely symmetrical about the mid-latitude ( $0^{\circ} \mathrm{E} / \mathrm{W}$, in this example). We know that the Rhumb Line track angle is $090^{\circ}(\mathrm{T})$ all the way, which is also the direction of the great circle track at mid-longitude. Therefore the great circle track must have left on a track of $080^{\circ}(\mathrm{T})$ at A and passed through B on a track of $100^{\circ}(\mathrm{T})$.

The difference between great circle direction and Rhumb Line direction joining two given points is called conversion angle

Conversion angle, in our example, is $10^{\circ}$. At A, it is the difference between $080^{\circ}(\mathrm{T})$ (the great circle track) and $090^{\circ}(\mathrm{T})$ (the Rhumb Line track). At B, it is the difference between $100^{\circ}(\mathrm{T})$ (the great circle track) and $090^{\circ}(\mathrm{T})$ (the Rhumb Line track). Conversion angle is always the same at each end of the 2 points connected by the great circle and the Rhumb Line tracks.

This leads to the following definition and formulae:


Figure 14.19 Conversion angle
Conversion angle is the difference between great circle direction and Rhumb Line direction joining two given points.

## Conversion angle is $1 / 2$ convergency.

Conversion angle $=1 / 2$ change of longitude $\times$ sin mean latitude


Figure 14.20 Same situation-different projection
It is equally possible to draw the diagram with convergent meridians. In this case, the great circle appears as a straight line, even though its track angle changes direction, and the Rhumb Line appears as a curve, even though it is a line of constant True direction. However, the great circle still runs nearer the near pole, exactly as on the globe.

## More Practical Examples

## Question 4

a. Determine the value of convergency between $J(5812 \mathrm{~N} 00400 \mathrm{~W})$ and $K$ (5812N 00600E).
b. What is the rhumb line track from J to K?
c. $\quad$ What is the initial great circle track from K to J ?

## Question 5

An aircraft is planned to fly a Great Circle track via the following turning points- $53^{\circ} \mathrm{N}$ $030^{\circ} \mathrm{W}$ to $53^{\circ} \mathrm{N} 020^{\circ} \mathrm{W}$ to $53^{\circ} \mathrm{N} 010^{\circ} \mathrm{W}$. The track change on passing the second turning point will be approximately
a. an $8^{\circ}$ increase
b. a $4^{\circ}$ decrease
c. zero
d. an $8^{\circ}$ decrease

## Question 6

Given that:
A is $\mathrm{N} 55^{\circ} \mathrm{E} / \mathrm{W} 000^{\circ}$
B is $\mathrm{N} 54^{\circ} \mathrm{E} 010^{\circ}$, if the initial true great circle track from A to B is $100^{\circ}(\mathrm{T})$, what is the true rhumb line track at $A$ ?
a. $\quad 096^{\circ}(\mathrm{T})$
b. $\quad 107^{\circ}(\mathrm{T})$
c. $\quad 104^{\circ}(\mathrm{T})$
d. $\quad 100^{\circ}(\mathrm{T})$

## Answer to Question 4

## a. $\quad 8.5^{\circ}$ <br> b. $\quad 090^{\circ}(T)$ On same latitude so a Rhumb Line Track <br> c. $\quad 274.25^{\circ}$

## Answer to Question 5

The diagram looks like this:


Figure 14.21 INS waypoint changeover
The Rhumb Line track all the way from WP 1 to WP 3 is $090^{\circ}(\mathrm{T})$. However, the INS will steer the aircraft along the great circle track from WP 1 To WP 2, then change over at WP 2 to a continuous great circle track steer from WP 2 to WP 3. Therefore the great circle track will start off at less than $090^{\circ}(T)$ and end up on more than $090^{\circ}(T)$ on each of the 2 legs. At WP 2 there will be a left turn at the waypoint changeover. Each individual leg follows the D-I-I-D rules, i.e. the track angle increases, going easterly in the Northern hemisphere, but at the changeover, the result is a left turn.

Taking either leg, the change of longitude is 10 degrees and the mean latitude is 53 N .

```
Conversion angle = 1/2 change of longitude }\times\mathrm{ sin mean latitude
    = 1/2 }\times10\times0.
    = 4
```

Therefore the great circle track leaves WP 1 on $086^{\circ}(\mathrm{T})$ and approaches WP 2 on $094^{\circ}(\mathrm{T})$. At the waypoint changeover, the INS reverts to a track steer of $086^{\circ}(\mathrm{T})$ at the start of the second leg, which increases to $094^{\circ}(\mathrm{T})$ on approaching WP 3 . There is therefore an $8^{\circ}$ decrease of track angle on passing WP 2.

Answer (d).

## Answer to Question 6

Here is the diagram:-


Figure 14.22 Question 6 diagram

| Conversion angle | $=\quad 1 / 2$ change of longitude $\times$ sin mean latitude |
| ---: | :--- |
|  | $=1 / 2 \times 10 \times 0.814 .($ Mean latitude is 54.5 N$)$ |
|  | $=4^{\circ}$ |

IT IS NECESSARY TO APPLY THE CONVERSION ANGLE AT THE POSITION WHERE THE great circle DIRECTION IS MEASURED.

In this case, the GC track is $100^{\circ}(\mathrm{T})$ so, with a CA of $4^{\circ}$, the RL track will be $104^{\circ}(\mathrm{T})$.
Answer (c).
Radio waves travel over the surface of the Earth along great circle paths. It is often necessary for navigational purposes to convert the great circle direction to a rhumb line direction and this is done by applying conversion angle to the great circle direction to obtain the rhumb line direction.

## Questions

1. The convergency of the meridians through $\mathbf{M}$ and $\mathbf{N}$ which are in the Southern hemisphere is $12^{\circ}$.
If the rhumb line track from $M$ to $N$ is $249^{\circ}(T)$, what is the great circle track:
a. from M to N ?
b. from N to M ?
2. The great circle bearing of position $B$ in latitude $30^{\circ} 00^{\prime} S$ from position $A\left(30^{\circ} 00^{\prime} S\right.$ $165^{\circ} 00^{\prime} \mathrm{E}$ ) is $100^{\circ}(\mathrm{T})$.
What is:
a. The great circle track from $B$ to $A$ ?
b. The longitude of position B ?
3. The rhumb line from position $D\left(30^{\circ} 00^{\prime} \mathrm{N} 179^{\circ} 00^{\prime} \mathrm{W}\right)$ to position C is $090^{\circ}(\mathrm{T})$.

The initial great circle track from $C$ to $D$ is $287^{\circ}(T)$.
What is:
a. The great circle track from $D$ to $C$ ?
b. The approximate latitude and longitude of position C ?
4. The great circle track from $A$ to $B$ measures $227^{\circ}(T)$ at $A$ and $225^{\circ}(T)$ at $B$. What is the convergency of the meridians through $A$ and $B$ and in which hemisphere are they?
5.
a. In what latitude is the convergency between two meridians on the Earth equal to twice their convergency in latitude $20^{\circ} \mathrm{N}$ ?
b. Is there a latitude where the convergency would be three times the value at $20^{\circ} \mathrm{N}$ ?
6.
a. $\quad A$ and $B$ are in the same hemisphere.

The initial great circle track from $B$ to $A$ is $268^{\circ}(T)$ and the initial great circle track from $A$ to $B$ is $092^{\circ}(\mathrm{T})$.
i) In which hemisphere are A and B?
ii) What is the rhumb line track from $A$ to $B$ ?
b. $\quad C$ and $D$ are in the same hemisphere.

The initial great circle track from $C$ to $D$ is $063^{\circ}(T)$ and the rhumb line track from $D$ to $C$ is $240^{\circ}(\mathrm{T})$.
i) In which hemisphere are C and D?
ii) What is the approximate initial great circle track from $D$ to $C$ ?
7. Position $\mathrm{X} 64^{\circ} 00^{\prime} \mathrm{S} 11^{\circ} 50^{\prime} \mathrm{W}$. Position Y $64^{\circ} 00^{\prime} \mathrm{S} 05^{\circ} 10^{\prime} \mathrm{W}$. Give:
a. The convergency between the meridians of $X$ and $Y$.
b. The approximate initial great circle track from $Y$ to $X$.
c. The rhumb line track from $X$ to $Y$.
8.
a. Calculate the convergency between positions $\mathrm{A}\left(55^{\circ} 30^{\prime} \mathrm{N} 04^{\circ} 35^{\prime} \mathrm{W}\right)$ and B ( $64^{\circ} 00^{\prime} \mathrm{N} 22^{\circ} 37^{\prime} \mathrm{W}$ ).
b. If the rhumb line track from $A$ to $B$ is $313^{\circ}(T)$, what is the approximate initial great circle track from $B$ to $A$ ?
9. The initial great circle track from $B$ to $A$ is $245^{\circ}(T)$ and the rhumb line track from $A$ to $B$ is $060^{\circ}(\mathrm{T})$.

If the mean latitude between $A$ and $B$ is $53^{\circ}$ and the longitude of $B$ is $02^{\circ} 15^{\prime} E$, what is the longitude of A?
10. A and $B$ are both in the Southern hemisphere and the convergency of their meridians is $8^{\circ}$. The initial great circle track from $A$ to $B$ is $094^{\circ}(T)$.

If the position of $B$ is $23^{\circ} 00^{\prime} S 20^{\circ} 00^{\prime} \mathrm{W}$, what is the position of $A$ ?

## Answers

1. a. $243^{\circ}(\mathrm{T})$
b. $\quad 075^{\circ}(\mathrm{T})$
2. a. $260^{\circ}(\mathrm{T})$
b. $\quad 155^{\circ}$ West
3. a. $073^{\circ}(\mathrm{T})$
b. $\quad 30^{\circ} \mathrm{N} 111^{\circ} \mathrm{W}$
4. $\quad 2^{\circ}$ of Convergency and in Northern hemisphere
5. a. Latitude $43^{\circ}$ North
b. No, because you cannot have a latitude whose sine is greater than
1.00
6. a. i) Southern hemisphere
ii) $\quad 090^{\circ}(\mathrm{T})$, as $A$ and $B$ on same latitude
b. i) Southern hemisphere
ii) $\quad 237^{\circ}(\mathrm{T})$
7. a. $6^{\circ}$
b. $\quad 267^{\circ}(\mathrm{T})$
c. $090^{\circ}(\mathrm{T})$
8. a. $15.5^{\circ}$ Convergency; call it $16^{\circ}$
b. $\quad 125^{\circ}(\mathrm{T})$
9. $010^{\circ} 15^{\prime} \mathrm{W}$
10. Position A is at $23^{\circ} 00 \mathrm{~S} 040^{\circ} 30^{\prime} \mathrm{W}$. (Rhumb Line is $090^{\circ}(\mathrm{T})$ )

## Chapter <br> 15

## Departure

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15 Departure

## Definition of Departure

Departure is the distance between two meridians along a specified parallel of latitude, usually in nautical miles.

Consider Figure 15.2 below. You can see that our 2 meridians represent the same change of longitude, whatever their latitude. However, because they converge from the Equator to the Pole, the same change of longitude does not represent the same East- West distance in nautical miles.


Figure 15.1 Departure

If we take the Equator as our specified parallel of latitude, the distance between 2 meridians will be greater than at some higher latitude.

Figure 15.2 Effect of latitude

## 15

Departure is maximum at the Equator where $1^{\circ}$ change of longitude (ch.long) equals 60' of arc of a Great Circle.


Figure 15.3 Maximum at equator

Departure is zero at both poles because the meridians converge and meet at these two points.


Figure 15.4 Zero at Poles
Departure therefore varies as the cosine of the latitude.
It can be found by applying the formula:

## Departure (NM) = ch.long (in minutes) $\times$ cos lat

Since this distance is always along a parallel of latitude it represents a Rhumb Line distance.

## Calculation of Departure

Consider 2 meridians on the Earth, joined by a parallel of latitude, as in Figure 15.5. The diagram is reduced to a sketch in Figure 15.6, below:


Figure 15.5 Meridians and a parallel

In this example, the change of longitude is 20 degrees.

Multiply by 60 to convert it into minutes.
Then multiply by the cosine of $52^{\circ}$.
Departure (NM) = ch.long (min) $\times$ cos lat
$=\quad 20 \times 60 \times \cos 52^{\circ}$
$=738.8$ nautical miles


Figure 15.6 Departure sketch

## Types of Departure Questions

There are 2 main types of question on departure:

- Variations on the basic departure formula
- Given departure at one latitude, calculate it at another

These are set out in more detail below:-

## Variations on the Basic Departure Formula

Example 1 An aircraft at position 6000N 00522W flies 165 km due East. What is the new position?

Firstly, convert 165 km to NM. 165/1.852 = 89 NM.
Now substitute into the departure formula:

$$
\begin{aligned}
\text { Departure }(N M) & =\text { ch.long }(\mathrm{min}) \times \cos \text { lat } \\
89 & =\text { ch.long } \times \cos 60^{\circ}
\end{aligned}
$$

Therefore, ch.long $=89 / .5=178 \mathrm{~min}$ of longitude
This is $2^{\circ} 58^{\prime}$ East from an initial longitude of 00522W.
The new position is 6000 N 00224 W .
Example 2 In which latitude is a difference of longitude of $44^{\circ} 11^{\prime}$ equivalent to a departure of 2000 NM?
$44^{\circ} 11^{\prime}=2651$ minutes of arc

Departure (NM) $=$ ch.long (min) $\times$ cos lat

$$
2000=2651 \times \cos \text { lat }
$$

cos lat. $=\frac{\text { Dep }}{\text { ch.long }}$
$=\frac{2000}{2651}=0.7544$
$\therefore$ Latitude is $41^{\circ}$ North or South

## Given Departure at One Latitude, Find It at Another

## Example 3

An aircraft leaves position ' $G$ ' in Latitude $40^{\circ} \mathrm{S}$ and flies the following Rhumb Line tracks and distances:

G to H: $180^{\circ}$ (T) 240 NM H to J: $270^{\circ}$ (T) 240 NM $J$ to $K: 000^{\circ}(T) 240 \mathrm{NM}$

What is the Rhumb Line bearing and distance of ' K ' from ' G '?


Figure 15.7 Diagram for example 3

There are 2 ways you can solve these types of question. One is by a double substitution into the basic departure formula. It works, but it is a bit slower. The other is by use of the following formula:-

$$
\frac{\text { Departure at Latitude } A}{\text { Departure at Latitude } B}=\frac{\cos A}{\cos B}
$$

The new formula is quicker and saves time. Let us do the question both ways, firstly using double substitution into the basic departure formula and then by use of the new formula. You should then see the advantage of the new formula.

## Method 1

| $240 \mathrm{NM}=$ change of lat of $4^{\circ} \mathrm{S}$ |  | $\therefore$ lat of $\mathrm{H}=44^{\circ} \mathrm{S}$ |
| ---: | :--- | :--- |
| Departure (NM) | $=$ | ch.long $(\mathbf{m i n}) \times \cos$ lat |
| 240 | $=$ | ch.long $\times \cos 44^{\circ}$ |
| Therefore ch.long. H to J | $=$ | $\frac{240}{\cos 44^{\circ}}$ |
| ch.long. H to J | $=$ | 333.6 minutes of longitude |
| Change of lat from J to K also | $=$ | $4^{\circ}$ |

Thus K will be at $40^{\circ} \mathrm{S}$ and change of long from G to K is the same as H to J, i.e. 333.6 min of long

| Departure (NM) | $=$ | ch.long $(\mathbf{m i n}) \times \cos$ lat |
| ---: | :--- | :--- |
|  | $=\quad 333.6 \times \cos 40^{\circ}$ |  |
|  | $=\quad 255.6 \mathrm{NM}$ |  |
| Rhumb Line bearing | $=270^{\circ}(\mathrm{T})$ |  |

## Method 2

Re-arrange this formula:

$$
\begin{aligned}
& \frac{\text { Departure at Latitude } A}{\text { Departure at Latitude } B}=\frac{\cos A}{\cos B} \\
& \text { into Departure at Latitude } A=\text { Departure at Latitude } B \times \frac{\cos A}{\cos B}
\end{aligned}
$$

Choose Latitude B to be the one for which you already know the departure ( 240 NM ), which is 44 S , which means that you can solve for Latitude A (40S) without any further rearrangement of the algebra.

$$
\text { Departure at Latitude } A=240 \times \frac{\cos 40^{\circ}}{\cos 44^{\circ}}=\quad 255.6 \mathrm{NM}
$$

## Inspection of the Answers

Do not forget that for some questions, it is not always necessary to calculate. If the 4 option answers are given, it may be possible to solve the question by elimination of the impossible answers and save calculation time.

## Example 4

An aircraft at position 2700 N 17000 W travels 3000 km on a track of $180^{\circ}(\mathrm{T})$, then 3000 km on a track of $090^{\circ}(\mathrm{T})$, then 3000 km on a track of $000^{\circ}(\mathrm{T})$, then 3000 km on a track of $270^{\circ}(\mathrm{T})$. What is its final position?
a. 2700 N 17000 W
b. $0000 \mathrm{~N} / \mathrm{S} 17000 \mathrm{~W}$
c. $\quad 2700 \mathrm{~N} 17318 \mathrm{~W}$
d. 2700 N 14300 W

## Answer:

If the aircraft starts at 2700 N and it travels the same distance ( 3000 km ) due S , then E , then N , then W, the change of longitude when flying westward (at the higher latitude) will be greater than when it flew eastward at the lower latitude. Therefore it will finish up at the same latitude that it started from, but further west. Answer (c) is the only one that fits. You can do the calculation if you like, and you will get the right answer, but it is not necessary.

## Questions

1. A flight is to be made along the parallel of latitude from A at $48^{\circ} 00^{\prime} \mathrm{N} 04^{\circ} 00^{\prime} \mathrm{W}$ to B at $48^{\circ} 00^{\prime} \mathrm{N} 02^{\circ} 27^{\prime} \mathrm{E}$. Calculate the distance.
2. An aircraft flies for 1000 nm along a rhumb line track of $090^{\circ}(\mathrm{T})$ from C at $36^{\circ} 00^{\prime} \mathrm{N}$ $174^{\circ} 45^{\prime} E$ to $D$. What is the longitude of $D$ ?
3. The following rhumb line tracks and distances are flown; starting from $E$ in latitude $50^{\circ} \mathrm{N}$.

E to $F \quad 000^{\circ}(\mathrm{T}) 300 \mathrm{NM}$
F to G $090^{\circ}(\mathrm{T}) 300 \mathrm{NM}$
G to $\mathrm{H} \mathbf{1 8 0}^{\circ}(\mathrm{T}) \mathbf{3 0 0} \mathbf{N M}$
What is the rhumb line bearing and distance of H from E ?
4. What is the track and distance measured along the parallel of latitude of $80^{\circ} \mathrm{S}$ from $176^{\circ} 15^{\prime} \mathrm{W}$ to $179^{\circ} 45^{\prime} \mathrm{E}$ ?
5. In which latitude is a difference in longitude of $44^{\circ} 10^{\prime}$ the equivalent of a departure of 2295 NM?
6. An aircraft leaves $\mathrm{J}\left(36^{\circ} 00^{\prime} \mathrm{S} 130^{\circ} 14^{\prime} \mathrm{E}\right)$ at 0946 UTC , flying a rhumb line track of $270^{\circ}$ at Flight Level 100 , temperature $0^{\circ} \mathrm{C}$, Mach No. 0.81 , wind component 35 kt tail. What is the aircraft's position at 1004 UTC?
7.
a. At which latitude is the departure in NM. between two points equal to:
i. their difference in longitude in minutes?
ii. half their difference in longitude in minutes?
b. At latitude $52^{\circ} \mathrm{N}$,
i. what is the departure between the meridians of $136^{\circ} 16^{\prime} \mathrm{W}$ and $43^{\circ} 44^{\prime} \mathrm{E}$ ?
ii. what is the shortest distance between the same points?
8. An aircraft leaves position $L$ in latitude $37^{\circ} S$ and makes good the following tracks and ground speeds:

L to $\mathrm{M} \mathbf{2 7 0}{ }^{\circ}(\mathrm{T}) \mathbf{3 1 0} \mathrm{kt} \mathrm{M}$ to $\mathrm{N} \mathbf{1 8 0}^{\circ}(\mathrm{T}) \mathbf{2 8 0}$ kt N to $\mathrm{P} \mathbf{0 9 0 ^ { \circ }}(\mathrm{T}) \mathbf{2 5 0}$ kt
$P$ is due south of $L$
$M$ is reached 80 minutes after leaving $L$
P is reached 93.5 minutes after leaving $\mathbf{N}$
Calculate the distance and time from $\mathbf{M}$ to N .
9. Aircraft Q, ground speed 301 kt flies parallel of latitude $46^{\circ} \mathrm{N}$ through $10^{\circ}$ of longitude. Aircraft R, ground speed 364 kt flies between the same meridians in the same time as aircraft Q .

What is the latitude of the track of aircraft $R$ ?
10. Aircraft S leaves position $36^{\circ} \mathrm{N} 10^{\circ} \mathrm{E}$ at 1522 hours, and flies a rhumb line track of $090^{\circ}(\mathrm{T})$ at ground speed 470 kt until it reaches the $13^{\circ} \mathrm{E}$ meridian when it turns onto a track of $180^{\circ}(\mathrm{T})$ at a ground speed of 460 kt .
Aircraft T leaves position $30^{\circ} \mathrm{N} 10^{\circ} \mathrm{E}$ at 1522 hours and flies a rhumb line track of $090^{\circ}(\mathrm{T})$ at ground speed 150 kt ,

1. Which aircraft reaches position $30^{\circ} \mathrm{N} 13^{\circ} \mathrm{E}$ first?
2. At what time does this occur?
3. Give the position of the other aircraft at this time.
4. An aircraft starts at position 0410S 17822W and heads true north for 2950 NM, then turns 90 degrees left, and maintains a rhumb line track for 314 kilometres. What is its final position?
a. 5500 N 17422 W
b. 4500 N 17422 W
c. 5500 N 17738 E
d. 4500N 17738E
5. An aircraft at position 50N 006E travels 300 NM due South, then travels 300 NM due East, then 300 NM due North and then 300 NM due West. What is the aircraft's final position relative to its start point?
a. North
b. East
c. West
d. South
6. An aircraft departs a point 0400N 17000 W and flies 600 NM South, followed by 600 NM East, then 600 NM North, then 600 NM West. What is its final position?
a. 0400 N 17000 W
b. 0600 S 17000 W
c. $\quad 0400 \mathrm{~N} 169^{\circ} 58.1^{\prime} \mathrm{W}$
d. $0400 \mathrm{~N} 170^{\circ} 01.8^{\prime} \mathrm{W}$
7. An aircraft is flying around the Earth eastwards along the 60 N parallel of latitude at a ground speed of 240 knots. At what ground speed would another aircraft have to fly eastwards along the Equator to fly once round the Earth in the same journey time?
a. 600 knots
b. $\quad 240$ knots
c. $\quad 480$ knots
d. 120 knots
8. Your position is 5833N 17400W. You fly exactly 6 NM eastwards. What is your new position?
a. $\quad 5833 \mathrm{~N} 17411.5 \mathrm{~W}$
b. $\quad 5833 \mathrm{~N} 17355 \mathrm{~W}$
c. $\quad 5833 \mathrm{~N}$ 17340W
d. $\quad 5833 \mathrm{~N} 17348.5 \mathrm{~W}$

## Answers

## Question 1

Change of longitude $6^{\circ} 27^{\prime}=387 \mathrm{~min}$ long. Distance $=387 \times \cos 48^{\circ}=259 \mathrm{NM}$.

## Question 2

Distance $=1000 \mathrm{NM}=\frac{1000}{\cos 36^{\circ}} \min$ long $=1236^{\prime}$
$=20^{\circ} 36^{\prime}$ East from $174^{\circ} 45^{\prime} \mathrm{E}$
$=164^{\circ} 39^{\prime} \mathrm{W}$

## Question 3

$55^{\circ} \mathrm{N}$ 300 NM $\frac{\text { Dep } 300 \times \cos 50^{\circ}}{\cos 55^{\circ}}=336 \mathrm{NM}$

F to G
$300 \mathrm{NM}=5^{\circ} \quad(\mathrm{F} / \mathrm{G}$ and $\mathrm{E} / \mathrm{H}$ have SAME change of Longitude)
$50^{\circ} \mathrm{N}$ $\qquad$ E to H E to H Departure Distance 336 NM

E to F $300 \mathrm{NM}=5^{\circ}$ Lat. Lat of $\mathrm{F}=55^{\circ}$
F to G $300 \mathrm{NM}=\frac{300}{\cos 55^{\circ}}=523^{\prime}$ Long.

G to H $300 \mathrm{NM}=5^{\circ}$ Lat. Lat of $\mathrm{H}=50^{\circ}$
At $50^{\circ} 523^{\prime}$ Long. $=523 \times \cos 50^{\circ}=336 \mathrm{NM}$.
Rhumb line bearing H from $\mathrm{E}=090^{\circ}$ at 336 NM .

## Question 4

$176^{\circ} 15^{\prime} \mathrm{W}$ to $179^{\circ} 45^{\prime} \mathrm{E}=4^{\circ}=240^{\prime}$ long.
$240^{\prime}=240 \times \cos 80^{\circ}=41.7 \mathrm{NM}$.
Track $=270^{\circ}(\mathrm{T})$ Dist 41.7 NM

## Question 5

$44^{\circ} 10^{\prime}=2650^{\prime}$ Long.
Dist $\quad 2295=\cos$ lat $=0.866$
Min Long. 2650
Lat $=30^{\circ} \mathrm{N}$ or S

## Question 6

M 0.81 Temp $0^{\circ}$ TAS $=520 \mathrm{kt}+35$ Tail $=555 \mathrm{kt}$ GS
18 min at $555 \mathrm{kt}=166.5 \mathrm{NM}$.
$\frac{166.5 \mathrm{NM}}{\cos 36^{\circ}}=206 \mathrm{~min}$ Long $=3^{\circ} 26^{\prime}$ Long
$130^{\circ} 14^{\prime} \mathrm{E}-3^{\circ} 26^{\prime}=126^{\circ} 48^{\prime} \mathrm{E}$
Position at $1004=36^{\circ} 00^{\prime} \mathrm{S} 126^{\circ} 48^{\prime} \mathrm{E}$

## Question 7

a. i) At Equator
ii) At $60^{\circ} \mathrm{N} / \mathrm{S}$
b. i) ch.long $=180^{\circ}=10800^{\prime}=10800 \times \cos 52^{\circ} \mathrm{NM}$ $=6653 \mathrm{NM}$
ii) Shortest distance $=$ great circle via pole.

$$
(90-52) \times 2=76^{\circ}=4560 \mathrm{NM}
$$

## Question 8

| M | L |
| :--- | :--- |
| N | P |

L to $\mathrm{M}=413.3 \mathrm{NM}=\frac{413.3}{\cos 37^{\circ}}=517^{\prime}=8^{\circ} 37^{\prime}$

N to $\mathrm{P}=93.5 \mathrm{~min}$ at $250 \mathrm{kt}=389.6 \mathrm{NM} ;$ also $=8^{\circ} 37^{\prime}$
$\frac{389.6}{517}=\cos$ lat of N and P Lat $\mathrm{N} \& \mathrm{P}=41^{\circ} 00^{\prime} \mathrm{S}$

Dist M to $\mathrm{N}=41^{\circ} 00^{\prime} \mathrm{S}-37^{\circ} 00^{\prime} \mathrm{S}=4^{\circ}=240 \mathrm{NM}$
Time M to $\mathrm{N}=240 \mathrm{NM}$ at 280
$=51.25 \mathrm{~min}$.

## Question 9

Q at $46^{\circ} \quad 10^{\circ}$ Long $=600^{\prime}=600 \times \cos 46^{\circ}=417 \mathrm{NM} @ 301 \mathrm{kt}=83 \mathrm{~min}$ R at GS $\quad 364 \mathrm{kt}$ in 83 min covers 503.5 NM.
$\begin{array}{lc}\text { Dist NM } & \frac{503.5}{600}=\text { cos lat of } R \\ \text { Min Long }\end{array}$

Lat of $R=33^{\circ} 00^{\prime} \mathrm{N} / \mathrm{S}$

## Question 10

S $\quad 10^{\circ} \mathrm{E}$ to $13^{\circ} \mathrm{E}=180^{\prime} \mathrm{Long}=145.6 \mathrm{NM} @ 470 \mathrm{kt}=18.5 \mathrm{~min}$ $36^{\circ} \mathrm{N}$ to $30^{\circ} \mathrm{N}=360 \mathrm{NM} @ 460 \mathrm{kt}+47.0 \mathrm{~min}$ Total $=65.5 \mathrm{~min}$

T $\quad 180^{\prime}$ Long @ $30^{\circ} \mathrm{N}=155.9 \mathrm{NM} @ 150 \mathrm{kt}=62.0 \mathrm{~min}$
T reaches $30^{\circ} \mathrm{N} 13^{\circ} \mathrm{E} 3.5 \mathrm{~min}$ before S
$3.5 \mathrm{~min} @ 460 \mathrm{kt}=27 \mathrm{NM}=27$ Lat
When T reaches $30^{\circ} \mathrm{N} 13^{\circ} \mathrm{E} S$ will be $27^{\prime}$ North
$=30^{\circ} 27^{\prime} \mathrm{N} 13^{\circ} 00^{\prime} \mathrm{E}$

| 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: |
| d | c | c | c | d |

## Chapter <br> 16 <br> Scale

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16 sale

## Scale

Scale is the relationship between the length of a line drawn between two positions on a chart and the distance on the Earth between the same points.

Three methods of expressing scale are common:
Statement in words. This is self-explanatory. The statement "One inch to ten nautical miles" means that a line one inch long on the chart represents a line ten nautical miles long on the Earth. Jeppesen charts often use this method; for instance, the Jeppesen $\mathrm{E}(\mathrm{HI}) 1$ / 2 uses a scale of 1 inch to 20 NM.

Graduated Scale Line. Graduated scale lines are normally depicted at the bottom of the chart as in Figure 16.1 and may be in nautical miles, statute miles or kilometres. Since one degree of latitude is equal to 60 nautical miles, a latitude scale on the chart in effect provides a graduated scale line as is shown in Figure 16.2.


Figure 16.1


Figure 16.2 Latitude scale
Representative Fraction. This is a statement in words put into mathematical form and is calculated from the equation:
Scale $=\frac{\text { Chart Length }}{\text { Earth Distance }}$

By convention, the chart length is always 1 and the dimensions above and below the line are the same and thus cancel each other to make the fraction dimensionless - representative. Thus on the ICAO 1:500 000 topo, 1 cm on the chart represents 500000 cm on the Earth and 1 inch on the chart represents 500000 inches on the Earth.

## Converting from a Statement in Words to a Representative Fraction

When converting from a statement in words to a representative fraction, you must ensure that you use the same units for both the numerator and the denominator. For instance, if converting 'One inch to ten nautical miles' into a representative fraction, substitution into the above equation gives:

Representative Fraction $\quad=\frac{1 \text { inch }}{10 \text { nautical miles }}$
Multiply by 6080 to get the denominator into feet:-
Representative Fraction $\quad=\frac{1 \text { inch }}{10 \times 6080 \text { feet }}$
And then by 12 to get it into inches:-
Representative Fraction $\quad=\frac{1 \text { inch }}{10 \times 6080 \times 12 \text { inches }}$
Now both numerator and denominator are in the correct units, i.e.:
Representative Fraction $\quad=\frac{1}{729600}$
so the RF becomes dimensionless - it will work with centimetres or any other units as long as the ratios remain the same.

## Reminder about Conversion Factors

You will find the following conversion factors useful in dealing with typical scale problems:
1 nautical mile $=6080$ feet or 1852 metres
1 metre $=\quad 3.28$ feet
1 inch $=2.54 \mathrm{~cm}$
5.4 NM = $\quad 10$ kilometres

## Example 1

On a particular chart, 5 cm represents 7 nautical miles. What is the scale?
Answer:
Representative Fraction $=\frac{\text { Chart Length }}{\text { Earth Distance }}$

$$
=\frac{5 \mathrm{~cm}}{7 \text { (nautical miles) } \times 1852 \times 100}
$$

(We multiply 7 by 1852 to convert to metres, then by 100 to convert to centimetres)

$$
=\frac{5}{1296400}
$$

However, we always express Representative Fractions with ' 1 ' as the numerator. We need to divide both numerator and denominator by 5 .

Representative Fraction $\quad=\frac{1}{259280}$

## Easy Calculator Operation

The easiest way to enter all this on to your calculator in one stage is to evaluate the denominator first then divide by the numerator, i.e. in the above example, as follows:

```
7\times1852\times100\div5=259280
```

The answer comes out as the denominator. You then have to write it down as $\frac{1}{259280}$

If you enter numerator first, you get the RF expressed as a decimal, like this:
$5 \div(7 \times 1852 \times 100)=0.000003856$
If you then want to use your $\mathrm{x}^{-1}$ key, fine. But it is quicker to do it the first way.

## Types of Scale Question

There are 2 basic sub-divisions of scale problems: Simple Scale and Mercator Scale. Mercator Scale problems are dealt with in a later chapter. All simple scale problems are dealt with by substituting into the one basic equation:

$$
R F=\frac{C L}{E D}
$$

Sometimes you are given the Chart Length and the Earth Distance and asked to find the RF, as in the previous example. Sometimes you are given the scale (the RF) and the ED and asked to find the CL. Sometimes you are given the RF and the CL and asked to find the ED. Just substitute what you know into the above equation, apply any conversion factors necessary to get both CL and ED in the same units, rearrange if necessary to make the unknown the subject of the equation, and solve using your calculator.

## Example 2

On a chart of scale $1 / 5000000$, how many nautical miles on the Earth are represented by 1.7 cm ?

$$
\begin{aligned}
\text { RF } & =\frac{C L}{E D} \\
\frac{1}{5000000} & =\frac{1.7 \mathrm{~cm}}{E D}
\end{aligned}
$$

Re-arranging:-
$E D=\frac{1.7 \times 5000000}{1852 \times 100}=45.89 \mathrm{NM}$

## Rounding Errors

The conversion factors we have quoted are, in many cases, only approximate. There is often more than one way to carry out the conversions, and sometimes a scale numerical problem may be solved by converting both numerator and denominator into metric units or alternatively, both to imperial units. This may give small differences in the answers. Look at the 4 options given in the question and choose the nearest.

## Large and Small Scale

Confusion often exists as to whether a chart is 'large scale' or 'small scale'.
A UK Ordnance Survey Map is an example of a large scale chart with a scale of 1:50000.
Two centimetres on this chart represents one kilometre on the Earth. The chart does not cover much area, but there is lots of detail. This is a LARGE scale chart.

A world atlas would be an example of a small scale chart - it covers a lot of area, but there is not much detail.

## LARGE SCALE - LOTS OF DETAIL

## SMALL SCALE - LOTS OF AREA

However, the Representative Fraction is in the right sense. A SMALL scale gives a SMALLER RF. A LARGE scale gives a LARGER RF. Consider 2 charts, one a $1 / 50000$ UK Ordnance Survey and the other a $1 / 5000000$ small scale chart.

| Scale | Representative Fraction | Expressed as a <br> Decimal |
| :---: | :---: | :---: |
| Large | $1 / 50000$ | 0.00002 |
| Small | $1 / 5000000$ | 0.0000002 |

You can see that 0.00002 is a LARGER number than 0.0000002 . The Representative Fractions are in the right sense. The confusion occurs because the denominators are easier to handle and so that is how we usually refer to charts. We call the first chart a "fifty thousand" in conversation. We call the second one a "five million". We do not refer to them as a "0.00002" or a " 0.0000002 ". But the logic of 'large' or 'small' is correct.

So:-
LARGE SCALE - SMALLER DENOMINATOR
SMALL SCALE - LARGER DENOMINATOR

## Questions

1. Give the scale, as a representative fraction, of the following charts:-
a. One centimetre represents five kilometres
b. one centimetre represents 5.4 nautical miles
c. one inch represents 15.4 kilometres
d. $\quad 3.5$ inches represents 70 kilometres
e. five inches represents eight nautical miles
2. The representative fraction of a chart is given as $\mathbf{1 : 5 0 0 0 0 0}$ (Half Mil Chart)
a. how many (1) centimetres and (2) inches would represent 30 kilometres on the ground?
b. how many (1) inches and (2) centimetres would represent 30 nautical miles on the ground?
3. The representative fraction of a chart is given as $1: 1000000$
a. What is the chart length of a line representing an Earth distance of 50 kilometres on this chart? give your answer in both centimetres and inches
b. is this scale larger or smaller than the one used in question two above?
4. On a chart 5 centimetres represents 7 nautical miles
a. Give the scale of this chart as a representative fraction
b. Determine the distance in inches on this chart which would represent the distance flown by an aircraft in 5 minutes at a groundspeed of 156 knots.
5. On a chart 14.8 centimetres represents 20 nautical miles on the ground
a. Give the chart scale as a representative fraction
b. Determine the length in inches on this chart which would represent the distance flown by an aircraft in 9 minutes at a ground speed of 185 Knots.
6. On a chart, one centimetre represents 3.5 kilometres
a. Give the scale as a representative fraction
b. What is the length in inches on the chart which would represent the distance flown in 4 minutes at a ground speed of 204 kilometres per hour?

## Answers

1. a. 1:500000
b. 1:1000000
c. 1:600000 (approx)
d. $\quad 1: 788000$
e. 1:116740
2. a. (1) 6 centimetres (2) 2.36 inches
b. (1) 4.38 inches (2) 11.12 centimetres
3. a. $5 \mathrm{~cm}=1.97 \mathrm{in}$
b. Smaller
4. a. 1:260000
b. $\quad 3.65$ inches
5. a. 1:250000
b. $\quad 8.1$ inches
6. a. 1:350000
b. $\quad 1.53$ inches

## Chapter 17 General Chart Properties

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## Chart Projections - General

The transfer of information from a globe on to a flat paper chart is achieved by 'projection', because, originally, the technique involved using a light source within the globe to project the latitude/longitude graticule on to a flat sheet of paper. Today, the projection is achieved mostly by using computer models. However, an understanding of the original projection techniques using light is more than adequate for understanding the properties of charts, whatever actual method of production is used.

## "Perspective" and "Non-perspective" Charts

Charts produced directly from a projection are called perspective or geometric projections. Charts produced by mathematical methods are called non-perspective charts. Most of the charts we use are in this latter, non-perspective category but they can be thought of as perspective projections which have been modified mathematically.

## The "Reduced Earth"

The term "Reduced Earth" (RE), means the scale model of the Earth on which the projection of the chart is based. To make a 1:1000000 chart, a one millionth scale model of the Earth will be used.

## Types of Projection

There are 3 general types of projection surfaces:

- Azimuthal/Plane
- Cylindrical
- Conical


## Azimuthal/Plane Projections

An azimuthal (or 'plane') projection is produced by placing a flat sheet of paper against a point on the Earth. A common use is to provide charts of the North/South polar regions (this is the only example taught and examined in the EASA syllabus). The resulting 'Polar Stereographic' charts are covered in more detail in a later chapter.

The principle of the projection is illustrated in Figure 17.1


Figure 17.1 Azimuthal projection
The graticule of the projection would appear as in Figure 17.2.


Figure 17.2 Azimuthal graticule
Note: Although this projection has produced a chart of the entire Southern hemisphere, the chart would only be used in the central area, in this case for flights over Antarctica.

## Cylindrical Projections

The earliest chart projections were produced in the 16th century by a Flemish navigator called Gerard de Kremer who used the Latin alias 'Gerardus Mercator'. His projections used cylinders of paper wrapped around the "Reduced Earth" and touching the RE at the Equator. The projection technique is illustrated in Figure 17.3.


Figure 17.3 Simple cylindrical projection and graticule

When the graticule has been projected onto the cylinder, the cylinder is opened to produce at flat sheet of paper. Cartographers then can add the geographic detail.

Note that the shapes look a little strange. This is explained in chapter 18 (Mercator Charts).
Again, notice that most of the world (except the Poles) has been projected and the user would only use a section of the chart (in this case, normally near the Equator).

## Conical Projections

Conical projections involve placing a cone of paper over the Reduced Earth and projecting the graticule onto the cone. Subsequently the cone is slit along one side and the cone can then be opened to produce a flat sheet of paper. The technique is illustrated in Figure 17.4.


Figure $17.4 a$ and $b$ Conical projections


Figure 17.5

In Figure 17.4(a), the cone is placed over the Earth. The graticule is projected onto the cone. The cone is then slit in Figure 17.4(b) (along the Greenwich meridian in this case). The cone is then 'developed' (opened) to give the flat sheet of paper and graticule illustrated in Figure 17.5. In this case, the chart will show the whole Northern hemisphere. The user will need only a small section of the chart, for example the rectangle suggested. In this section of the chart, the graticule will look more familiar (similar to the ICAO 1:500000 topographical chart) and this is illustrated in Figure 17.6.


Figure 17.6 Typical conical graticule
The conical graticule in Figure 17.6 is typical of the Lambert's Conical Chart which is discussed in Chapter 20.

## Properties of an Ideal Chart

Having developed methods of projecting the spherical Earth onto a flat sheet of paper, we should now consider what properties we require of the chart. The following is a list of properties we might like on an 'ideal' (perfect) chart. (The list is not comprehensive).

## Representation of the Earth's Surface

- Angles on the Earth's surface should be represented by the same angles on the chart.
- Scale should be constant and 'correct'.
- Areas should be represented with their true shape on the chart.
- Equal areas on the Earth's surface should be shown as equal areas on the chart.


## Navigation Requirements

- Rhumb Lines should be straight lines.
- Great Circles should be straight lines.
- Latitudes/Longitudes should be easy to plot
- Adjacent sheets should fit correctly.
- Coverage should be worldwide.

Two of these properties can never be obtained, except on the globe.
Scale can never be constant and correct. We can modify charts mathematically to give nearly constant scale in small areas, but not over large areas.

The shapes of large areas cannot be represented perfectly. However, it is possible to represent the shapes of small areas reasonably accurately.

All the other ideal properties can be obtained on charts, but unfortunately not together on the same chart. However, not all of them are essential for navigation. For instance, it really does not matter to a pilot whether areas are correctly represented or not. You do not make comparisons of area when flying. Even a reasonable amount of distortion of shape is acceptable, provided it is not too great and landmarks can be recognized.

## Orthomorphism / Conformality

Of all the ideal properties, the only essential one is that navigation bearings must be "correct" and the critical property is that angles on the Earth must be represented correctly on the chart.

This property is critical to aviation - or indeed for navigation generally. If you draw a line joining 2 points on the chart and measure the angle but then find that this does not correspond to the true direction on the Earth, the chart is useless for navigation. You might previously have thought that if you measure a track off any map, it will correspond to Earth direction but this is not true for most charts. Those charts that do have this property are in the minority and are known as orthomorphic or conformal charts.

Note: Your ICAO 1:500 000 Topographical chart is called a "Lambert's Conformal Conic" - just above the graduated scale

Remember that people produce charts for many reasons - not just navigation. For example, if your field of expertise is farming, then you might be interested in whether 1000 hectares of land in Kansas produces a greater or a lesser tonnage of wheat than 1000 hectares of land in Azerbaijan. In that case, the property you would look for in a map would be an accurate representation of area - but it would not matter to you whether bearings were represented accurately.

Therefore, from the vast range of projections available, we have to select only those which are orthomorphic (or conformal).

There are two fundamental conditions which must be met to achieve orthomorphism/ conformality.

Condition 1 The meridians and parallels on the chart must intersect at right angles (as they do on the Earth). This is illustrated in Figure 17.7.


Figure 17.7 Orthomorphism 1
Consider a rectangle on the Earth's surface as depicted in Figure 17.7a. Also, consider the bearing of position X from position 0, which is $056^{\circ}$. In Figure 17.7 b , the chart graticule has been distorted and the meridian/parallel intersection is not $90^{\circ}$. Not only is the shape incorrect but the bearing from 0 to X is now $028^{\circ}$ rather than $056^{\circ}$. The chart is not conformal.

Look at the graticules so far discussed. They all meet the $90^{\circ}$ intersect rule.

Condition 2 The second condition relates to the scale at a point on the chart and is illustrated.


Figure 17.8 Scale at a point
A square on the Earth has been illustrated in Figure 17.8(a) along with the line 0 Y which has a bearing of $045^{\circ}(\mathrm{T})$. In Figure $17.8(\mathrm{~b})$ representing the chart, the $\mathrm{N}-\mathrm{S}$ scale has changed but the E-W scale has remained the same. The overall effect is that the shape has changed to a rectangle and the bearing 0 Y is now $035^{\circ}(\mathrm{T})$ rather than the correct $045^{\circ}(\mathrm{T})$ on the Earth. The chart is not orthomorphic/conformal.

On an orthomorphic chart, scale at a point should be the same in all directions.
The above diagram implies that a chart should be constant scale which we know is only true on a globe. On a chart, scale will change. The condition 2 is modified to be:

At any point on a chart, scale should be the same in all directions, or should change at the same rate in all directions.

Refer back to Figure 17.3. The shapes on the simple cylindrical projection have been distorted, in this case stretched in a N-S direction because the N-S scale is changing at a greater rate that the E-W scale. Mercator, the Flemish cartographer, recognized this problem and mathematically adjusted the N-S scale change to produce the conformal Mercator chart which is the subject of the next chapter.

## Chapter <br> 18

## Mercator Charts - Properties

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## Introduction

The earliest method of transferring the graticule of meridians and parallels from a globe to a flat sheet of paper was achieved using cylindrical projections, described in the previous chapter. A scale model of the Earth, the Reduced Earth (RE), was made at an appropriate scale. A cylinder of paper was wrapped around the RE, touching the RE at the Equator. Using a light source at the centre of the RE, the graticule was projected onto the cylinder. The cylinder was then 'developed' or opened up to a flat sheet of paper.

The technique described above would produce a perspective projection. This simple cylindrical projection technique is illustrated below.


Figure 18.1 Simple Cylindrical Projection
The projected graticule had one significant advantage for early navigators - the meridians were equally spaced parallel lines. Thus a straight line drawn on the chart would have a constant direction - the straight line on the chart would be a rhumb line. With their basic compass systems, early navigators preferred to sail constant directions and had to accept the fact that they would be sailing a rhumb line. Modern navigators use more advanced guidance systems and normally aim to fly the great circle track (more of this later).

## Mercator's Projection - General

In the 16th century, a Flemish cartographer called Gerard de Kremer, who used the Latin alias 'Mercator', recognised the limitations of the simple cylindrical projection. The projected graticule met one of the requirements for an orthomorphic/conformal chart - the meridians and parallels crossed at right angles. Therefore, a straight line was a line of constant direction - a rhumb line. Unfortunately, it was not the correct direction. The shapes were clearly not correct and therefore angles on the chart were not correct.

Note the shapes in Figure 18.1 - they are stretched in a N-S direction. Mercator realized that this was caused by failure to meet the second requirement of orthomorphism/conformality, namely that - at any point on a chart, scale should be the same in all directions, or should change at the same rate in all directions. On the simple cylindrical projection, the N-S scale was changing at a different rate from the E-W scale. Mercator determined that the E-W scale was changing such that, at any latitude, the scale was proportional to the secant of the latitude (secant = 1/cosine). However, the N-S scale was changing such that, at any latitude, the scale was proportional to the tangent of the latitude, resulting in the N-S stretching of shapes. This can be seen below in Figure 18.2.


Figure 18.2 Simple Cylindrical Projection
Latitude spacing is a function of $\tan$ (lat)
Mercator solved the problem by adjusting the positions of the parallels of latitude. The parallels had been projected as parallel lines with separation between the parallels increasing in proportion to the tangent of the latitude. Mercator adjusted the parallels of latitude so that their separation increased only at a rate proportional to the secant of latitude, matching the E-W scale change.

Briefly, Mercator mathematically adjusted the positions of the parallels of latitude to make the chart orthomorphic/conformal.

Because the chart has been mathematically produced, it is a non-perspective chart.
Mercator solved the problem in 1569. Because his solution was so simple, elegant, and correct, we still use his projection today, over 400 years later. A modern Mercator chart looks very different from his own because we have subsequently discovered and explored so many more countries. However, the basic principles of the graticule have not changed. He got it right.

An example of the adjusted Mercator projection is given in Figure 18.3.


Figure 18.3 Mercator Projection
This Mercator projection is often called a normal or direct Mercator. The projection surface touches the Reduced Earth at the Equator. The geographic poles cannot be projected (they are on the axis of the cylinder).

## Scale

Mercator scale expands as the secant of the latitude.
This arises out of the departure formula. You will remember that:

$$
\text { Departure }=\text { change of longitude }(\min ) \times \cos \text { latitude }
$$

Mercator realized that, by the use of a cylindrical projection, the meridians were produced as parallel lines. This meant that a change of longitude on the chart was always represented by the same distance on the chart (so many centimetres), whatever the latitude. Therefore, we are holding the change of longitude (the Chart Length) constant and we need to see how the departure (the Earth Distance) changes with latitude. This tells us how the scale changes with latitude. We therefore re-arrange the formula as follows;

$$
\text { change of longitude }=\frac{\text { Departure }}{\text { cos latitude }}
$$

However, in mathematics there is a trigonometrical function called the secant. Its definition is that:

$$
\operatorname{secant} \theta=\frac{1}{\operatorname{cosine} \theta}
$$

Therefore we can re-write the above equation as:

$$
\text { change of longitude }=\quad \text { Departure } \times \text { secant latitude }
$$

As the E/W scale has to change as a function of the secant of the latitude (because the meridians are parallel at all latitudes), then, as Mercator realized, the N/S scale has to change by the same amount.

We therefore come up with the following equation:
scale at any latitude $(\lambda)=$ scale at Equator $\times \sec (\lambda)$
Let us see how this works with a practical example. Take a case where the scale at the Equator is $1 / 1000000$. Now let us calculate what the scale on a Mercator chart will be at $8^{\circ} \mathrm{N}$ (or S) latitude.

$$
\begin{aligned}
\text { scale at } 8^{\circ} N & =\text { scale at Equator } \times \sec 8^{\circ} N \\
& =\frac{1}{1000000} \times \frac{1}{\cos 8^{\circ}}
\end{aligned}
$$

There is no point in evaluating secant as a numerator. In multiplying out representative fractions, we want ' 1 ' on the top line. We are better off writing secant as $1 / \cos$, because then we shall be multiplying denominators.

So:-

$$
\begin{aligned}
\text { scale at } 8^{\circ} N & =\frac{1}{1000000} \times \frac{1}{0.990268} \\
& =\frac{1}{990268}
\end{aligned}
$$

This shows that the scale at $8^{\circ} \mathrm{N}$ (or S ) latitude is $99 \%$ of the scale at the Equator, or within $1 \%$ of correct scale.

This $1 \%$ figure is of importance in navigation. Up to a scale error of $1 \%$, we can regard a chart as being of "constant scale", which means that you can measure distances using a ruler. Once the scale error increases to more than $1 \%$, we must find distances either by calculation or by using the local latitude scale and measuring small distances at a time with a pair of dividers.

The most important figures to remember are that Mercator scale is:
Within $1 \%$ up to $8^{\circ}$ from the Equator.

In examination questions, $8^{\circ}$ is often approximated to 500 nautical miles. If you see this number in questions, it is often a clue that the examiner is using it to mean $8^{\circ}$, and is referring to the $1 \%$ scale error band.

To summarize:

- Mercator scale is correct (same as the Reduced Earth) at the Equator.
- Mercator scale expands as the secant of the latitude.
- Mercator scale is within $1 \%$ up to $8^{\circ}$ from the Equator.


## Orthomorphism

All charts used for navigation must be orthomorphic. This chart is orthomorphic/conformal by mathematical construction (Mercator's adjustment of the parallels of latitude). The projection is non-perspective.

## Graticule

The graticule is rectangular. Meridians are equally spaced parallel lines. Parallels of latitude are parallel lines with the space between them increasing as the secant of the latitude.

## Shapes

Mercator produced an orthomorphic projection. However, looking again at Figure 18.3, (or Figure 18.4) the student should notice that the areas are not correctly represented. Correct representation of area is not required on an orthomorphic chart.

Thus, Greenland appears as large as Africa on the chart, despite the fact that the land area of Africa is approximately 18 times that of Greenland. In reality, the land area of Greenland is only the same as the small NE corner of Africa, as illustrated in Figure 18.4. Similarly, the land area of Scandinavia illustrated is only one third of the land area of India, but on the chart they appear to have similar areas. Note also the chart length equivalent to 3000 NM at 60N, which is twice the chart length for the same distance at the Equator.

This distortion of area and change of scale also leads to change of shape. Land masses at high latitudes appear too wide for their height compared with the same land masses on a globe or on a chart with convergent meridians. Compare the shape of the North American continent, for instance, on a Mercator with its shape on a globe (Figure 18.5b and Figure 18.5a). The continent appears too wide. The E/W distance from Labrador to Western Alaska seems too great compared with the distance from Northern Alaska to the Gulf of Mexico.


Figure 18.4 Examples of Mercator Scale Expansion


Figure 18.5

These distortions of shape are insignificant over small distances and therefore have no effect on a pilot's ability to map-read. They have no implications for rhumb line navigation at all, but they can give a false impression of the most direct routing, especially at high latitudes. This point was covered in Chapter 2.

## Chart Convergence

The concept of Earth Convergence, or Convergency, was covered in Chapter 14. It is the angle of inclination of the meridians on the Earth, (or the change in direction of a great circle), between 2 longitudes. However, for each type of projection, we also have the concept of Chart Convergence. This is the angle of inclination between meridians on the chart, (or the change in direction of a straight line), between 2 longitudes.

For a Mercator chart, as all meridians are parallel, their mutual inclination is zero. The change in direction of a straight line drawn on the map is also zero. It will always cut all meridians at the same angle. That is why Mercator produced the projection in the first place - so that a straight line on the chart gives a single track angle.

Earth convergency is also zero at the Equator, but nowhere else. Therefore Mercator convergence is correct at the Equator (only) but constant everywhere (always zero).

## Rhumb Lines

Because the meridians are parallel lines, a straight line track drawn on the chart will cut all meridians at the same angle. A straight line track on a Mercator chart is a rhumb line.

## Great Circles

The rhumb line between two points will always be nearer to the Equator than the corresponding great circle. Conversely, the Great circle between two points will always lie nearer the Pole than the rhumb line.

This was covered in detail in Chapter 2. The statement can be simplified visually in Figure 18.6.


Figure 18.6 Mercator Chart - rhumb lines and Great circles

Apply these rules to the Mercator chart at Figure 18.7. Consider a round-the-world trip from London to Los Angeles, USA to Auckland, New Zealand to Singapore and returning to London. The route is drawn on a Mercator chart in Figure 18.7.


Figure 18.7 London/LA/Auckland/Singapore/London - Rhumb lines

All the tracks drawn are rhumb line tracks. From London to LA, the rhumb line track is approximately $257^{\circ}(\mathrm{T})$. From LA to Auckland, the rhumb line track is approximately $221^{\circ}(\mathrm{T})$. From Auckland to London, the rhumb line track is approximately $301^{\circ}(\mathrm{T})$. Note that the Auckland/London rhumb line track passes through Singapore.

Now draw in the great circle tracks for the same route.


Figure 18.8 London/LA/Auckland/Singapore/London-Great circles
Note the shapes of the great circles. The Equator and the meridians (which are great circles) project as straight lines. All other great circles project as curved lines concave to the Equator (convex to the nearer Pole).

## Mathematical Calculations

In the above discussion, the rules about the shapes of great circles have been stated. Given the rhumb line direction, the student also should be able to calculate the direction of the great circle. The angle between the great circle and the rhumb line is conversion angle (CA) where:

```
Conversion Angle (CA) = 1⁄2 Earth Convergency (EC)
or
Conversion Angle (CA) = 1/2 }\times\mathrm{ ch.long }\times\operatorname{sin}(mean lat
```

Now review Figure 18.8 and consider the route from London to Los Angeles.

As stated previously, the rhumb line direction (a straight line on a Mercator chart) is approximately $257^{\circ}(\mathrm{T})$. The angle between the rhumb line and the great circle is conversion angle (CA). In this case:

```
Conversion angle = 1/2 }\times\mathrm{ ch.long }\times\operatorname{sin}(\mathrm{ mean lat )
```

Using approximate values

```
CA = 1/2 }\times120\times\operatorname{sin}4\mp@subsup{5}{}{\circ
    = 60\times.707 = approx 42 }\mp@subsup{}{}{\circ
```

Therefore the great circle track from London to LA, measured at London, is approximately $257^{\circ}+42^{\circ}=299^{\circ}(\mathrm{T})$

Now calculate (using equivalent values) the great circle track direction from LA to London, measured at LA.

Answer: approximately $035^{\circ}(\mathrm{T})$
$R / L$ track is $077^{\circ}(\mathrm{T})$ - reciprocal of $257^{\circ}(\mathrm{T})$.
Conversion angle is approx. $42^{\circ}$.
Therefore, great circle track is approx. $035^{\circ}(\mathrm{T})$.
Similar exercises could be carried out for all the other sectors. However, a problem would occur on the sectors crossing the Equator. For example, on the Los Angeles to Auckland sector, it could be argued that the conversion angle is zero because the mean latitude is $0^{\circ}$ (the Equator). In these situations the sector would have to be divided into two - the first sector from LA to the Equator (mean lat approx $18^{\circ}$ ) and the second sector from the Equator to Auckland (mean lat approx $18^{\circ}$ ).

There are 2 exceptions to this rule.
The Equator and meridians are straight lines (because they are also rhumb lines).

## Summary of Mercator Properties

A summary of Mercator properties is set out in Figure 18.9 below. All of these are asked in examinations and they should be learnt.

| PROPERTIES OF A MERCATOR CHART |  |
| :--- | :--- |
| Scale | Correct on the Equator. <br> Elsewhere increases as the secant of the latitude. <br> Within 1\% up to $8^{\circ}$ from the Equator. |
| Orthomorphic | Yes. All charts used for navigation must be. |
| Graticule | Meridians are straight parallel lines, evenly spaced. <br> Parallels are straight parallel lines with the space between them <br> increasing with the secant of the latitude. |
| Shapes | Reasonably correct over small areas. <br> Distortion over large areas, especially at high latitudes. |
| Chart | Convergence <br> Great circles |
| Zero everywhere. |  |
| Constant across the chart. |  |

Figure 18.9 Summary of Mercator Chart Properties

## Questions

1. A normal Mercator chart is a $\qquad$ projection?
(i) Cylindrical
(ii) Perspective
(iii) Non-perspective
(iv) Conformal
(v) Conical
(vi) Azimuthal

The correct combination of the above statements is:
a. (i), (ii) and (iii)
b. (ii), (iv) and (v)
c. (i), (iii) and (iv)
d. (iii), (iv) and (vi)
2. A direct Mercator graticule is:
a. Rectangular
b. Square
c. Circular
d. Convergent
3. On a normal Mercator chart, rhumb lines are represented as:
a. Curves concave to the Equator
b. Curves convex to the Equator
c. Complex curves
d. Straight lines
4. On a direct Mercator, great circles can be represented as:
a. Straight lines
b. Curves
c. Straight lines and curves
5. On a direct Mercator, with the exception of the meridians and the Equator, great circles are represented as:
a. Curves concave to the Nearer Pole
b. Curves convex to the Equator
c. Curves concave to the Equator
d. Straight lines
6. The angle between a straight line on a Mercator chart and the corresponding great circle is:
a. Zero
b. Earth convergency
c. Conversion angle
d. Chart convergence
7. If the rhumb line track from Turin (45N 008E) to Khartoum ( 15 N 032 E ) is $\mathbf{1 4 5}^{\circ}(\mathrm{T})$, what is the direction of the great circle track measured at Turin?
a. $\quad 133^{\circ}(\mathrm{T})$
b. $\quad 139^{\circ}(\mathrm{T})$
c. $\quad 145^{\circ}(\mathrm{T})$
d. $\quad 151^{\circ}(\mathrm{T})$
8. In Question 7, what is the direction of the great circle track from Khartoum to Turin?
a. $\quad 319^{\circ}(\mathrm{T})$
b. $\quad 325^{\circ}(\mathrm{T})$
c. $\quad 331^{\circ}(\mathrm{T})$
d. $\quad 337^{\circ}(\mathrm{T})$
9. On a Mercator chart, the rhumb line track from Durban (30S 032E) to Perth (30S 116 E ) is $090^{\circ}(\mathrm{T})$. What is the great circle track from Perth to Durban?
a. $\quad 291^{\circ}(\mathrm{T})$
b. $\quad 312^{\circ}(\mathrm{T})$
c. $\quad 228^{\circ}(\mathrm{T})$
d. $\quad 249^{\circ}(\mathrm{T})$

Note: the examiner may not tell you the rhumb line track if, as in this case, the two points are on the same parallel of latitude
10. At 60S on a Mercator chart, chart convergence is:
a. greater than Earth convergency
b. "correct"
c. less than Earth convergency
d. equal to ch. long $\times 0.866$

Answers

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| c | a | d | c | c | c | b | c | d | c |

4 A tricky question! Answers a. and b. are correct. But answer c. is more correct. Great circles on a Mercator chart can be represented as straight lines but only in the case of the Equator and all meridians. See Question 5.

5 This is a question involving "concave" and "convex" and the Equator and the Poles.
7 This is a convergency/conversion angle problem but often appears as a Mercator problem. A Mercator graticule can be a good way to visualize this problem.

8 as above
9 Comments as Q7/8. Notice that the question involves two places at the same latitude on the same parallel of latitude (30S). The rhumb line track has been given as $090^{\circ}(\mathrm{T})$ but an examiner may not necessarily give this information. The examiner may expect the student to recognize that the track between two places on the same parallel of latitude must be East/West $090^{\circ}(T) / 270^{\circ}(T)$

## Chapter <br> 19 <br> Mercator Charts - Scale

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## Mercator Scale - General Principles

We covered the general principles of Mercator scale in the previous chapter. A cylindrical projection must produce the meridians as parallel lines. Therefore, the scale in the E/W sense at any latitude is defined by the departure equation. The change of longitude (which is a fixed distance on the chart, whatever the latitude) remains constant. By re-arrangement of the equation, we find that this means that the E/W scale depends on the secant of the latitude. Mercator, therefore, devised a projection in which the N/S spacing of the parallels of latitude is also a function of the secant of the latitude. Now that both the N/S and the E/W scale expansion were at the same rate, the projection became orthomorphic - though non-perspective.

On a Mercator chart, scale expands away from the Equator. The scale expansion is proportional to the secant of the latitude.

We therefore finish up with the following equation:-

## Scale at any latitude $(\lambda)=$ Scale at Equator $\times \sec (\lambda)$

This is the starting point for all Mercator scale problems. Questions generally come in one of 4 basic types:

- Given scale at the Equator, find Mercator scale at some other latitude.
- Given Mercator scale at some latitude, find scale at the Equator.
- Given Mercator scale at some latitude, find Mercator scale at some other latitude.
- Given a fixed chart distance between meridians, find Mercator scale at a specific latitude.


## Given Scale at the Equator, Find Mercator Scale at Some Other Latitude

## Example 1

If the scale of a Mercator chart at the Equator is 1:1000000, what is the scale at 60 N (or S)?
a. 1:2000000
b. $1: 1000000$
c. 1:866000
d. 1:500000

Whenever solving problems using representative fractions (RF) for scale, remember that the scale must be written mathematically in fraction form.

$$
\begin{aligned}
\text { Scale at } 60 \mathrm{~N} & =\text { Scale at Equator } \times \text { secant } 60 \mathrm{~N} \\
& =\frac{1}{1000000} \times \text { secant } 60^{\circ}
\end{aligned}
$$

But it is easier if we write secant as $1 / \cos$. That way, we get 1 as the numerator, which is what we want, and we only have to multiply out the denominators.

```
Scale at \(60 \mathrm{~N}=\frac{1}{1000000} \times \frac{1}{\cos 60^{\circ}}\)
\[
=\frac{1}{1000000} \times \frac{1}{0.5}
\]
\[
=\frac{1}{500000}
\]
```


## Answer:1:500 000 (d)

The trap in this question is that at $60^{\circ} \mathrm{N} / \mathrm{S}$, the scale on a Mercator chart is double the scale at the Equator. If the scale doubles, the denominator of scale is halved. Remember, if scale expands, the denominator decreases; if scale contracts, the denominator increases.

## Given Mercator Scale at Some Latitude, Find Scale at the Equator

## Example 2

If the scale of a Mercator chart at 52 S is $1: 2000000$, what is the scale at the Equator?
a. 1:3250000
b. 1:1000000
c. 1:866000
d. 1:500000

Start in the usual way:-
Scale at 52S $=$ Scale at Equator $\times$ secant 52 S
Now substitute in what you already know:-

$$
\frac{1}{2000000}=\text { Scale at Equator } \times \frac{1}{\operatorname{cosine} 52^{\circ}}
$$

Re-arranging:-
Scale at Equator $=\frac{\text { cosine } 52^{\circ}}{2000000}$

Again, as in some of the basic scale calculations, it is easier to enter this into your calculator denominator first, then divide by the numerator. This gives you the value of the denominator with ' 1 ' as the numerator, which is what we want for a representative fraction.

Key in: $2000000 \div \cos 52=$
You will then get the answer: 3248538
You write it down as $\frac{1}{3248538}$

The answer is therefore (a).

There are 2 learning points to note from this answer. One is that the options on scale questions are often approximated to the nearest round number. The correct answer to the scale question was $1: 3258538$, but the closest option available is $1: 3250000$. This is not unusual.

The other is that you do not need to do the calculation at all. Scale must expand away from the Equator. The denominator must be biggest at the Equator and then reduce as you move away from the Equator. Since you knew that the denominator at 525 was 2000000 , the denominator at the Equator must be a bigger number, i.e. the scale must be smaller. In this particular set of options, answer (a) is the only one which is smaller. This would have saved you time.

## Given Mercator Scale at Some Latitude, Find Mercator Scale at Some Other Latitude

There is a short cut formula which gets rid of the need to use secants and the need to remember to write the representative fraction as a fraction. You simply need to remember that the large figure in the RF is the denominator. (It is also the figure we most commonly remember). Thus a $1: 500000$ chart (ICAO topo) has a denominator (D) of 500000.

The following derivations need not be remembered but the final formula is important.
When comparing scales at two different latitudes on a Mercator chart, we can write
At latitude A:

$$
\text { Scale at } A=\text { Scale at Equator } \times \text { secant } A
$$

At latitude B:

$$
\text { Scale at } B \quad=\text { Scale at Equator } \times \text { secant } B
$$

Divide the two scale formulae:

$$
\frac{\text { Scale at } A}{\text { Scale at } B}=\frac{\text { Scale at Equator }}{\text { Scale at Equator }} \times \frac{\text { secant } A}{\text { secant } B}
$$

This can be simplified to:

$$
\frac{\text { Scale at } A}{\text { Scale at } B}=\frac{\text { secant } A}{\text { secant } B}
$$

but even further simplified to:


The equation to remember is this one:

$$
\frac{\text { Denominator }_{A}}{\text { Denominator }_{B}}=\frac{\cos A}{\cos B}
$$

You may recall that this is also the formula used to compare the departure between two longitudes at different latitudes. The only difference now is that $D$ represents the denominator of scale rather than the departure.

## Example 3

On a Mercator chart, the scale at 54S is $1: 2000000$. What is the scale at 25N?
a. 1:2000000
b. 1:3084000
c. 1:1121000
d. 1:3825000

Firstly, use your knowledge to discount answers a and c. At 25N, you are closer to the Equator than at 54S. Therefore, scale must contract between 54 S and 25 N - the denominator at 25 N must be larger than at 54S. In some cases, there may only be one appropriate answer given and the answer can be spotted without calculation. In this case, answers $b$ and $d$ could be correct and we need to revert to the simplified formula.

$$
\begin{aligned}
& \frac{\text { Denominator }_{A}}{\text { Denominator }_{B}}
\end{aligned}=\frac{\cos A}{\cos B}, \begin{aligned}
\text { Let } D_{A} & =\text { scale denominator at } 25^{\circ} \mathrm{N} \\
D_{B} & =\text { scale denominator at } 54^{\circ} \mathrm{S} \\
\frac{D A}{2000000} & =\frac{\cos 25^{\circ}}{\cos 54^{\circ}} \\
\text { Therefore } D_{A} & =\frac{\cos 25^{\circ} \times 2000000}{\cos 54^{\circ}}
\end{aligned}
$$

$$
=3083806
$$

Answer: 1:3084000-b

## Given a Fixed Chart Distance between Meridians, Find Mercator Scale at a Specific Latitude

## Example 4

On a Mercator chart, the chart length between two meridians, 160E and 160W, is 30 cm at 30S. What is the scale of the chart at 30S?

There are 2 ways of doing this, and there is very little to choose between them.
The first is to treat it as a departure problem.

$$
\begin{aligned}
\text { Scale } & =\frac{\text { Chart Length }}{\text { Earth Distance }} \\
\text { Chart Length } & =30 \mathrm{~cm} \\
\text { Earth Distance } & =\text { Departure } \\
\text { Departure } & =\text { ch.long }(\mathrm{min}) \times \cos \text { lat } \\
& =\left(40^{\circ} \times 60\right) \times 0.866 \mathrm{NM} \\
& =2078 \mathrm{NM}
\end{aligned}
$$

$$
\begin{aligned}
\text { Scale } & =\frac{30 \mathrm{~cm}}{2078 \mathrm{NM}} \\
& =\frac{1 \mathrm{~cm}}{69.28 \mathrm{NM}} \\
& =\frac{1}{69.28 \times 1852 \times 100} \\
& =\frac{1}{12831000}
\end{aligned}
$$

Scale is $1: 12831000$
Note: This has little to do with Mercator charts and is essentially a Scale/Departure problem.
Alternatively, you can find the scale at the Equator as a normal RF, then convert to scale at 30S, like this:

Scale at Equator

$$
=\frac{\text { Chart Length }}{\text { Earth Distance }}
$$

The Earth distance at the Equator is $40^{\circ}$ of ch.long $\times 60 \mathrm{NM}$ (the Equator is a great circle). This comes to 2400 NM. Now get numerator and denominator in the same units and work out the RF.

Scale at Equator $\quad=\frac{30 \mathrm{~cm}}{2400 \times 1852 \times 100 \mathrm{~cm}}=\frac{1}{14816000}$

Now we just have a normal scale change problem, just like Example 1.
Scale at $30^{\circ} \mathrm{S}=$ Scale at Equator $\times$ secant $30^{\circ} \mathrm{S}$
Scale at Equator $=\frac{1}{14816000} \times \frac{1}{\cos 30^{\circ}}$
Scale is $\quad 1: 12831000$
Example 5 (This appears to be a more complex problem but is as straightforward as Example 4)

At 40N, the scale of Mercator chart is $1: 10000000$. What is the distance in centimetres between the 160E and 160W meridians at 20S?

An immediate problem appears to be that the scale is given at 40 N but the distance between the two meridians has to be calculated at 20S. Initially, the student might be tempted to change the scale from 40 N to 20 S using the scale conversion formula previously given. This is unnecessary. Think about the Mercator chart - it has parallel meridians and so the distance between two meridians will be the same at all latitudes. The problem can be solved completely at 40N.

Scale at Equator $\quad=\frac{\text { Chart Length }}{\text { Earth Distance }}$
but Earth distance at $40^{\circ} \mathrm{N}=$ Departure

$$
\begin{aligned}
& =\text { ch.long }(\mathrm{min}) \times \text { cos lat } \\
& =2400 \times \cos 40^{\circ} \\
& =1838.4 \mathrm{NM} \\
\frac{1}{10000000} & =\frac{\text { Chart length }}{1838.4 \mathrm{NM}} \\
\text { Chart length } & =\frac{1838.4 \mathrm{NM}}{10000000}
\end{aligned}
$$

but this needs to be converted to centimetres

$$
\begin{aligned}
\text { Chart length } & =\frac{1838.4 \times 1852 \times 100}{10000000} \\
\text { Answer } & =34 \mathrm{~cm}
\end{aligned}
$$

## Area of "Constant" Scale

For practical navigation, charts may be considered to be constant scale in any area where the scale is within $1 \%$ of correct scale.

On a Mercator chart, scale is constant and "correct" only along the Equator.
The scale is within $1 \%$ of "correct" scale within a band of $8^{\circ}$ latitude either side of the Equator (see Note 1).

Note 1 The $8^{\circ}$ figure can be quoted in exams as 480/500 NM.

## Questions

1. On a Mercator chart, the scale at the Equator is $1: 3500000$. What is the scale at $37^{\circ} \mathrm{S}$ ?
2. On a Mercator chart, the scale at $50^{\circ} \mathrm{N}$ is $1: 4500000$. What is the scale at the Equator?
3. On a Mercator chart, the scale at $15^{\circ} \mathrm{N}$ is $1: 1500000$. What is the scale at $47^{\circ} \mathrm{S}$ ?
4. The chart length between meridians $1^{\circ}$ apart is 3.2 inches. What is the scale of the chart as a representative fraction at 50N ?
5. A Mercator is to be constructed at a scale of 1:2 237000 in latitude 44 N . What is the required spacing in centimetres between meridians $1^{\circ}$ apart?
6. The scale of a Mercator chart is $1: 3500000$ at 52 N . Give the length in inches of a straight line on the chart from $A(58 N 42 E)$ to $B(58 N 34 E)$.
7. On a Mercator chart the spacing of meridians $1^{\circ}$ apart is 2.82 cm . In which latitude will the scale be 1:2319000?
8. On a Mercator chart the spacing of meridians $1^{\circ}$ apart is 5.42 cm . What is the scale of the chart as a representative fraction in latitude 56N ?
9. The scale of a Mercator chart is $1: 3500000$ in latitude 52 N . What is the spacing, in centimetres, between meridians $1^{\circ}$ apart?
10. The scale of a Mercator chart is 1:5 107200 in latitude 47N.
a. What distance in NM does one inch on the chart represent in 47N ?
b. $\quad$ What is the spacing in cm between meridians $10^{\circ}$ apart ?
c. Would the scale at 30N have a larger or smaller denominator than 5107000 ?
Give a reason for your answer.
a. Give the scale at 60 N as a representative fraction.
b. How many nautical miles will be represented by a line drawn along the same parallel and measuring 7 cm in length ?
11. The scale of a Mercator chart is $1: 2179000$ at the Equator.
a. What is the spacing in centimetres between meridians $10^{\circ}$ apart ?
b. In which latitude does one inch represent 18.4 NM ?
12. A Mercator chart has a scale of $1: 3000000$ at $60 N$.
a. In what latitude is the scale:
i. 1:6000000 ?
ii. 1:5300000?
b. Is there a latitude on this chart where the scale is 1:6500000?
13. On a particular direct Mercator wall chart, the $180^{\circ} \mathrm{W}$ to $180^{\circ} \mathrm{E}$ parallel of latitude at $53^{\circ} \mathrm{N}$ is 133 cm long. What is the scale of the chart at $30^{\circ} \mathrm{S}$ ?
a. 1:3000000
b. 1:18000000
c. 1:21000000
d. 1:27000000

## Answers

1. $1: 2795224$
2. 1:7000757
3. 1:1059085
4. $60^{\prime}$ of long @ $50 \mathrm{~N}=60 \times \cos 50^{\circ} \mathrm{NM}=38.58 \mathrm{NM}$
3.2 in $=38.58 \mathrm{NM} .1 \mathrm{in}=12.05 \mathrm{NM}$
$R F=1: 12.05 \times 72960=1: 879333$ or
$\frac{60^{\prime} \cos 50^{\circ} \times 6080 \times 12}{3.2}=1: 879624$
5. At 44 N 60 @ long $=60 \times \cos 44^{\circ} \mathrm{NM}=43.14 \mathrm{NM}=80 \mathrm{~km}$

Chart Length $=$ Earth Length $\times \frac{1}{\mathrm{RF}}$
$\frac{8000000 \mathrm{~cm}}{2237000}=3.576 \mathrm{~cm}$
6. ch.long $=8^{\circ}=480^{\prime}=480 \times \cos 52^{\circ}=295.7 \mathrm{NM}$

Chart Length $=$ Earth Length $\times \frac{1}{\mathrm{RF}}$
$=\frac{295.7 \times 72960}{3500000}=6.16$ inches (Chart Length same at 52 and 58 N )
7. As the RF is 1:2319000, Earth distance $=2.82 \times 2319000 \mathrm{~cm}=65.39 \mathrm{~km}=35.28 \mathrm{NM}$.

As $N M=\min$ long $\times \cos$ lat
Then cos lat $=\frac{\mathrm{NM}}{\min \text { long }}=\frac{35.28}{60}=.588=\cos 54^{\circ} \mathrm{N} / \mathrm{S}$
8. $\quad 1^{\circ} @ 56^{\circ}=60 \times \cos 56^{\circ} \mathrm{NM}=33.55 \mathrm{NM}=62.19 \mathrm{~km}$
$5.42 \mathrm{~cm}=62.19 \mathrm{~km} \quad 1 \mathrm{~cm}=11.47 \mathrm{~km}$

RF for Chart = 1:1147000
9. At $52 \mathrm{~N} 1^{\circ}=60 \times \cos 52^{\circ} \mathrm{NM}=36.94 \mathrm{NM}=68.47 \mathrm{~km}$.

Chart dist $=$ Earth dist $\times \frac{1}{\mathrm{RF}}$
$=\frac{68.47 \times 1000 \times 100}{3500000}=1.956 \mathrm{~cm}$
10. a. $1^{\prime \prime}=5107200=\frac{5107200}{72960}=70 \mathrm{NM}$
b. $\quad 10^{\circ}=600 \times \cos 47^{\circ}=409.2 \mathrm{NM}$

$$
\frac{409.2 \times 72960}{72960}=5.845^{\prime \prime}=14.85 \mathrm{~cm}
$$

c. At 30 N the scale is less than at 47 N . Mercators chart EXPANDS away from the Equator. Therefore scale denominator would be larger.
11. a. $4^{\prime \prime}=200 \mathrm{~km} \mathrm{1"}=50 \mathrm{~km}=26.97 \mathrm{NM}$
$S F=1: 26.97 \times 72960=1: 1968000$
OR $\quad 10.16 \mathrm{~cm}=200 \mathrm{~km} \quad 1 \mathrm{~cm}=19.68 \mathrm{~km}$
SF = 1:1968000
(Note that any unit of measurement can be used as scale is same for both)
b. $\quad 7 \mathrm{~cm}=7 \times 19.68 \mathrm{~km}=137.79 \mathrm{~km}=74.336 \mathrm{NM}$
12. a. At Equator $10^{\circ}=600 \mathrm{NM}=1112.2 \mathrm{~km}$

$$
\text { at } \quad \text { SF 1:2 } 179000=\frac{1112.2}{21.79}=51.04 \mathrm{~cm}
$$

b. $\quad 1^{\prime \prime}=18.4 \mathrm{NM}$
$S F=1: 18.4 \times 72960=1: 1342464$
cos lat $=\frac{1342464}{2179000}=0.616$
$\cos 0.616$ is equal to lats 52 N or S
13. a
$\begin{aligned} \text { (i) Scale at } 60 \mathrm{~N} & =1: 3000000 \\ \text { Scale at Equator } & =1: 3000000 \times \frac{1}{\cos 60^{\circ}} \\ & =1: 6000000\end{aligned}$
(ii) $\quad$ cos lat $=\frac{5300000}{6000000}=.833 \therefore$ Lat $28 N / \mathrm{S}$
b. NO. Scale on the Mercator is always minimum at the Equator, and expands both North \& South away from the Equator.
14. d (the correct answer is nearer to $1 / 26000000$, but $1 / 27000000$ was the nearest option)

# Chapter <br> 20 <br> Mid Course Test 

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20 Questions

## Introduction

All the questions are of the same standard as the Gen Nav ATPL exam - it is just that, for this test, they are drawn only from those parts of the syllabus which have already been covered. There are 54 questions in 2 hours and most of them are from previous EASA papers, or are very similar. It is a representative and realistic exam.

## Practice Test

Practice Test - General Navigation 54 Questions 2 Hours 80 Marks

1. What is the shortest distance in kilometres between San Francisco (38N 123W) and Dubai (25N 057E)?
a. $\quad 7020$
b. 8073
c. 13001
d. 11250
2. A great circle has a northern vertex of 50 N 100 W . the southern vertex is:
a. 40 S 100 W
b. $\quad 40 \mathrm{~S} 080 \mathrm{E}$
c. 50 S 100 W
d. $\quad 50 \mathrm{~S} 080 \mathrm{E}$
3. An ICAO nautical mile is defined as:
a. 6080 feet
b. $\quad 1852$ metres
c. 1863 feet
d. 6062 feet
4. How much is the polar diameter of the Earth different from the equatorial diameter?
(2 Marks)
a. less by 40 km
b. greater by 27 statute miles
c. less by 27 statute miles
d. greater by 27 nautical miles
5. The maximum difference between geodetic and geocentric latitude occurs at about?
a. $\quad 45^{\circ}$ North and South
b. $\quad 90^{\circ}$ North and South
c. $\quad 60^{\circ}$ North and South
d. $\quad 0^{\circ}$ (Equator)
6. Required course $045^{\circ}(\mathrm{T}), \mathrm{W} / \mathrm{V}=190^{\circ}(\mathrm{T}) / 30, \mathrm{FL}=55 @ \mathrm{ISA}$, Variation $=15^{\circ} \mathrm{E} . \mathrm{CAS}=$ 120 knots. What is magnetic heading and $\mathrm{G} / \mathrm{S}$ ?
(3 Marks)
a. $\quad 052^{\circ}(\mathrm{M}) \quad 154$
b. $\quad 067^{\circ}(\mathrm{M}) \quad 154$
c. $\quad 037^{\circ}(\mathrm{M}) \quad 154$
d. $\quad 037^{\circ}(\mathrm{M}) \quad 113$
7. What is the shortest distance in kilometres between Cairo ( $30^{\circ} 17^{\prime} \mathrm{N} 030^{\circ} 10^{\prime} \mathrm{E}$ ) and Durban( $\left.29^{\circ} 48^{\prime} \mathrm{S} 030^{\circ} 10^{\prime} \mathrm{E}\right)$ ?
a. 3605
b. 4146
c. 4209
d. 6676
8. An aircraft at latitude $02^{\circ} 20^{\prime} \mathrm{N}$ tracks $180^{\circ}(\mathrm{T})$ for 685 km . What is its latitude at the end of the flight?
(2 Marks)
a. $03^{\circ} 50^{\prime} \mathrm{S}$
b. $02^{\circ} 50^{\prime} \mathrm{S}$
c. $02^{\circ} 10^{\prime} \mathrm{S}$
d. $\quad 08^{\circ} 55^{\prime} \mathrm{S}$
9. An aircraft is at latitude 10 N and is flying south at $444 \mathrm{~km} /$ hour. After 3 hours the latitude is:
a. 10 S
b. $\quad 02 \mathrm{~N}$
c. 025
d. $0 N / S$
10. The circumference of the Earth is approximately?
(1 Mark)
a. 43200 NM
b. $\quad 10800 \mathrm{NM}$
c. $\quad 21600 \mathrm{NM}$
d. $\quad 5400 \mathrm{NM}$
11. $\mathrm{SAT}=+35^{\circ} \mathrm{C}$. Pressure alt (corrected for barometric error) $=\mathbf{5 0 0 0}$ feet. What is true alt?
(2 Marks)
a. $\quad 4550$ feet
b. $\quad 5550$ feet
c. 4920 feet
d. 5320 feet
12. Given that the value of ellipticity of the Earth is $1 / 297$ and that the semi-major axis of the Earth, measured at the axis of the Equator is 6378.4 km , what is the semimajor axis of the Earth measured at the axis of the Poles?
(2 Marks)
a. $\quad 6399.9 \mathrm{~km}$
b. $\quad 6356.9 \mathrm{~km}$
c. $\quad 6378.4 \mathrm{~km}$
d. $\quad 6367.0 \mathrm{~km}$
13. You plan to fly a track (course) of $348^{\circ}(\mathrm{T})$, Drift is $17^{\circ}$ port, Variation is $32^{\circ} \mathrm{W}$.

Deviation is $4^{\circ} \mathrm{E}$. What compass heading should you fly?
(2 Marks)
a. $\quad 041^{\circ}$
b. $033^{\circ}$
c. $\quad 016^{\circ}$
d. $359^{\circ}$
14. At a specific location, the value of magnetic variation:
a. depends on the value of magnetic heading
b. depends on the value of true heading
c. varies slowly over time
d. depends on the type of compass installed
15. At the magnetic equator:
a. dip is zero
b. variation is zero
c. deviation is zero
d. the isogonal is an agonic line
16. The value of magnetic variation on a chart changes with time. This is due to:
a. movement of the magnetic poles, causing an increase
b. increase in the magnetic field, causing an increase
c. reduction in the magnetic field, causing a decrease
d. movement of the magnetic poles, which can cause either an increase or a decrease
17. One purpose of a compass calibration is to reduce the difference, if any, between:
(1 Mark)
a. Compass North and True North
b. Compass North and Magnetic North
c. True North and Magnetic North
d. Compass North and the lubber line
18. Given:

True Track $=352^{\circ}(\mathrm{T})$
Variation $=11^{\circ} \mathrm{W}$
Deviation $=-5^{\circ}$
Drift $\quad=\quad 10^{\circ} \mathrm{R}$
What is heading ${ }^{\circ}(\mathrm{C})$ ?
(1 Mark)
a. $\quad 078^{\circ}(\mathrm{C})$
b. $\quad 346^{\circ}(\mathrm{C})$
c. $\quad 358^{\circ}(\mathrm{C})$
d. $\quad 025^{\circ}(\mathrm{C})$
19. Pressure altitude is 27000 feet, OAT $=-35^{\circ} \mathrm{C}$, Mach No. $=0.45, \mathrm{~W} / \mathrm{V}=\mathbf{2 7 0} / 85$, Track $=200^{\circ}(\mathrm{T})$. What is drift and ground speed?
(1 Mark)
a. 18L / 252 knots
b. 15R / 310 knots
c. 17L / 228 knots
d. 17R / 287 knots
20. You plan to take off from Khamis Mushayt, Saudi Arabia, elevation 6500 ft . The ambient temperature is $+25^{\circ} \mathrm{C}$. What is your density altitude?
(1 Mark)
a. $\quad 3500 \mathrm{ft}$
b. $\quad 6500 \mathrm{ft}$
c. $\quad 9500 \mathrm{ft}$
d. 12500 ft
21. You are flying at 400 kt TAS, Indicated temperature $-50^{\circ} \mathrm{C}$. What is your COAT?
(1 Mark)
a. $\quad-67^{\circ} \mathrm{C}$
b. $\quad-50^{\circ} \mathrm{C}$
c. $\quad-33^{\circ} \mathrm{C}$
d. $\quad-17^{\circ} \mathrm{C}$
22. You are flying at FL330 at Mach No. 0.9. Ambient temperature is ISA +15. What is your TAS?
a. 600 knots
b. $\quad 595$ knots
c. $\quad 540$ knots
d. 505 knots
23. You plan to land on R/W 14. The met forecast wind velocity is $110 / 30$. Variation is $30^{\circ} \mathrm{W}$. What crosswind do you expect?
a. $\quad 15 \mathrm{kt}$
b. $\quad 0 \mathrm{kt}$
c. $\quad 26 \mathrm{kt}$
d. $\quad 30 \mathrm{kt}$
24. Given: IAS $\mathbf{1 2 0}$ knots, FL80, OAT $+20^{\circ} \mathrm{C}$. What is the TAS?
(1 Mark)
a. $\quad 132 \mathrm{kt}$
b. $\quad 141 \mathrm{kt}$
c. $\quad 102 \mathrm{kt}$
d. $\quad 120 \mathrm{kt}$
25. Given: TAS 200 kt , $\operatorname{Track} 110^{\circ}(\mathrm{T})$, W/V 015/40, calculate heading $\left({ }^{\circ} \mathrm{T}\right)$ and ground speed.
(2 Marks)
a. $097^{\circ}$ - 201 kt
b. $099^{\circ}-200 \mathrm{kt}$
c. $\quad 121^{\circ}-207 \mathrm{kt}$
d. $\quad 121^{\circ}$ - 199 kt
26. Given: True Hdg $145^{\circ}$, TAS 240 kt , True Track $150^{\circ}$, G/S 210 kt , calculate the W/V.
(2 Marks)
a. $360 / 35$
b. $295 / 35$
c. $180 / 35$
d. $115 / 35$
27. Given: TAS 140 kt , heading $005^{\circ}(\mathrm{T})$, W/V 265/25, calculate the drift and ground speed.
(2 Marks)

|  |  | $11 R$ |
| :--- | :--- | :--- |
| a. | -140 kt |  |
| b. | 10 R | -146 kt |
| c. | $9 R$ | -140 kt |
| d. | 11 R | -142 kt |

28. An aircraft is maintaining a 5.2\% gradient on a flat terrain. Its height at 7 NM from the runway is approximately:
a. $\quad 3640$ feet
b. $\quad 1890$ feet
c. $\quad 2210$ feet
d. $\quad 680$ feet
29. $\mathrm{G} / \mathrm{S}=\mathbf{2 4 0}$ knots, distance to $\mathbf{g o}=\mathbf{5 0 0} \mathrm{NM}$. What is time to go?
a. $\quad 20$ minutes
b. $\quad 29$ minutes
c. $\quad 2 \mathrm{~h} 05 \mathrm{~m}$
d. $\quad 2 \mathrm{~h} 12 \mathrm{~m}$
30. Course $040^{\circ}(\mathrm{T})$, TAS 120 kt , wind speed $=30$ knots. From which direction will the wind give the greatest drift?
(1 Mark)
a. $\quad 215^{\circ}(\mathrm{T})$
b. $\quad 230^{\circ}(\mathrm{T})$
c. $\quad 235^{\circ}(\mathrm{T})$
d. $\quad 240^{\circ}(\mathrm{T})$
31. 



ATA $X$ is 1420. ETA $Y$ is 1447. ATA $Y$ is 1450 . What is new ETA $Z$ ?
a. 1506
b. 1512
c. $\quad 1510$
d. 1515
32. Heading is $156^{\circ}(\mathrm{T})$, TAS is 320 knots, W/V 130/45. What is your true track?
(2 Marks)
a. 160
b. 152
c. $\quad 104$
d. 222
33. You are flying at a True Mach No. of .72 in a SAT of $-45^{\circ} \mathrm{C}$. At $\mathbf{1 0 0 0}$ hours you are 100 NM from the CPT DME and your ETA at CPT is 1012. ATC ask you to slow down to be at CPT at 1016. What should your new Mach No be if you reduce speed at 100 NM to go?
(3 Marks)
a. . 67
b. . 63
c. 54
d. . 51
34. Track $=090^{\circ}(\mathrm{T}), \mathrm{TAS}=460$ knots, $\mathrm{W} / \mathrm{V}=360^{\circ}(\mathrm{T}) / 100$, Variation $=10^{\circ} \mathrm{E}$, Deviation $=-2$. What is compass heading and ground speed?
a. $079^{\circ}-470 \mathrm{kt}$
b. $069^{\circ}-450 \mathrm{kt}$
c. $068^{\circ}$ - 460 kt
d. $070^{\circ}$ - 455 kt
35. You leave $A$ to fly to $B, 475$ NM away, at 1000 hours. Your ETA at $B$ is 1130. At 1040, you are 190 NM from A. What ground speed is required to arrive on time at B?
(2 Marks)
a. $\quad 317$ knots
b. $\quad 330$ knots
c. $\quad 342$ knots
d. $\quad 360$ knots
36. The wind velocity is $359 / 25$. An aircraft is heading $180^{\circ}(\mathrm{T})$ at a TAS of 198 knots. (All directions are True). What is its track and ground speed?
(2 Marks)
a. $180^{\circ}-223 \mathrm{kt}$
b. $\quad 179^{\circ}-220 \mathrm{kt}$
c. $180^{\circ}-220 \mathrm{kt}$
d. $179^{\circ}-223 \mathrm{kt}$
37. An aircraft is climbing at a constant CAS in ISA conditions. What will be the effect on TAS and Mach No.?
a. TAS increases and Mach No. decreases
b. Both increase
c. Both decrease
d. TAS decreases and Mach No. increases
38. Convert 70 metres/sec into knots.
a. $\quad 136$ knots
b. $\quad 36$ knots
c. $\quad 146$ knots
d. $\quad 54$ knots
39. Airfield elevation is 1000 feet. The QNH is 988 . Use $\mathbf{2 7}$ feet per hectopascal. What is pressure altitude?
(1 Mark)
a. $\quad 675$ feet
b. $\quad 325$ feet
c. $\quad 1675$ feet
d. 825 feet
40. You are flying from A to B, planned track $245^{\circ}(\mathrm{M})$, distance 225 NM. Your ground speed is 180 knots. After 15 minutes flying, you fix your position as 3 NM left of planned track. Which of the following statements is correct?
(2 Marks)
a. Your drift is $2 P$
b. Your drift is 4 P
c. You should turn 5 right to go direct to B
d. You should turn 8 right to go direct to B
41. A pilot receives the following signals from a VOR DME station.

Radial $=180^{\circ}+/-1^{\circ}$, distance $=200 \mathrm{NM}$ What is the approximate maximum error?
(1 Mark)
a. $\quad+-2 \mathrm{NM}$
b. $\quad+/-3.5 \mathrm{NM}$
c. $\quad+/-7 \mathrm{NM}$
d. $\quad+/-1 \mathrm{NM}$
42. An aircraft has planned to fly from $A$ to $B$ a total distance of 135 NM. After flying 45 NM the aircraft is 3 NM left of the planned track. Using the 1 in 60 Rule, what would be the alteration of heading to flying directly to $B$ ?
(1 Mark)
a. $\quad 6^{\circ}$ right
b. $\quad 4^{\circ}$ right
c. $\quad 8^{\circ}$ right
d. $\quad 4^{\circ}$ left
43. By what amount must you change your rate of descent given a 10 knot increase in headwind on a $3^{\circ}$ glide slope?
a. $\quad 50$ feet per minute increase
b. $\quad 30$ feet per minute increase
c. $\quad 50$ feet per minute decrease
d. $\quad 30$ feet per minute decrease
44. You are on an ILS 3-degree glide slope which passes over the runway threshold at 50 feet. Your DME range is $\mathbf{2 5}$ NM from the threshold. What is your height above the runway threshold elevation? (Use the 1 in 60 rule and 6000 feet = 1 nautical mile).
a. 8010 feet
b. $\quad 7450$ feet
c. $\quad 6450$ feet
d. 7550 feet
45. An aircraft flies directly from 56 N 020 W to 56 N 030W. The initial Great Circle track from 020W is:
(2 Marks)
a. $\quad 086^{\circ}(T)$
b. $\quad 082^{\circ}(\mathrm{T})$
c. $\quad 274^{\circ}(\mathrm{T})$
d. $\quad 278^{\circ}(\mathrm{T})$
46. An aircraft starts at position 0410S 17822 W and tracks true north for 2950 NM, then turns 90 degrees left, and maintains a rhumb line track for 314 kilometres. What is the final position?
(3 Marks)
a. 5500 N 17422 W
b. 4500 N 17422 W
c. 5500 N 17738 E
d. 4500 N 17738 E
47. On a chart, 49 nautical miles is represented by 7.0 centimetres. What is the scale?
(1 Mark)
a. $\quad 1 / 700000$
b. $\quad 1 / 2015396$
c. $\quad 1 / 1296400$
d. $\quad 1 / 1156600$
48. On a particular direct Mercator wall chart, the 180 W to 180 E parallel of latitude at 53 N is 133 cm long. What is the scale of the chart at 30S?
(2 Marks)
a. 1:30 000000
b. $1: 15000000$
c. $1: 13000000$
d. 1:26000000
49. Which of the following differences in latitude will give the biggest difference in the initial great circle track and the mean great circle track between 2 points separated by $10^{\circ}$ change of longitude?
(1 Mark)
a. $\quad 60 \mathrm{~N}$ and 605
b. $\quad 60 \mathrm{~N}$ and 55 N
c. 30 S and 30 N
d. 30 S and 25 S
50. On a chart, meridians at 43 N are shown every 10 degrees apart. This is shown on the chart by a distance of 14 cm . What is the approximate scale?
(2 Marks)
a. $\quad 1 / 200000$
b. $\quad 1 / 4000000$
c. $\quad 1 / 5000000$
d. $1 / 6000000$
51. A non-perspective chart:
a. is produced directly from a light projection of a Reduced Earth
b. cannot be used for navigation
c. is produced by mathematically adjusting a light projection of the Reduced Earth
d. is used for a polar stereographic projection
52. On a Mercator chart, a rhumb line appears as a:
a. small circle concave to the nearer pole
b. curve convex to the nearer pole
c. complex curve
d. straight line
53. Mercator charts use $\qquad$ projections.
a. cylindrical
b. conical
c. plane/azimuthal
d. complex
54. How does scale change on a normal Mercator chart?
a. expands as the secant ${ }^{2}$ ( $1 / 2$ co-latitude)
b. expands directly with the secant of the latitude
c. correct on the standard parallels, expands outside them, contracts within them
d. expands as the secant of the E/W great circle distance

## Answers

| Question | Answer | Marks | Your Score |
| :---: | :---: | :---: | :---: |
| 1 | c | 2 |  |
| 2 | d | 1 |  |
| 3 | b | 1 |  |
| 4 | c | 2 |  |
| 5 | a | 1 |  |
| 6 | c | 3 |  |
| 7 | d | 2 |  |
| 8 | a | 2 |  |
| 9 | c | 2 |  |
| 10 | c | 1 |  |
| 11 | b | 2 |  |
| 12 | b | 2 |  |
| 13 | b | 2 |  |
| 14 | c | 1 |  |
| 15 | a | 1 |  |
| 16 | d | 1 |  |
| 17 | b | 1 |  |
| 18 | c | 1 |  |
| 19 | c | 1 |  |
| 20 | c | 1 |  |
| 21 | a | 1 |  |
| 22 | c | 1 |  |
| 23 | b | 1 |  |
| 24 | b | 1 |  |
| 25 | b | 2 |  |
| 26 | d | 2 |  |
| 27 | b | 2 |  |
| 28 | c | 2 |  |
| 29 | c | 1 |  |
| 30 | d | 1 |  |
| 31 | c | 1 |  |
| 32 | a | 2 |  |
| 33 | d | 3 |  |
| 34 | b | 2 |  |
| 35 | c | 2 |  |
| 36 | a | 2 |  |
| 37 | b | 1 |  |
| 38 | a | 1 |  |


| Question | Answer | Marks | Your Score |
| :---: | :---: | :---: | :---: |
| 39 | c | 1 |  |
| 40 | c | 2 |  |
| 41 | b | 1 |  |
| 42 | a | 1 |  |
| 43 | c | 1 |  |
| 44 | d | 1 |  |
| 45 | c | 2 |  |
| 46 | d | 3 |  |
| 47 | c | 1 |  |
| 48 | d | 2 |  |
| 49 | b | 1 |  |
| 50 | d | 2 |  |
| 51 | c | 1 |  |
| 52 | a | 1 |  |
| 53 | b | 1 |  |
| 54 |  |  |  |

If you have scored less than $75 \%$, you need to do some more revision.

## Chapter

## 21

## Lambert's Conformal Chart - 1

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## Why Conical Charts?

We have seen that the Mercator chart has many powerful properties, but it does have 2 limitations. These are:

- Great Circles are not projected as straight lines.
- The chart is not constant scale. Indeed, scale changes quite rapidly on a Mercator chart.

Navigators wished to maintain Rhumb Line tracks for the first 400 years or so or the life of the Mercator chart, because they were steering by compass, and therefore needed to have a constant track direction.

However, that situation started to change from about 1960 onwards, when automatic computing became available. It is possible to calculate a desired Great Circle track direction by spherical trigonometry formulae, and the computers built into INS, IRS, FMS and GPS do this as a matter of course.

If the aircraft is going to be steered along a Great Circle, it would be helpful to have a chart on which a Great Circle is a straight line. Otherwise the aircraft will appear to go off track in the middle of the leg, then recover back to track again.

If the scale were to remain constant on a chart, we could measure distances with a ruler instead of having to use a pair of dividers, and having to open the dividers to different distances at different latitudes. For aviation, it is useful to be able to use a ruler - most aircraft do not have a large plotting table like a ship.

## Background to the Lambert Projection

The basis of the Lambert projection is the simple conical. However, the simple conical projection has some limitations, not least that it is not orthomorphic. A Swiss mathematician called Johann Heinrich Lambert made some modifications to the basic conic in 1777. The result is the orthomorphic, near constant scale Lambert's Conformal Conic Chart.

The approach taken in the initial part of this chapter will be firstly to derive some of the properties of the simple conic projection. Some of these, particularly the chart convergence, are also found in the Lambert projection. We will next examine why Lambert made his changes and what effect they had. We will then finish up with the properties of the Lambert projection.

## The Simple Conic Projection - Convergence

It is possible to project the graticule of the Earth on to the inside surface of a cone. This process gives the conical family of projections from which Lambert's is derived.

A cone is placed over a Reduced Earth, in such a way that the cone is tangential with the Reduced Earth along a parallel of latitude. The apex of the cone will lie on the extended line of the Earth's axis as shown in Figure 21.1.


Figure 21.1 The principle of simple conical projection
A light source at the centre of the Reduced Earth casts shadows of the graticule on the inside surface of the cone.

These shadows could be marked in, the cone removed, cut down its slant side and rolled out flat to give a simple conical projection as illustrated in Figure 21.2.

On the simple conical projection, scale is correct on the parallel of tangency, in this example, $45^{\circ} \mathrm{N}$. This parallel, on which scale is correct, is called the 'Standard Parallel'. Scale expands on either side of the Standard Parallel.

Figure 21.2 illustrates that when the cone is flattened $360^{\circ}$ of longitude are represented in a sector of a circle of 255 degrees, in this case. The size of the segment is controlled by the parallel of latitude chosen to be the parallel of tangency - the higher the latitude chosen the larger will be the sector.


Figure 21.2 A simple conical projection-graticule

Look at Figure 21.3. The parallel of tangency is 45 N . You can see that if you take this parallel as the base of a triangle formed by the cross-section of the cone, you get an isosceles triangle, with the angle at the apex being $90^{\circ}$. On the other hand, if we had taken a parallel of tangency of 60 N , the angle at the apex would have been $120^{\circ}$. The ultimate case is a parallel of tangency of 90 N - the North Pole. In this case we would get a flat sheet of paper and the angle at the North Pole would be $180^{\circ}$. The angle at the apex of the triangle is always twice the parallel of tangency.


Figure 21.3 Apex angle is twice parallel of origin
The apex angle is also known as the angle of the cone. It is the angle of the cone which determines the arc of the sector formed by laying the cone flat. In the ultimate case, a $90^{\circ}$ parallel of tangency gives a $180^{\circ}$ cone angle, which gives no missing gap at all. If the parallel of tangency is at a high latitude, say, 60N, then the paper sector forms quite a large arc and the 'missing' sector is small. At a parallel of tangency of 45 N the sector is $255^{\circ}$, making the 'missing' sector $105^{\circ}$.

The relationship of the sector formed to the original $360^{\circ}$ of longitude change is determined by the sine of the parallel of origin.

```
arc of sector = change of longitude }\times\mathrm{ sine of parallel of origin
    = 360}\times\mathrm{ * sine 45'
    = 255 
```

What we are saying here is that $360^{\circ}$ (which is the change of longitude between the $180^{\circ} \mathrm{E}$ and the $180^{\circ} \mathrm{W}$ meridian in Figure 21.2 - going the long way round - not across the gap) is represented by $255^{\circ}$ of angle of inclination between the meridians as drawn on the simple conic chart.

## angle of inclination of meridians $=$ ch.long $\times$ sine of parallel of origin

The angle of inclination of the meridians is known as chart convergence. Therefore:

```
chart convergence = change of longitude }\times\mathrm{ sine of parallel of origin
```

This gives a general relationship, which is illustrated as follows:


Figure 21.4 Ch.long v chart convergence over $100^{\circ}$

In Figure 21.4, a change of longitude of $100^{\circ}$ is represented on the chart by an angle of inclination of the meridians of $\left(100^{\circ} \times \sin 45^{\circ}\right)$, which is $70.71^{\circ}$. In Figure 21.5 , a change of longitude of $10^{\circ}$ is represented on the chart by an angle of inclination of the meridians of ( $10^{\circ}$ $\times \sin 45^{\circ}$ ), which is $7.071^{\circ}$.

The sine of the parallel of origin is called "the constant of the cone" and is represented by the symbol "n".


Figure 21.5 Ch.long v chart convergence over $10^{\circ}$

## The Simple Conic Projection - Scale

On the simple conic projection the scale is correct at the parallel of tangency - the paper touches the Reduced Earth at this point.


Figure 21.6 Correct scale
However, the scale expands away from the parallel of tangency rather rapidly:-


Figure 21.7 Scale expansion

## Lambert's Conical Orthomorphic Projection

Lambert modified the simple conic in order to reduce this rapid rate of scale change.

Instead of doing this:

(a) Simple Conic

He did this:

(b) Lambert

Figure 21.8

Lambert made the cone of his projection go inside the Reduced Earth.

The scale is now correct at these 2 points.
These are now called the Standard Parallels.


Figure 21.9 Lambert - correct scale

The scale expands here: (the yellow parts).


Figure 21.10 Lambert - expanded scale


Figure 21.11 Lambert - contracted scale

It contracts here: (the yellow part).

This has had the effect of evening out the scale error, making it more of a constant scale chart.
This new projection has two standard parallels on which the scale is correct. The old parallel of tangency of the simple conic is renamed the parallel of origin of the new projection, and scale is least on this parallel, just as it was on the simple conic.

Outside the standard parallels scale is greater (more expanded) than it is on the standard parallels, whilst between them scale is less (contracted), being least of all on the parallel of origin.

In Figure 21.12 the standard parallels are $43^{\circ} \mathrm{N}$ and $47^{\circ} \mathrm{N}$ and the parallel of origin is $45^{\circ} \mathrm{N}$. The upper standard parallel appears one sixth of the way from the top of the chart and the lower standard parallel, one sixth of the way from the bottom of the chart.

This is the 'one sixth rule', and ensures minimum scale variation over the sheet.


Figure 21.12 Part of a Lambert conical orthomorphic chart

## Lambert Chart Convergence

In the simple conic the angle of the cone was twice the latitude of the parallel of tangency. This fixed the chart convergence, making it the change of latitude times the sine of the parallel of tangency.

In making the cone go inside the Reduced Earth, Lambert did not change the cone angle. You can either think of it as keeping the cone angle constant and reducing the size of the cone, or, equally validly, just pushing the same cone downwards a bit.

If the cone angle has not changed, then neither has the ' $n$ ' factor, or constant of the cone. The chart convergence remains the same. This was originally defined by the parallel of tangency, so in the Lambert chart it is defined by the parallel of origin.

Thus the scale is correct at the standard parallels, but the convergence factor is defined by the parallel of origin.

## Non-perspective Chart

Having brought the cone inside the Reduced Earth, it was necessary to make some mathematical adjustments in order to make the chart orthomorphic.

The Lambert projection is a non-perspective chart.

## Lambert Chart Properties

## Scale

This is least on the parallel of origin. It expands away from the parallel of origin, until it is correct on the standard parallels. Scale is greatest on the top and bottom parallels of the projection.

## Orthomorphism

The chart is orthomorphic by mathematical construction.

The meridians converge:


But the parallels curve:


When you put them together, they cut at right angles.


## Graticule

Meridians are straight lines radiating from the pole. Parallels of latitude are arcs of concentric circles, all of which are centred at the pole. The pole is usually off the map sheet which you are using. The map sheet is shown as a red broken rectangle here.

## Parallel of Origin



Figure 21.13
The parallel of origin is the mathematical basis of the projection. It defines the chart convergence. It is half-way between the 2 standard parallels. The sine of the parallel of origin is called 'the constant of the cone' and is denoted by the symbol ' $n$ '.

## Chart Convergence

Unlike on the Earth, the meridians of a Lambert projection are straight lines.
Therefore, also unlike the Earth, the convergence between 2 given meridians does not change with latitude.


Figure 21.14 Chart convergence
The formula for calculating chart convergence is: -
Chart Convergence $=$ change in longitude $\times$ sine parallel of origin.

Since the meridians are straight lines, the chart convergence between two selected meridians will not change with latitude. In other words, chart convergence is constant on a Lambert chart. This is shown in Figure 21.14, where the meridians through, A, C and E, have been paralleled through B, D and F, respectively, forming angles marked ' $C C^{\prime}$ ' which are all equal to each other.

## Rhumb Lines

Except for meridians which appear as straight lines, Rhumb Lines are curves concave to the pole of the projection (parallels of latitude).

## Great Circles

Except for the meridians which appear as straight lines, Great Circles appear as curves concave to the parallel of origin. A straight line in an 'east-west' direction most nearly represents a Great Circle when drawn between two positions on the parallel of origin. The following series of diagrams explain why:-

Imagine a spherical Earth, (i.e. the real Earth or a Reduced Earth).
The meridians converge towards each other with increasing latitude.
Now imagine 3 Great Circle tracks at different latitudes.


Figure 21.15 Great circles on real Earth
What will happen when we project this real Earth situation onto a Lambert chart with a parallel of origin of $45^{\circ} \mathrm{N}$ ?

The Earth meridians are straightened out into straight-line chart meridians. This means that the formerly straight-line Great Circles are stretched outwards at latitudes higher or lower than the parallel of origin.


Chart Convergence = Earth Convergence at the Parallel of Origin

Figure 21.16 Earth convergence compared with chart convergence

So Great Circles are straight lines at the parallel of origin and curves concave to the parallel of origin at any other latitude.


Figure 21.17 Great circles on a Lambert chart
However, the amount of curvature from the straight line is exaggerated in Figure 21.17 simply to make the explanation clearer. In fact there is very little curvature compared with a straight line and, for all practical purposes, including plotting, Great Circles on a Lambert chart may be treated as straight lines.

The amount of curvature on a Rhumb Line is far greater than a Great Circle. We will return to this point in the next chapter.

## Summary of Lambert Properties

A summary of Lambert properties is set out below. All of these are asked in examinations and should be learnt.

| PROPERTIES OF A LAMBERT CHART |  |
| :--- | :--- |
| Scale | Correct on the standard parallels. <br> Contracted within the standard parallels. <br> (least at parallel of origin). <br> Expanded outside standard parallels. |
| Orthomorphic | Yes. All charts used for navigation must be. |
| Graticule | Meridians are straight lines, originating from the pole. <br> Parallels are arcs of circles, centred at the pole. <br> (The pole is always off the map). |
| Parallel of Origin | Mathematical basis of projection. Assumed to be halfway <br> between the 2 standard parallels. |
| Chart Convergence | Constant across the chart. <br> Chart convergence $=$ ch.long $\times$ sin parallel of origin |
| Rhumb Lines | Meridians are straight lines. All other Rhumb Lines are <br> concave to the pole (i.e. parallels of latitude). |
| Great Circles | Meridians are straight lines. <br> At the parallel of origin - near-straight line. <br> At any other latitude, a curve concave to the parallel of <br> origin. |

## Chapter

## 22

## Lambert's Conformal Chart - 2

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## Constant Scale

As we have seen, the Lambert chart is not completely constant scale. Scale expands outside the standard parallels and contracts inside them. However, the whole point of Lambert's modification was to reduce the rate of scale change.

For practical purposes, we regard a chart as being 'constant scale' if the amount of scale distortion does not exceed $1 \%$. This means that you can measure distances with a ruler and they will be $1 \%$ or less in error. This is good enough for most applications in aviation.

Some published Lambert charts meet this specification and others do not. You can measure distances with a ruler on the UK CAA 1:500000 topographical maps very accurately (there is about $0.1 \%$ maximum error), but there is about $3 \%$ scale error on the Jeppesen ED-6.

The amount of scale error depends on the separation of the standard parallels. You are not required to know the formula or the amount of scale error but, just to give you a feel for what are and are not constant scale charts, the following table lists the scale error for various latitude differences.


Figure 22.1 ch.lat of standard parallels

| Change of Latitude between Standard Parallels | \% Scale Error |
| :---: | :---: |
| $51_{3} 3^{\circ}$ | 0.1 |
| $16^{\circ}$ | 1 |
| $23^{\circ}$ | 2 |
| $28^{\circ}$ | 3 |
| $32^{\circ}$ | 4 |

The UK CAA 1:500 000 topographical maps have SPs at 4940 N and 5420 N - a separation of $5^{\circ}$. The SPs on the Jeppesen series which includes the ED-6 are 37 N and 65 N , a separation of $28^{\circ}$. This explains the percentage scale errors.

## Earth Convergence and Chart Convergence

Earth convergence (also known as convergency) is the difference in inclination between 2 meridians on the Earth:


Figure 22.2 Earth convergence
or the change in direction of a great circle track between 2 meridians on the Earth.


Figure 22.3 Convergency as the change in great circle direction

Chart convergence is the difference in inclination between 2 meridians on the chart:


Figure 22.4 Chart convergence
or the change in direction of a straight line track between 2 meridians on the chart.

The difference between a rhumb line and a great circle is known as conversion angle and is half earth convergence.
the formula is $c a=1 / 2 c h . l o n g \times \sin$ mean lat.


Figure 22.5 Conversion angle
The difference between a rhumb line and a straight line on the chart is half chart convergence.


Figure 22.6 Half chart convergence
The formula is:
Difference between rhumb line and straight line $=1 / 2 \mathrm{ch}$. long $\times \boldsymbol{\operatorname { s i n }}$ parallel of origin

## Great Circle Curvature on a Lambert Chart

This difference between chart convergence and earth convergence explains why great circles are concave to the parallel of origin on the Lambert projection.

Imagine a Lambert chart. Take 2 meridians, say, $20^{\circ}$ apart. Now consider 3 different latitudes, say, 47N, 45N (parallel of origin) and 43N.


Figure 22.7
Draw straight lines joining the meridians at these latitudes.


Figure 22.8 Straight lines
Now consider the rhumb line in each case.
the difference between the rhumb line and the straight line will be $1 / 2$ chart convergence and is equal to $1 / 2$ ch.long $\times \sin$ parallel of origin.

$$
1 / 2 \times 20^{\circ} \times .7071=7.071^{\circ} \quad \text { in all } 3 \text { cases }!
$$

Now calculate the angle that the great circle makes with the rhumb line. this is conversion angle, and is $1 / 2 \mathrm{ch}$.long $\times \sin$ mean latitude for each line

Calculating this value gives us 3 different answers, one for each latitude.


Figure 22.9 Conversion angle

```
CA47N \(=1 / 2 \times 20^{\circ} \times \sin 47^{\circ}=7.314^{\circ}\)
CA45N \(=1 / 2 \times 20^{\circ} \times \sin 45^{\circ}=7.071^{\circ}\)
CA43N \(=1 / 2 \times 20^{\circ} \times \sin 43^{\circ}=6.820^{\circ}\)
```

Look at the conversion angles in Figure 22.9. You can see that they are different for each latitude.

Plotting the conversion angles in at the other end and joining them up gives us the great circle track paths.
this shows why a great circle appears as a straight line at the parallel of origin but is concave to the parallel of origin at any other latitude. the straight lines always make the same angle to the rhumb line, whatever their latitude, but the angle that the great circle makes to the rhumb line depends on its latitude.


Figure 22.10 Great circles

## Lines Parallel at Mid-meridian

If two positions are joined by a straight line, a great circle, and rhumb line on a Lambert chart, they are all parallel to each other at the mid-meridian.


Figure 22.11 Measuring at mid-meridian

## The Advantages of a Lambert Chart

Great circles can be treated as straight lines for all practical purposes.
Radio bearings are great circles, so can be plotted without application of conversion angle. You still need to apply convergence by paralleling the NDB's meridian from the aircraft's DR meridian position, but no calculation is involved - this is explained below.

If scale error is small, a graduated ruler can be used to measure distances.

## The Disadvantages of the Lambert Chart

Flying by compass gives rhumb line tracks, so if great circle tracks are to be flown the aircraft must have a system which provides automatic continuous computation of desired track (INS / IRS / FMS / GPS), or the aircraft must be steered by gyro (without transport wander correction).

The graticule is not rectangular and the plotting of positions is not as simple as on a Mercator.
NDB position lines have to be plotted using meridian transfer. This is explained below.

## Plotting on a Lambert Chart

Plotting bearings on a Lambert chart is less complicated than on a Mercator because great circles (radio waves) are straight or near-straight lines, and so there is no conversion angle to apply. However, there may be a correction for chart convergence to apply. It depends on whether the bearing is measured at the ground station or at the aircraft.

The bearing measurement process for VDF (QDMs and QTEs) and for VOR takes place at the ground station. It may appear with VOR that the bearing is displayed in the aircraft and has been decoded in the aircraft, but it has not been measured in the aircraft. The different between the VOR reference phase and the bearing phase is encoded at the moment of transmission on that particular bearing at the ground station. All that the aircraft VOR receiver does is de-modulate the encoded signal and display it. It does not actually measure a bearing, unlike ADF.

For bearings measured at the ground station (VDF or VOR), the plotting process is simple. You simply correct for variation at the point where it has been added (the ground station), and plot the true bearing from the ground station. This will be the correct great circle track at the ground station. the straight-line bearing will change direction with respect to north as it crosses meridians and encounters convergence and therefore will not be the same great circle track direction at the aircraft, but it is not being plotted from the aircraft, so this does not matter. It is the correct radio wave path.

For bearings measured at the aircraft (ADF / NDB and AWR (airborne weather radar)), the situation is complicated because there is convergence between the meridian where the bearing is measured (the aircraft) and from where it will be plotted (the NDB). Consider the situation at Figure 22.12. An aircraft measures a bearing of an NDB of $245^{\circ}(\mathrm{R})$ while on a heading of $025^{\circ}(\mathrm{T})$. It is required to plot the position line on a Lambert chart.

Adding together the relative bearing of $245^{\circ}$ and the true heading of $025^{\circ}$ gives the true great circle bearing of the NDB from the aircraft, $270^{\circ}$.

But if the reciprocal, $090^{\circ}$, is plotted from the meridian of the NDB, the position line will not pass through the aircraft position, as the meridian through the aircraft is not parallel to that through the NDB. See Figure 22.12(a).


Figure 22.12 Plotting an ADF bearing
If, however, the angle of $090^{\circ}$ is measured from a line parallel to the aircraft's meridian drawn through the NDB, the position line will pass through the aircraft as shown in Figure 22.12(b).

Chart convergence has been automatically allowed for by drawing a line parallel to the aircraft's meridian through the NDB. This method is always used in plotting.

If it is necessary to obtain the bearing to plot from the meridian of the NDB, reference to Figure $22.12(b)$ will show that in this case it is equal to $090^{\circ}$ minus chart convergence.

## Example

An aircraft flying, in the Northern hemisphere, on a heading of $330^{\circ}(\mathrm{T})$ measures a bearing of $090^{\circ}(R)$ of an NDB.

What is the bearing to plot on a Lambert chart:
a. from a line parallel to the aircraft's meridian drawn through the NDB?
b. from the meridian of the NDB? Chart convergence is $3^{\circ}$.

## Solution

a. $330^{\circ}+090^{\circ}(R)=060^{\circ}(T)$
(True great circle bearing of NDB from aircraft) Plot $240^{\circ}$ from a line parallel to the aircraft's meridian drawn through the NDB. See Figure 22.13(a).
b. Bearing to plot from meridian of NDB

$$
=240^{\circ}+3^{\circ}
$$



Figure 22.13 Plotting an ADF bearing

Note that, in the case of bearings measured at the aircraft, it is the aircraft's compass which is used to add to relative bearing in order to obtain true bearing. Therefore, if the heading is magnetic and it is necessary to correct it to True, we always use variation at the aircraft.

Bearings measured at ground station (VOR, VDF) - variation at ground station
Bearings measured at aircraft (NDB/ADF, AWR) - variation at aircraft

## Example 1

Two points $A$ and $B$ are plotted on a Lambert chart and joined by a straight line, which from the meridian of A measures $250^{\circ}(\mathrm{T})$. Given that the chart convergence of the meridians through A and $B$ is $6^{\circ}$, what is:-
a. the rhumb line track from $A$ to $B$ ?
b. the approximate great circle bearing of $A$ from $B$, as given by the straight line, measured from the meridian of $B$ ?

Assume Northern hemisphere.

## Solution

The angle between the straight line and the rhumb line is $1 / 2$ chart convergence $=3^{\circ}$
a. $\quad 250^{\circ}-3^{\circ}=247^{\circ}(\mathrm{T})$.
b. The rhumb line bearing of $A$ from $B$ is $247^{\circ}(T)-180^{\circ}=067^{\circ}(T)$.

The 'straight line' bearing therefore is $067^{\circ}-3^{\circ}=064^{\circ}(\mathrm{T})$.

## Example 2

The straight line from A to B, both of which are in the Northern hemisphere, is plotted on a Lambert chart and cuts the mid-meridian in the direction of $300^{\circ}(\mathrm{T})$. Measured from the meridian through $A$, the direction of the straight line is $302^{\circ}(\mathrm{T})$. What is:-
a. the rhumb line track from $A$ to $B$ ?
b. the chart convergence of the meridians through $A$ and $B$ ?
c. the approximate GC bearing of A from B , as given by the straight line?

## Solution

a. Rhumb line and straight line are parallel at the mid-meridian. Therefore rhumb line track from A to B $\quad=300^{\circ}(\mathrm{T})$
b. Straight line at $\mathrm{A} \quad=\quad 302^{\circ}(\mathrm{T})$
Rhumb line $=300^{\circ}(\mathrm{T})$

So $1 / 2$ chart convergence $=2^{\circ}$
So chart convergence $=4^{\circ}$
c. Rhumb line bearing of $A$ from $B=120^{\circ}(T)$
$1 / 2$ chart convergence $=2^{\circ}$
So approx. great circle bearing $\quad=\quad 118^{\circ}(\mathrm{T})$

## Questions

1. A Lambert's conformal chart extends from $29^{\circ} \mathrm{N}$ to $45^{\circ} \mathrm{N}$. Standard parallels are at $31^{\circ} \mathrm{N}$ and $43^{\circ} \mathrm{N}$. Chart convergency is ch.Long $\times 0.6$. On this chart is the scale at $43^{\circ} \mathrm{N}$ the same, greater or less than the scale at:
i. $\quad 45^{\circ} \mathrm{N}$
ii. $\quad 37^{\circ} \mathrm{N}$
iii. $\quad 31^{\circ} \mathrm{N}$
2. Using the same chart as Question 1, a straight line track on this chart from $\mathrm{A}\left(40^{\circ} \mathrm{N}\right.$ $12^{\circ} \mathrm{E}$ ) to C which is at $18^{\circ} \mathrm{W}$ passes through B . The direction of this track is $292^{\circ}(\mathrm{T})$ at $A$ and $283^{\circ}(T)$ at $B$.
a. What is the longitude of $B$ ?
b. Give the approximate values of the rhumb line track angle on this route:
i. at $A$
ii. at B
3. A Lambert's Conformal chart extends from $42^{\circ} \mathrm{S}$ to $64^{\circ} \mathrm{S}$. Chart convergency is ch.Long $\times 0.8$.(Convergency Factor). A straight line track on this chart from $\mathrm{K}\left(48^{\circ} \mathrm{S}\right.$ $178^{\circ} \mathrm{E}$ ) to M at longitude $152^{\circ} \mathrm{W}$ passes through L .
The direction of this track at $K$ is $102^{\circ}(T)$ and at $L$ is $096^{\circ}(T)$.
a. What is the longitude of position L?
b. What is the approximate value of the rhumb line track from K to M ?
c. What is the approximate value of the rhumb line track from $K$ to $L$ ?
d. What is the latitude of $M$ ?
4. A Lambert's conical conformal chart has standard parallels at 63 N and 41 N . What is the constant of the cone?
a. . 891
b. 788
c. 656
d. . 707
5. Scale on a Lambert's conformal conic chart
a. is constant
b. is constant along a meridian of longitude
c. varies slightly as a function of latitude and longitude
d. is constant along a parallel of latitude
6. On a conformal chart, the standard parallels are $41^{\circ} 20^{\prime} \mathrm{N}$ and $11^{\circ} 40^{\prime} \mathrm{N}$. What is the constant of the cone?
a. 660
b. . 202
c. 446
d. . 895
7. Scale on a Lambert conformal chart is:
a. constant along a line of latitude
b. constant along a line of longitude
c. constant everywhere
d. correct at the parallel of origin
8. On a Lambert chart, the constant of the cone is .78585. What is the parallel of tangency?
a. $\quad 51^{\circ} 02^{\prime}$
b. $\quad 51^{\circ} 36^{\prime}$
c. $\quad 51^{\circ} 15^{\prime}$
d. $\quad 51^{\circ} 48^{\prime}$
9. On a Lambert's chart the constant of the cone is 0.80 . A is at 53 N 04 W . You plan to fly to $B$. The initial Lambert's chart straight-line track is $070^{\circ}(\mathrm{T})$ and the rhumb line track from $A$ to $B$ is $082^{\circ}(T)$. What is the longitude of $B$ ?
a. 26 E
b. $\quad 34 \mathrm{~W}$
c. 11 E
d. 15E

## Answers

1. i. 45 N (Less)
ii. 37 N (Greater)
iii. 31 N (Same)
2. a. $003^{\circ} 00^{\prime} \mathrm{W}$
b. i) $283^{\circ}(\mathrm{T})$
ii) $283^{\circ}(\mathrm{T})$
3. a. $174^{\circ} 30^{\prime} \mathrm{W}$
b. $\quad \mathrm{K}$ to $\mathrm{M}\left(090^{\circ}(\mathrm{T})\right)$
c. $\quad \mathrm{K}$ to $\mathrm{L}\left(099^{\circ}(\mathrm{T})\right)$
d. 48 S

| 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $b$ | d | c | a | d | a |

## Chapter <br> 23 <br> The Polar Stereographic Chart

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## Introduction

The polar stereographic projection is, of those discussed, the only geometric projection, i.e. true perspective projection. It is constructed using the principle shown in Figure 23.1.

A flat surface is used, touching the North Pole (the point of tangency). The light source is positioned at the South Pole (diametrically opposed), creating a graticule, by geometrical projection, which is shown in the lower part of the diagram.


Figure 23.1 A Polar stereographic projection

## Polar Stereographic Graticule.

The meridians are straight lines, radiating from the Pole. The parallels are concentric circles. However, they are not evenly spaced. Their spacing from the Pole depends on the scale expansion from the Pole.

To determine this, we need to introduce the concept of the co-latitude. The co-latitude is simply ( $90^{\circ}$ - the latitude). Therefore, if the latitude is, say, $60^{\circ}$, the co-latitude will be $30^{\circ}$.

Now consider the light paths in Figure 23.1. Take the light which passes through the Equator. The Equator's latitude is $0^{\circ}$, so the co-latitude is $90^{\circ}$. Half of the co-latitude is $45^{\circ}$. In the triangle formed by the light path above, the opposite and the adjacent are both the radius of the Earth, so we have a right-angled triangle with a bottom angle of $45^{\circ}$.

It can be shown by a geometrical proof that the angle that the light path makes with the axis of the Earth is always a function of half the co-latitude. You are not required to understand the proof, but you should remember the result, which is that the scale expands away from the Pole at a rate of the secant squared of half the co-latitude.

Therefore: $\quad$ scale expansion $=\sec ^{2}(1 / 2$ co-latitude)
In Figure 23.2 you can see that the meridians are straight lines, and the parallels are concentric circles. However, the spacing between the parallels is increasing as the distance from the Pole increases.

The Equator can be projected so that a hemisphere or more can be shown on one chart.


Figure 23.2 Polar stereographic graticule

## Polar Stereographic Properties

The properties of the Polar Stereographic chart are analysed below:-

## Scale

The scale is correct at the Pole (where the paper touches the Reduced Earth). Elsewhere it expands as the $\sec ^{2}$ ( $1 / 2$ co-latitude).

The equation to find scale at any latitude is as follows: (Not examinable in the EASA ATPL exam).
scale at any latitude $=$ scale at Pole $\times \sec ^{2}(1 / 2$ co-latitude)
We will now substitute a particular value of latitude, $78^{\circ}$, and see how this changes the scale.
Assume that the scale at the Pole is $1 / 1000000$.

$$
\text { scale at } 78^{\circ}=\frac{1}{1000000} \times \sec ^{2}(1 / 2 \text { co-latitude })
$$

If the latitude is $78^{\circ}$, then the co-latitude is $12^{\circ}$. Half the co-latitude is $6^{\circ}$. The equation therefore becomes:

$$
\text { scale at } 78^{\circ}=\frac{1}{1000000} \times \sec ^{2}\left(6^{\circ}\right)
$$

This can be written as:

$$
\text { scale at } 78^{\circ}=\frac{1}{1000000} \times \sec \left(6^{\circ}\right) \times \sec \left(6^{\circ}\right)
$$

or, alternatively:

$$
\begin{aligned}
\text { scale at } 78^{\circ} & =\frac{1}{1000000} \times \frac{1}{\cos 6^{\circ}} \times \frac{1}{\cos 6^{\circ}} \\
& =\frac{1}{1000000} \times \frac{1}{0.989074}=\frac{1}{989074}
\end{aligned}
$$

In other words, between latitudes $90^{\circ}$ and $78^{\circ}$, the scale is within $1 \%$ of the scale at the Pole, which can be regarded as a constant scale chart.

A similar exercise can be carried out for the latitude of $70^{\circ}$. Out to $70^{\circ}$ the scale is within $3 \%$ of the scale at the Pole.

To summarize:

## Between latitudes $90^{\circ}$ and $78^{\circ}$ - scale within $1 \%$ of scale at Pole. <br> Between latitudes $78^{\circ}$ to $70^{\circ}$ - scale between $1 \%$ and $3 \%$ of scale at Pole.

This constancy of scale makes the polar stereographic projection an almost ideal map for flying in polar regions. With the breakdown of former political rivalries and the advent of aircraft of long range and endurance, ETOPS procedures, and inertial navigation systems, it has now become common to use polar routes, which are often shorter, and some go very close to the Pole indeed. Consider the situation at Figure 23.3.


Figure 23.3 Track near Pole

Take the $1 \%$ scale zone. The co-latitude of $78^{\circ}$ is $12^{\circ}$. Therefore, the distance from the Pole to latitude $78^{\circ}$ is $60 \mathrm{NM} \times 12=720 \mathrm{NM}$. If the aircraft is entering at one side and leaving more or less on the opposite side of the circle, the total distance in the $1 \%$ zone is around 1400 NM . At typical jet-liner speeds of around 450 knots TAS, that is approximately 3 hours flying time.

A chart which can be considered constant scale and allows measurement of distances with a ruler for over 3 hours flying is a very useful projection.

## Orthomorphism

Meridians are straight lines originating from the pole. Parallels of latitude are arcs of circles centred at the pole. The expansion is at the same rate in any direction from any point. Therefore the projection is orthomorphic.

## Graticule

Meridians are straight lines originating from the pole. Parallels of latitude are arcs of circles centred at the pole.

## Shapes

Because of scale expansion, shapes and areas will be distorted away from the pole.

## Chart Convergence

The meridians converge at the pole at exactly the same rate as they do on the Earth's surface. Therefore, on the projection the convergency remains the same, whereas on Earth it decreases away from the Pole.

This gives the Polar stereographic chart a convergency factor, or ' $n$ ', of 1 . The chart convergence is always exactly equal to the change of longitude between 2 points, whatever the latitude.

Chart convergence is constant all over the chart.
Chart convergence is 'correct', i.e. same as the Earth only at the Pole.

## Rhumb Lines and Great Circles



Figure 23.4 Great circle, rhumb lines and straight lines
In Figure 23.4 a Rhumb Line, a Great Circle and a straight line on the chart have been drawn between longitudes $000^{\circ} \mathrm{E} / \mathrm{W}$ (the Greenwich Meridian) and $090^{\circ} \mathrm{E}$, at 2 different latitudes, $50^{\circ} \mathrm{N}$ and $70^{\circ} \mathrm{N}$.

As with the Lambert chart, the Rhumb Line has the greatest curvature. A parallel of latitude must cut meridians at right angles. Rhumb Lines are always concave to the pole of projection.

As with the Lambert chart, the Great Circle is concave to the parallel of origin. It is just that, on this chart, the parallel of origin is the pole. Therefore, Great Circles are also concave to the pole of projection, but the amount of curvature is less than the Rhumb Line in the same hemisphere.

It is possible to calculate the difference between a Great Circle and a straight line at any latitude, in order to see how closely a straight line approximates to a Great Circle. The angle between the straight line and the Rhumb Line is half chart convergence, whilst the angle between the Great Circle and the Rhumb Line is conversion angle (half Earth convergence).

At $50^{\circ} \mathrm{N}$, the angle between a straight line and a Rhumb Line is half chart convergence. Over a $90^{\circ}$ change of longitude, this is $45^{\circ}$ for a Polar Stereographic chart (' $n$ ' $=1$ ). The angle between the Great Circle and the Rhumb Line is conversion angle, which is $1 / 2$ ch.long $\times \sin$ mean latitude $-1 / 2 \times 90 \times 0.766=341 / 2^{\circ}$.

Difference between straight line and Great Circle $=45^{\circ}-341 / 2^{\circ}=101 / 2^{\circ}$.
This is a large initial difference and therefore it would not be possible to approximate a straight line to a Great Circle at this latitude over this change of longitude. However, if we were flying directly over the Pole, then both the straight line track and the Great Circle track would initially be $000^{\circ}$, then $180^{\circ}$ and there would be no difference. So the amount of change of longitude obviously makes a difference.

At $70^{\circ} \mathrm{N}$, the angle between a straight line and a Rhumb Line over a $90^{\circ}$ change of longitude is also $45^{\circ}$. The angle between the Great Circle and the Rhumb Line is conversion angle, which is $1 / 2$ ch.long $\times \sin$ mean latitude $-1 / 2 \times 90 \times 0.9397=42.3^{\circ}$.

Difference between straight line and Great Circle $=45^{\circ}-42.3^{\circ}=2.7^{\circ}$.
Therefore, at $70^{\circ} \mathrm{N}$, even in the worst case $\left(90^{\circ}\right.$ change of longitude) there is less than $3^{\circ}$ difference between the straight line and the Great Circle. This shows that the difference between a straight line and a Great Circle becomes smaller as the latitude increases.

With any other change of longitude at $70^{\circ} \mathrm{N}$, the difference will be even smaller, falling to zero for a track directly over the pole. A straight line may be taken to be a Great Circle on a Polar Stereographic chart at latitudes greater than $70^{\circ}$.

## Uses of a Polar Stereographic Chart

The chart is used mainly for plotting in Polar regions. Grid and gyro steering technique is usually used as well, not only because of the projections but also because of proximity to the North Magnetic Pole.

Some meteorological charts are on this projection or on a modified polar stereographic.
The chart is also used for planning purposes (shows the Pole and Equator if desired).

## Summary

| PROPERTIES OF A POLAR STEREOGRAPHIC CHART |  |
| :--- | :--- |
| Scale | Correct at Pole. <br> Elsewhere expands as sec ${ }^{2}(1 / 2$ co-latitude $)$ <br> Within 1\% from latitudes $90^{\circ}$ to $78^{\circ}$. <br> Within 3\% from latitudes $78^{\circ}$ to $70^{\circ}$. |
| Orthomorphic | Yes. All charts used for navigation must be. |
| Graticule | Meridians are straight lines radiating from the Pole. <br> Parallels are concentric circles drawn from the Pole. |
| Shapes | Become more distorted as distance increases from the <br> Pole. |
| Chart Convergence | Correct at the Pole. <br> Constant across the chart. <br> Convergence = change of longitude <br> 'n' $=1$ <br> Convergence Factor = 1 |
| Rhumb Lines | Curves concave to the pole of projection |
| Great Circles | Curves concave to the pole of projection, but <br> with less curvature than Rhumb Lines in the same <br> hemisphere. <br> Can be taken as straight lines at latitudes greater than 70 |

## Straight-line Track Problems

## Example 1: What is the initial straight-line from $A(75 N 60 W)$ to $B(75 N 60 E)$ on a Polar

 Stereographic chart?a. $\left.\quad 090^{\circ} \mathrm{T}\right)$
b. $\quad 030^{\circ}(\mathrm{T})$
c. $\quad 120^{\circ}(\mathrm{T})$
d. $\quad 330^{\circ}(\mathrm{T})$

Answer: Start by drawing a diagram of the situation.
The ' $n$ ' factor of a Polar Stereographic chart is 1 - meridians which are 1 degree apart in longitude will have an angle of inclination to each other of 1 degree. As the longitude of $A$ is 60 W and the longitude of $B$ is 60 E , meridians $A$ and $B$ are inclined to each other at $120^{\circ}$.

We also know that $A$ and $B$ are both at latitude $75 N$. Therefore the co-latitude in each case is $15^{\circ}$, i.e. the distance from the Pole to $A$ is the same as the distance from the pole to $B$. So we have an isosceles triangle.


Figure 23.5 Polar stereo situation
The internal angles of a triangle must add up to $180^{\circ}$. If 120 of them are at the apex, that leaves 60 of them to be split equally between $A$ and $B$ (it is an isosceles triangle, remember). So angles $A$ and $B$ are both $30^{\circ}$.

Now, the direction from A (or any other point on the Earth's surface, for that matter) to the North Pole is due north, $000^{\circ}(\mathrm{T})$. Therefore the direction from A to B is $030^{\circ}(\mathrm{T})$. The answer is (b).

Although the question did not ask it, we can also work out the straight line track angle from $B$ to A. In this case, the direction from B to the North Pole is also $000^{\circ}(\mathrm{T})$, also known as $360^{\circ}(\mathrm{T})$ and the direction from $B$ to $A$ is $30^{\circ}$ left of $360^{\circ}(\mathrm{T})$, so it must be $330^{\circ}(\mathrm{T})$.

If the track angle from $B$ to $A$ is $330^{\circ}(T)$ at $B$, going back to $A$, then it must be the reciprocal, $150^{\circ}(T)$, going from $A$ to $B$ as it passes over $B$. In other words, the straight-line track $A$ to $B$ starts with a track angle of $030^{\circ}(\mathrm{T})$ and finishes on a track angle of $150^{\circ}(\mathrm{T})$. We have shown from the geometry of the triangle that the track angle increases by $120^{\circ}$. We should expect this - the chart convergence from $A$ to $B$ is $120^{\circ}$, due to the longitude change of $120^{\circ}$ and the ' n ' factor of 1 .

There is another way of solving this problem - by use of the Rhumb Line. If latitude A is 75 N and latitude $B$ is 75 N then the Rhumb Line track between them is the parallel of latitude of 75 N and therefore must have a Rhumb Line track of $090^{\circ}(\mathrm{T})$.

The chart convergence between 60 W and 60 E is $120^{\circ}$. Therefore half chart convergence is $60^{\circ}$. This is the difference between the Rhumb Line and the straight line. Therefore $60^{\circ}$ anticlockwise from the Rhumb Line track of $090^{\circ}(\mathrm{T})$ gives us an initial straight-line track of $030^{\circ}(\mathrm{T})$.


Figure 23.6 Polar stereo situation - rhumb line track
The same method would work for finding the track from B to A, measured at B. At B, the Rhumb Line track to $A$ is $270^{\circ}(\mathrm{T})$. Sixty degrees clockwise from $270^{\circ}(\mathrm{T})$ is $330^{\circ}(\mathrm{T})$.

Example 2: At what longitude does the straight-line track from A (70N 40W) to B (70N 80E) on a Polar Stereographic chart reach its highest latitude ie the Great Circle Vertex?
a. 040 W
b. 030W
c. 020 E
d. 040E

Answer: Start by drawing a diagram of the situation.


Figure 23.7 Polar stereo situation

The highest latitude occurs at the shortest co-latitude distance, i.e. the shortest distance from the Pole. This shortest distance to the Pole must be when the line between the Pole and the track is at right angles to the track, as in Figure 23.8.

This must occur at mid-meridian - at 020E.
Answer (c).


Figure 23.8 Right angles to track

Example 3: On a Polar Stereographic map, a line is drawn from position A (70N 102W) to position $B(80 N 006 E)$. The point of highest latitude along this line occurs at 035 W . What is the initial straight-line track angle from $A$ to $B$, measured at $A$ ?
a. $\quad 049^{\circ}(\mathrm{T})$
b. $\quad 077^{\circ}(\mathrm{T})$
c. $\quad 229^{\circ}(\mathrm{T})$
d. $\quad 023^{\circ}(\mathrm{T})$

Again, start by drawing a diagram:
This is clearly a new sort of problem, because A and B are at different latitudes. There appears to be no isosceles triangle or right-angled triangle.


Figure 23.9
However, we have not used all the information in the question yet. The point of highest latitude must have some significance otherwise presumably the examiner would not have mentioned it. So let us draw that in:

Now we have a right-angled triangle, between A, the North Pole, and the point of highest latitude. Angle $b$ is the difference between 035 W and 102 W , so it is $67^{\circ}$. Angle c is $90^{\circ}$. The internal angles of a triangle must add up to $180^{\circ}$. So angle a must be $23^{\circ}$.

Answer 'd'.


Figure 23.10

## Chapter <br> 24 <br> Time (1)

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## Time General

Why do pilots need to study time? There are 2 main applications:
You should know why different countries keep different times and how to find what the standard time is in any country you may land in. Most laymen (and some pilots when using the Public Address to passengers) call this the 'local time', but the correct term is 'Standard Time'.

You need to be able to work out when it gets dark in different parts of the world at different times of the year (or when it gets light in the morning), especially if the airport of destination does not have airfield lighting.

## Time Basics

Our measures of time are based on the Earth's rotation about its own axis, the Earth's rotation around the Sun, and the movement of the solar system in our galaxy and the movement of our galaxy in the Universe. We therefore need to know something about elementary astronomy.

## Solar System - Planetary Orbits - Kepler's Laws

The solar system consists of the Sun and major planets (of which the Earth is one).
Planetary orbits (and therefore the Earth's orbit) are governed by Kepler's laws of planetary motion which are:

- The orbit of each planet is an ellipse with the Sun at one of the foci.
- The line joining the planet to the Sun, known as the radius vector, sweeps out equal areas in equal time.
- The square of the sidereal period of a planet is proportional to the cube of its mean distance from the Sun.

The important laws, the first 2 above, are illustrated in the following diagram:


Figure 24.1 Kepler's laws

## Kepler's First Law

A planet travels around the Sun in an elliptical orbit as shown. The Sun is at one of the foci, F1, of the ellipse as indicated (the second focal point is shown at F2 but may be ignored). Because of its elliptical orbit, the planet will be at its closest point at P, known as 'perihelion' (Greek peri - near, helios - sun). At position A, the planet is at a point furthest from the Sun known as 'aphelion'. Perihelion occurs in early January (approx 4th). Aphelion occurs in early July (approx 4th).

## Kepler's Second Law

At position B, the planet approaches 'aphelion' (A) and the radius vector SB will sweep out the area SBA in the time between $B$ and $A$. However, a corresponding situation at $Q$ as the planet approaches 'perihelion' $(P)$ requires the radius vector SQ (which is shorter than $S B$ ) to move faster than SB to ensure that the area SQP has the same area as SBA. Thus the planet moves faster around its orbit near perihelion than it travels around its orbit near aphelion.

In summary, in an elliptical planetary orbit, the orbital speed is fastest at perihelion and slowest at aphelion.

The Earth's orbital situation as viewed from the North Celestial Pole (NCP), i.e. the point in the sky directly above geographic North Pole, is illustrated in Figure 24.2.


Figure 24.2 Earth orbit
Notes: The Earth rotates about its geographic N/S axis in an anticlockwise direction when viewed from the NCP. This rotation determines our measurement of a 'day', (to be discussed later.)

The Earth orbits the Sun in an anticlockwise direction when viewed from the NCP. The orbital
period determines our measurement of a 'year'. The changing orbital speed affects our measurements of a day (to be discussed later).

## The Seasons

It is a common misconception that the changes of seasons are governed by the Earth's distance from the Sun. This is not true. As the Sun's heat is transmitted radially, the effect of the distance change between 91.4 million statute miles at perihelion and 94.6 million statute miles at aphelion would only be to change the amount of heat received by the Earth by about 3\%, not enough to explain the marked changes between the summer and winter.
The predominant cause of the Seasons is the inclination (tilt) of the Earth. The Earth's axis is inclined (tilted) at an angle of $66.5^{\circ}$ to its orbital plane and this is often stated as $23.5^{\circ}$ to the normal to the orbital plane $\left(90^{\circ}-66.5^{\circ}=23.5^{\circ}\right)$.


Figure 24.3 The Earth's orbit
Adding the inclination of the Earth to a diagram of the Earth's orbit as in the figures below illustrates the annual seasonal changes.


The Sun appears vertically above $231 / 2^{\circ} S$ on the 21 st of December. This is known as:-
NH - Winter Solstice SH - Summer Solstice

The Sun will appear above $231 / 2^{\circ} \mathrm{N}$ on the 21 st of June at the:-
NH - Summer Solstice SH - Winter Solstice
The Sun crosses the Equator from South to North on about 21st March. This is known as:-
NH - Spring Equinox SH - Autumn Equinox
Six months later it crosses the Equator from North to South on about 21st September.
NH - Autumn Equinox SH - Spring Equinox
During the period of one day the Sun appears to move along a parallel of latitude from East to West.

A simplified diagram is given below for definition purposes.


Figure 24.5 Earth's orbit (side view)
In the diagram above, the plane of the Earth's orbit around the Sun is labelled the 'Plane of the Ecliptic'. This is an adequate definition although some textbooks refer to the Ecliptic as the plane of the annual path of the Sun.

The plane of the Equator is called the 'Plane of the Equinoctial'. (Equal day/night but see later).

The Plane of the Ecliptic and the Plane of the Equinoctial are inclined to each other at an angle of $23.5^{\circ}$. (This angle is called the obliquity of the ecliptic).

At a given time of year, the angle that the Sun is above or below the Equator determines the season and affects the length of daylight/night. The angle is known as DECLINATION. Declination is analogous in the sky to latitude on the Earth.

The Sun's declination changes annually between $23.5^{\circ} \mathrm{N}$ (Sun overhead the tropic of Cancer) through $0^{\circ}$ (Sun overhead the Equator) to $23.5^{\circ}$ S (Sun overhead the Tropic of Capricorn) and then back through $0^{\circ}$ to $23.5^{\circ} \mathrm{N}$. The annual cycle of the Sun's declination is illustrated in the diagram below. When the sun, or any planet, is overhead an observer at an altitude of $90^{\circ}$ it is at its Zenith.


Figure 24.6 Declination of the Sun
The declination of the Sun may be considered to follow a sine wave with its peak amplitude being $23.5^{\circ}$ and its cyclic period being one 'year'.

You may notice that the dates of the solstices and equinoxes may be quoted in some textbooks as about 21st, 22nd, of the appropriate month. This occurs simply because the precise date depends on the relationship between the year considered and the leap year cycle. Students are not expected to be able to calculate the precise date of these solar events.

The length of daylight/night at a given latitude varies with the declination of the Sun. The rate of change of the length of daylight will therefore be greatest when the rate of change of declination is greatest. The diagram shows that this situation occurs at the equinoxes (about Mar 21 and Sep 21).

The above situation is not true in several special cases, which will be discussed later. These are when the latitude considered is either the Equator or is above $66^{\circ} \mathrm{N} / \mathrm{S}$.

## Measurement of Days and Years

A 'day' may be defined as the length of time taken for the Earth to rotate once about its axis measured against a celestial body, (the Sun or a star). Measurements against a star are called 'sidereal' and against the Sun are called 'solar'.

## 'Civil' Day

A 'civil' day should be:

- Related to periods of light and darkness so that 1200 hrs is always about halfway between sunrise and sunset. The civil day should therefore be based on the Sun.
- Of a constant length.


## Sidereal Day

A sidereal day is measured against a distant star and is of nearly constant length. However, it is not related to light and dark and is not suitable as a civil day.

## Apparent Solar Day

An apparent solar day is measured against the real or apparent Sun (the one that 'appears' to you). However, using the apparent (real) Sun introduces the problem of the apparent solar day not being a constant length.

Consider the situation including the Earth's orbit around the Sun, illustrated in the diagram below showing the Earth's orbit viewed from the North Celestial Pole (NCP). The NCP is an imaginary point at a distance along the continuation of the Earth's axis from the $S$ pole to the N pole projected out into space.


Figure 24.7 Sidereal day/apparent solar day

Consider the Earth at position A but in a solar system where the Earth was stationary. An observer at position Z would have the Sun and a distant star directly over his meridian. After one complete anticlockwise rotation of the Earth, the Sun and the star would be over the observer's meridian again. The apparent solar day and the sidereal day (based on the star) would be equal. This is a false situation.

In the period of one $360^{\circ}$ revolution, the Earth travels around its orbit to position B. After a $360^{\circ}$ revolution, the distant star is again over the observer's meridian (a sidereal day) but an additional rotation and further orbit to position C is needed to put the Sun again over the observer's meridian. Thus:

An apparent solar day is longer than a sidereal day.
But, as stated before, the Earth's orbital speed changes throughout the year.
Thus, an apparent solar day cannot be of constant length.

## Mean Solar Day

The mean solar day is the average length of an apparent solar day (averaged over the year).
It is of constant length and related to light and darkness; it is used as the 'civil' day and is divided into hours, minutes and seconds of 'mean' time.

It is sometimes helpful to think of the Sun travelling westwards around the Earth rather than the Earth spinning eastwards. In the case of mean time, we consider the mean (average) Sun circling the Earth every 24 hours. This is the basis of Local Mean Time (LMT).

The maximum difference between mean time and apparent (real) Sun time is about 16 minutes and occurs in mid-November. A second maximum occurs in mid-February at about 14 minutes difference. In between these maxima, the difference reduces. The difference is known as the Equation of Time.


Figure 24.8 The equation of time

In November, the real Sun crosses an observer's meridian at 1144 whereas the mean Sun crosses the meridian 16 minutes later at 1200 LMT (by definition).

In February, the real Sun crosses a meridian at 1214 whereas the mean Sun crosses 14 minutes earlier at 1200 LMT (by definition).

Year
A Sidereal Year is the time taken by the Earth to complete an orbit of the Sun measured against a distant star. Its length is 365 days 6 hrs.

A Tropical Year (an apparent solar year) is the length of one cycle of the seasons. Its length is 365 days, 5 hrs and 48.75 minutes.

A Calendar Year is normally 365 days. It is kept in step with the tropical year by adding a day every 4th year, a 'leap' year. A fine adjustment is made on 3 occasions every 400 years. (At a centennial, when the first 2 numbers of the century are not divisible by 4 , the leap year is omitted.)

## Hour Angle

The Declination of a celestial body (in our case, the Sun) has been shown to be similar (analogous) to latitude. In the same way, Hour Angle can be shown to be analogous to longitude.

The Earth spins in an easterly direction, $360^{\circ}$ in every $\mathbf{2 4}$ hours. Thus, a celestial body (the Sun or a star) will transit across a given meridian at 24 hour intervals. It is convenient to simplify this situation by considering that the celestial bodies circle the Earth in westerly directions. The Sun rises in the East and travels in a westerly direction to set in the West and continues westerly to eventually rise again in the East.

The Hour Angle of a celestial body is defined as the arc of the Equator (equinoctial) intercepted between the meridian of a datum and the meridian of the body, measured westwards from $0^{\circ}$ to $360^{\circ}$.

Thus when a celestial body transits a given meridian, its Hour Angle is $000^{\circ}$. When the body transits the anti-meridian, its Hour Angle is $180^{\circ}$. If the given meridian is Greenwich, the Hour Angle is known as the Greenwich Hour Angle (GHA) which is directly analogous to longitude. A body with a GHA of $050^{\circ}$ will be transiting the 050W meridian. A body with a GHA of $180^{\circ}$ will be transiting the 180 W meridian. But a body with a GHA of $270^{\circ}$ will be transiting the 270W meridian, or the 090E meridian.


Figure 24.9 Local hour angle/Greenwich hour angle

## Questions

1. When does perihelion occur?
a. early January
b. mid March
c. early July
d. September 21
2. When does aphelion occur?
a. early January
b. mid March
c. early July
d. September 21
3. Viewed from the North Celestial Pole (above the North Pole), the Earth orbits the Sun:
a. clockwise in a circular orbit
b. anticlockwise in a circular orbit
c. clockwise in an elliptical orbit
d. anticlockwise in an elliptical orbit
4. When do the 'equinoxes' occur?
a. December and June
b. February and November
c. March and September
d. January and July
5. When it is the Winter Solstice in the Southern hemisphere, the Declination of the Sun is:
a. $\quad 0^{\circ} \mathrm{N} / \mathrm{S}$
b. $\quad 231^{1} 2^{\circ} \mathrm{N}$
c. $\quad 661^{1} 2^{\circ} \mathrm{N}$
d. $\quad 231 / 2^{\circ} S$
6. In the situation given above in Question 5, the Sun will be overhead:
a. the Arctic Circle
b. the Tropic of Capricorn
c. the Equator
d. the Tropic of Cancer
7. What is the angle between the Equinoctial and the Ecliptic?
a. $\quad 661^{1} 2^{\circ}$
b. $\quad 23 \frac{1}{2^{\circ}}{ }^{\circ}$
c. varies between $231^{1} 2^{\circ} \mathrm{N}$ and $231^{1 / 2}{ }^{\circ} \mathrm{S}$
d. varies between $661 / 2^{\circ} \mathrm{N}$ and $661 / 2^{\circ} \mathrm{S}$
8. The declination of a celestial body (the Sun) measured on the Celestial Sphere is analogous (equivalent) to $\qquad$ on the Earth?
a. latitude
b. longitude
c. altitude of the body measured from the sensible horizon
d. co-latitude
9. 'The length of daylight/night depends upon the declination of the Sun and the latitude of the observer'. When is the rate of change of the length of daylight greatest?
a. February/November
b. January/July
c. at the Equinoxes
d. at the Solstices
10. A sidereal day is:
a. longer than an apparent solar day
b. longer than a real solar day
c. shorter than an apparent solar day
d. equal to a real solar day
11. The maximum difference between mean noon (1200LMT) and real/apparent noon occurs in $\qquad$ ?
a. January/July
b. March/September
c. November/February
d. December/June
12. The maximum difference between mean time and apparent time is:
a. $\quad 21$ minutes
b. $\quad 16$ minutes
c. $\quad 30$ minutes
d. there is no difference
13. What is the length of a Sidereal Year?
a. $\quad 365$ days
b. $\quad 366$ days
c. $\quad 365$ days 6 hrs
d. $\quad 365$ days 5 hrs 48.75 minutes
14. 'The Calendar Year and the Tropical Year are of different lengths. The difference is adjusted partly by using leap years every fourth calendar year. However, some years are not designated as leap years'. Which of the following years will be a leap year?
a. 2001
b. 2100
c. 2300
d. 2400
15. The Hour Angle (Greenwich Hour Angle) of a celestial body is analogous/ equivalent on the Earth to $\qquad$ ?
a. latitude
b. longitude
c. co-latitude
d. UTC
16. A star has a Greenwich Hour Angle (GHA) of $220^{\circ}$. Which meridian is the star transiting (crossing)?
a. 040 W
b. 040E
c. 140 W
d. 140E

Answers

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a | c | d | c | b | d | b | a | c | c | c | b |


| 13 | 14 | 15 | 16 |
| :---: | :---: | :---: | :---: |
| c | d | b | d |

## Chapter

## 25 <br> Time (2)

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25
Time (2)

## Conversion of Arc (Angle) to Time

To solve time problems we need to be able to convert angular arcs to time in hours and minutes.

Because the Earth rotates $360^{\circ}$ in 24 hrs , we can convert angular arc to time as follows:

```
360}=24 hrs
15
1 }\mp@subsup{}{}{\circ}=4\mathrm{ minutes
15' of a degree of arc = 1 minute of time
15"(seconds of arc) = 1 second of time
```

Note: You are only required to work to the nearest minute of time. Conversion of angular arc to time can be achieved using the table on the last page of the Air Almanac extract (reproduced at the end of this chapter). Angular arcs from $0^{\circ}$ to $360^{\circ}$ are converted to hours and minutes of time in the first 5 columns. The final column is used to convert minutes of angular arc into minutes and seconds of time.

Example: Convert $137^{\circ} 36^{\prime}$ of arc to time. Use the arc/time table (see page 414).
$137^{\circ}$ of arc converts to 9 hours 08 minutes of time.
36 ' of arc converts to 2 minutes 24 seconds
The final solution is, 9 hours 10 minutes and 24 seconds, but rounding the answer gives:
Answer: 9 hours 10 minutes
You may not have the arc/time page from the Air Almanac. In this case, you must calculate the arc to time conversion. In most cases, the arc of longitude will be in whole degrees.

Example 1: Convert $127^{\circ}$ of arc into time.
Divide $127^{\circ}$ by $15=8.4667$ hours
The answer is now in hours and decimals of hours but the question may required the answer to be in hours and minutes
Convert the decimals of hours (.4667) to minutes by multiplying by $60-.4667 \times 60=28$
Answer: 8 hours 28 minutes
Example 2: Convert 096 ${ }^{\circ} 17^{\prime}$ of arc into time.
Express the arc in decimal form by dividing the minutes by $60 .(17 \div 60)=.283$
Arc $=096.283^{\circ}$
Divide arc by 15 to give decimal time
$096.283^{\circ} \div 15=6.4189$ hrs.
Convert decimal hours back to minutes
$.4189 \times 60=25$ minutes (to the nearest minute)
Answer: 6 hours 25 minutes

These problems are most easily solved using the dms function of a scientific calculator. We recommend that, in problems that appear in this book, students calculate the arc/time conversion and practise using the dms function on the calculator. Answers can then be checked against the arc/time table from the Air Almanac.

Example: Convert $137^{\circ} 36^{\prime}$ of arc to time.
The actual buttons to be pressed depend on which particular calculator you have bought. With the Casio series that Oxford recommends, the dms button is marked ${ }^{\circ}$ ' ". To convert arc to time, press the following sequence:

137 (dms) You will then see $137^{\circ}$ in the window.
36 (dms) You will then see $137^{\circ} 36^{\circ}$ in the window.
$\div 15=$ You will then see $9^{\circ} 10^{\circ} 24-9$ hours, 10 minutes, and 24 seconds.

## Local Mean Time

The Earth rotates daily around its geographic axis, anticlockwise if observed from above the North Pole. It is convenient sometimes to consider the Earth as stationary and the Sun travelling around the Earth once a day in a clockwise direction. This alternative fits well with our perception that the Sun rises in the East and sets in the West. For further discussions, this alternative will be used and, unless otherwise stated, the Sun referred to is the 'mean' Sun.

When the mean Sun transits (crosses) a particular meridian, the Local Mean Time (LMT) at all places on that meridian is $\mathbf{1 2 0 0}$ hrs (midday, noon).

Similarly, when the mean Sun transits the anti-meridian of a point, the LMT at the point is 0000hrs ( 2400 hrs ) or midnight. Conventionally, midnight of a particular night, say the night of the 6 th/7th, is regarded as 2400 hrs LMT on the 6 th or 0000 hrs on the 7 th.


Figure 25.1 Local Mean Time (LMT)

Figure 25.1 illustrates the situation when the mean Sun is transiting (crossing) the meridian of 45E on 16th May (the date is chosen simply for illustrative purposes). Thus the LMT at all places on the 45E meridian (Baghdad, Aden, Madagascar) is 1200 LMT on 16th May (local mean date).

Remember, it takes the Sun 1 hour to travel $15^{\circ}$ of longitude.
At 45E, the LMT is 1200 hrs on 16th May, (the Sun is crossing the meridian of 45E).
At 90E (approx India) the LMT is 1500 hrs on 16th May (the Sun passed the meridian 3 hours ago : $45=3 \times 15=3 \mathrm{hrs}$ )

At 135E (approx Japan) the LMT is 1800 hrs on 16th May.
At 180E (mid-Pacific in the Eastern hemisphere) the LMT is 2100 hrs on 16th May.
Now working to the west from the $045^{\circ} \mathrm{E}$ meridian.
At 0E/W (the Greenwich Meridian), the LMT is 0900 LMT on 16th May.
LMT at the Greenwich Meridian is known as Greenwich Mean Time (GMT) also called Zulu ' $Z$ ', which used to be the Earth's standard time. The present standard time is called Coordinated Universal Time (UTC) which for all practical purposes is the same as GMT.

At 45W (Newfoundland) the LMT is 0600 LMT on the 16th May.
At 90W (mid USA), the LMT is 0300 LMT on the 16th May.
At 135W (Alaska), the LMT is 0000 LMT on the 16th May. This time may be considered to be also 2400 hrs LMT on the 15th May.

At 180W (mid Pacific in the Western hemisphere), LMT is 2100 LMT on the 15th May. But, from the previous argument, at $180^{\circ} \mathrm{E}$, the LMT is 2100 LMT on 16th May.

Thus:
At the $180^{\circ}$ meridian, the LMT date changes by a day although the LMT time remains the same. Thus:

Travelling westward across the $180^{\circ}$ meridian, the LMT date increases by one day. Travelling eastward across the $180^{\circ}$ meridian, the LMT date decreases by one day.

This situation is illustrated in Figure 25.2 using an LMT at Greenwich of 0300 LMT on the 19th (0300 UTC on UTC date 19th).


Figure 25.2 LMT 2

## Co-ordinated Universal Time (UTC)

Co-ordinated Universal Time (UTC) changes at a constant rate and is regulated against International Atomic Time (IAT) derived from atomic clocks. UTC is regularly corrected to match GMT (the LMT at the Greenwich Meridian) but the corrections are very small. For all practical navigation purposes, UTC equals GMT. UTC is the datum for world time.

It is often necessary to convert LMT into UTC or vice versa. Based on the fact that the Sun appears to travel across the Earth from east to west, the following rule applies:

Longitude east, UTC least (less advanced)
Longitude west, UTC best (more advanced)

## Example:

Find the UTC if the LMT in Cairo (longitude 30E) is 0900 hours.

| LMT Cairo | 0900 |
| :--- | :--- |
| Arc/Time (Long.30E) | -00200 |
| UTC | $\underline{\mathbf{0 7 0 0}}$ (UTC LEAST) |

Find the UTC if the LMT in Madrid (longitude 04W) is 0400 hours.

| LMT Madrid | 0400 |
| :--- | :--- |
| Arc/Time (Long.04W) | $+\frac{0016}{\mathbf{0 4 1 6} \text { (UTC BEST) }}$ |
| UTC |  |

Find the LMT in Berlin (longitude 13E) if the UTC is 1345 hours.

| UTC | 1345 (UTC LEAST) |
| :--- | ---: |
| Arc/Time | $+\frac{0052}{1437 \text { (Long. East) }}$ |

Find the LMT in San Francisco (longitude 122W) if the UTC is 2115 hours.

| UTC | 2115 (UTC BEST) |
| :--- | :---: |
| Arc/Time | $-\frac{0808}{1307}$ (Long. West) |

## Local Mean Time/UTC Problems

Note that in basic LMT problems, the latitude is unimportant. For instance, all points on the 75W meridian have the same LMT. This example includes Ottawa (Canada), Philadelphia (USA), Kingston (Jamaica), Bogota (Colombia) and Lima (Peru).

We recommend that students lay out problems logically and always work through UTC as in the examples below.

## Example 1

If the LMT at ' $A^{\prime}, 40^{\circ} \mathrm{N} 137^{\circ} 50^{\prime} \mathrm{W}$, is 1812 LMT on 18 August, what is the LMT at ' $\mathrm{B}^{\prime}, 30^{\circ} \mathrm{S}$ $121^{\circ} 12^{\prime} \mathrm{E}$ ?

|  | Day (D) | Hour (H) | Minute (M) |  |
| :--- | :--- | :--- | :--- | :--- |
| LMT at A | 18 | 18 | 12 | LMT |


| Arc/Time conversion $137^{\circ} 50^{\prime} \mathrm{W}$ <br> (Last page of AA) |  | +9 | 11 |  |
| :--- | :--- | :---: | :---: | :--- | :--- |
| UTC (long. West, UTC best) | 19 | 03 | 23 | UTC |
| Arc/Time conversion $121^{\circ} 12^{\prime} \mathrm{E}$ |  |  |  |  |

Answer: 1128 LMT on 19th August

Note: This question could have been completed without UTC. The ch.long between A and $B$ is $259^{\circ} 02^{\prime}$ minutes which converts as 17 hours 16 minutes later than A's LMT. The method through UTC seems longer but it is too easy to make a mistake if you try to take short cuts, especially if the date changes. Always use the method above and work through UTC.

## Example 2

If it is 1012 UTC on 15 Dec at 50N 030W, what is the LMT in:
a. Singapore $\left(00^{\circ} 30^{\prime} \mathrm{N} 105^{\circ} 00^{\prime} \mathrm{E}\right)$
b. Hawaii $\left(22^{\circ} 00^{\prime} \mathrm{N} 155^{\circ} 00^{\prime} \mathrm{W}\right)$

Note: UTC is the same all over the world. Giving the initial position as $50^{\circ} \mathrm{N} 030^{\circ} \mathrm{W}$ is a 'red herring'.

| a. | D | H | M |  |
| :--- | :--- | :--- | :--- | :--- |
| Arc/Time $105^{\circ} \mathrm{E}$ | 15 | 10 | 12 | UTC |
|  |  | +7 |  |  |
| LMT Singapore (long. E, UTC least) | 15 | 17 | 12 | LMT |
| b. | D | H | M |  |
| Arc/Time $155^{\circ} \mathrm{W}$ | 15 | 10 | 12 | UTC |
| LMT Hawaii (long. W, UTC best) | 14 | 23 | 52 | LMT |

## Zone Time (ZT)

Although UTC is the worldwide time datum for aviation purposes, it has little civil use far from the Greenwich meridian. Humans tend to rise some time in the morning and go to bed some time in the late evening. On a cruise ship in mid Pacific, it would be disappointing if you chose to go up on deck to sunbathe when your watch was reading 1000 hrs UTC! This problem was recognized centuries ago and the system of Zone Time (ZT) was used to keep ships' time in synchronization with the periods of light and dark.

Zone Time was based on the fact that the Sun transits 15 degrees in one hour. The datum for the Zone Time system was the Greenwich Meridian ( $0 \mathrm{E} / \mathrm{W}$ ). All ships within longitudes 7.5 W and 7.5 E would keep a Zone Time the same as GMT (UTC). The correction from Zone Time to GMT (UTC) would be zero.

Ships between longitudes 7.5E to 22.5E would have a Zone Time one hour later than GMT (UTC). This zone was numbered -1 , the correction to be applied to Zone Time to calculate GMT (UTC). Similarly, ships between longitudes 22.5 E to 37.5 E would be in the zone numbered -2 . The numerical labels continue as minus values up to 180 E .

For ships in the Western hemisphere, a similar system applied. Thus, ships at longitudes between 7.5 W and 22.5 W would be in the zone numbered +1 , i.e. the zone time is earlier than UTC and 1 hour must be added to Zone Time to calculate UTC. The numerical labels increase as plus values to 180 W .

The system is illustrated in the following diagram:


Figure 25.3 Zone time

- Earth divided into 25 zones of $15^{\circ}$ longitude ( 1 hour), except the 2 semi-zones M \& Y adjacent to the International Date Line.
- Each zone is $7.5^{\circ}$ about a central meridian, firstly the Greenwich Meridian then every $15^{\circ}$. Zones are assigned a letter (see above).
- Zone time varies from UTC (Z time) by increasing periods of 1 hour counting east (+) or west (-) from 'Z' (eg Singapore = zone G \{+7 hr].


## Zone Time Calculations

You may be required to calculate the zone number for a given longitude or to convert between Zone Time and UTC.

To calculate the zone number, divide the longitude given by 15 and round to the nearest whole number with .5 rounding up. Easterly longitudes have negative (minus) zone numbers from -1 to -12 . Westerly longitudes have positive (plus) zone numbers form +1 to +12 . Remember, the zone number is applied to the Zone Time to give UTC. To convert from UTC to Zone Time, the +/- signs need to be reversed. These rules can be confusing and it may be easier to remember:

If it's midday in London, it's evening in the Far East and morning in the Wild West.
Or we are having
'Breakfast at Tiffany's (New York), Lunch in London, Tea in India'.
Mathematically:
UTC $=$ Zone Time + Zone Number

## Example 1

What is the zone number for position $\mathrm{A}, 40 \mathrm{~S} 137 \mathrm{E}$, and position $\mathrm{B}, 60^{\circ} 00^{\prime} \mathrm{N} 097^{\circ} 30^{\prime} \mathrm{W}$ ?
Note: latitude is irrelevant in these cases.

## Position A Zone number $=137^{\circ} \div 15$

$$
\begin{aligned}
& =9.13 \mathrm{hrs} \\
& =9 \mathrm{hrs} \text { (rounded to nearest whole number). }
\end{aligned}
$$

## Longitude E is negative

## Zone Number of position $A=-9$

Position B Longitude $097^{\circ} 30^{\prime} \mathrm{W}=97.5 \mathrm{~W}$

$$
\begin{aligned}
\text { Zone number } & =97.5 \div 15=6.5 \mathrm{hrs} \\
& =7 \mathrm{hrs}(.5 \mathrm{~s} \text { round up })
\end{aligned}
$$

## Longitude W is positive

## Zone Number of position $B=+7$

## Example 2

At position 10S 130E, the Zone Time is 0600 ZT on Zone Date 01 Jan 00. What is the UTC?

$$
\text { Zone Number }=130 \div 15=8.66
$$

$$
=-9 \text { (East) }
$$

UTC = Zone Time + Zone Number
$=010600(Z \mathrm{~T}) \mathrm{Jan}+(-9)$

## = 312100 UTC December 99

## Example 3

At 0300 UTC on 17 May, what is the Zone Time and date at 20N 155W?
Zone Number = 10.33
= +10 (West)

UTC = Zone Time + Zone Number
Therefore: Zone Time = UTC - Zone Number

$$
\begin{aligned}
& =170300 \text { May }-(+10) \\
& =161700 \text { May }
\end{aligned}
$$

Examples 2 and 3 have been completed above using an algebraic notation in the equations. Common sense should tell you that in Example 2, the UTC should be less than the Zone Time. Similarly, in Example 3, the Zone Time should be less than the UTC.

## Standard Time

Zone Time (ZT) gives a convenient standard for ships at sea, keeping the hours of the day approximately synchronized with daylight and darkness. However, the system would not be appropriate for use on land. For example, the 7.5E longitude line runs through the middle of Nigeria - it would be inconvenient if western Nigeria kept a time 1 hour in advance of eastern Nigeria.

Similarly, Local Mean Time (LMT) is inappropriate for use in a country as a standard time. In UK, we would have a situation where at 1200 LMT in Greenwich, the time at Oxford $\left(001^{\circ} 20^{\prime} \mathrm{W}\right.$ ) would be 1155 LMT and the time in Penzance ( 005 W ) would be 1140 LMT. This situation did exist up until the mid-1800s in the UK when lines of communication were slow. With the advent of the railway system, the need for a standardized system was recognized. Indeed, in those very early days, train guards leaving London carried a watch set to Greenwich time. At each station out of London, the station clock would be reset to the guard's watch (GMT) and, subsequently, church wardens would set church clocks to the station clock, i.e. to London time. Thus, each town in the UK set its clocks to a standard time for the country; this was the foundation of the Standard Time (ST) system.

For the UK, the Standard Time (ST) is UTC (GMT) at all times of the year. In summer, we adjust our clocks forward 1 hour and refer to the time as British Summer Time (BST), a time which is one hour added on to our Standard Time (UTC). Summer Time/Daylight Saving Time corrections are discussed later in these notes.

Each country nominates its own Standard Time (ST), normally notified in the Air Almanac as a correction to UTC. Clearly, each country aims to fix a Standard Time such that 1200 hr ST occurs about the middle of the period of light and 0000 hr ST in about the middle of the period of darkness. Thus Standard Time will be close to both Local Mean Time (LMT) and to Zone Time (ZT) but will not be exactly the same.

Standard Time corrections for most countries are listed in the Air Almanac on pages 1 to 4 inclusive. Countries are listed in 3 general areas:

| List 1 (pages 1,2) | Countries generally to the east of the Greenwich meridian. |
| :--- | :--- |
| List 2 (top of page 3) | Countries close to or on the Greenwich meridian. |
| List 3 (pages 3,4) | Countries generally to the west of the Greenwich meridian. |

Note: The Air Almanac extract issued for exam purposes is a copy from a 1992 Air Almanac. Country names, (for example USSR) may have changed and the corrections may also have changed. For exam purposes, the extract should be treated as valid.

## List 1

List 1 (pages 1,2 ) gives Standard Time Corrections for countries mainly east of Greenwich, whose Standard time will be 'fast' on UTC, i.e. Standard Time is later than UTC time. The note under the title shows how the corrections should be applied; time corrections given should be added to UTC to give Standard Time, or subtracted from Standard Time to give UTC.

Countries with a large E-W extent may be subdivided. Thus, Standard Time corrections for Australia change depending on the state involved. EASA questions about countries like Australia will specify the state involved. Also, the USSR is divided by states; EASA time questions involving the USSR will give the state of the place concerned.

Some countries do not carry an exact number of hours as a correction. This choice is entirely at that country's discretion. Also there are a few unusual corrections: Chatham Island at 12 hrs 45 min. and Tonga Islands at 13 hrs . These latter countries are actually in the Western hemisphere, and the corrections are made because the countries involved wish to keep the same day as their neighbours in the Eastern hemisphere. This is discussed later in this section.

## List 2

List 2 countries (top of page 3) are places close to the Greenwich Meridian which normally keep UTC as their Standard Time. Note that this list includes Great Britain.

## List 3

List 3 countries (pages 3,4 ) are those countries generally to the west of Greenwich whose Standard Time is 'slow' on UTC, i.e. less than UTC. The corrections given should be subtracted from UTC to give Standard Time or added to Standard Time to give UTC.

Some countries, (for example Canada) are divided by province, and even within some of those provinces (Northwest Territories) the correction is further subdivided by longitude. ATPL questions involving these areas will provide all the necessary detail. The USA is divided by states with Alaska being sub-divided by longitude. You do not need to use terms such as 'Eastern Standard Time' which are not given in the Air Almanac.

An anomaly does exist in the USA listing for the Aleutian Islands given as 10 hrs , but this will be discussed later.

## SummerTime/Daylight Saving Time (DST)

Some countries in all lists may carry Summer Time or Daylight Saving Time (DST) (USA) in the summer months. This is normally indicated by an asterisk * against the country listed. Summer time is normally 1 hour in advance of Standard Time. Thus if the Standard Time in UK in June is 1200 (UTC), the British Summer Time is 1300 BST. If the Daylight Saving Time in New York, USA in June is 0900 DST, the Standard Time is 0800 ST.

Exam questions will clearly indicate if Summer Time/Daylight Saving Time corrections are to be applied. However, if a country is listed with an asterisk (*), read the notes at the bottom of the page.

## Standard Time Anomalies

Remember, Local Mean Time and Zone Time always change by a day at the $180^{\circ} \mathrm{E} / \mathrm{W}$ meridian. Thus, it would be logical to conclude that Standard Time changes by 1 day at the $180^{\circ} \mathrm{E} / \mathrm{W}$. This is not always so.

In List 1, Chatham Island ( $44^{\circ} \mathrm{S} 176^{\circ} \mathrm{W}$ ) approximately 400 NM south-east of New Zealand, is listed with a correction of 12 hours 45 minutes. The island is administered by New Zealand and populated by New Zealanders (and sheep). However, it lies in the Western hemisphere whereas New Zealand lies in the Eastern hemisphere. Thus, Chatham Island's date should be one day earlier than New Zealand's date, e.g. when it is Sunday on Chatham Island, it will be Monday in New Zealand. This is administratively inconvenient. To correct the problem the 'dateline' which should be on the 180E/W meridian is moved to the east so that, instead of being 176W, the island is considered, for time, to be at 184E. The adjusted dateline is known as the International Date Line - a line generally following the 180E/W meridian but occasionally adjusted to east or west for political/administration purposes.

A similar situation occurs with the Tonga Islands. The Tonga Islands ( $22^{\circ} \mathrm{S} 170^{\circ} \mathrm{W}$ ) are part of the same Pacific Island group as Fiji ( $17^{\circ} \mathrm{S} 178^{\circ} \mathrm{E}$ ) and wish to keep the same day despite being in different hemispheres. Using the dateline of $180 \mathrm{E} / \mathrm{W}$, Tonga would have to keep a time one day earlier than Fiji. To overcome this problem, the International Dateline is moved further east to keep Fiji and Tonga on the same Standard Time day.

## An Interesting Example

## Example:

If the Local Mean Time (LMT) in the Tonga Islands $\left(22^{\circ} \mathrm{S} 170^{\circ} \mathrm{W}\right)$ is 0800 LMT on local mean date 10 May, what is the Standard Time and Standard Date in the Tonga Islands?

Using the recommended logical approach for this type of problem and working through UTC

|  | Day | Hour | Minute |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 10 | 08 | 00 | LMT |
| Local Mean Time Tonga |  | +11 | 20 |  |
| Arc/Time Correction for 170W | 10 | 19 | 20 | UTC Best |
| UTC | +13 |  |  |  |
| Standard Time Correction Tonga (list 1) |  | 11 May | 08 | 20 | ST

Note: Tonga's Standard Date is one day different (in advance) from its Local Mean date. A similar situation exists with Chatham Island mentioned previously and with the 2 other anomalies of this type - the extreme far east of Russia and Wrangell Island. In these other cases, the International Dateline has also been adjusted. A corresponding situation exists with the far western islands of the Aleutian chain which lie in the Eastern hemisphere. Here, the International Dateline is adjusted further west to keep the whole Aleutian chain on the same day as the USA.


Figure 25.4 International Date Line

## Example 1

Find the LMT in Tokyo, Japan (longitude 140E) if the Standard Time is 1425 hours on the $16^{\text {th }}$ January.

|  | D |  | H | M |
| :---: | :---: | :---: | :---: | :---: |
| Standard Time Tokyo | $16^{\text {th }}$ |  | 14 | 25 |
| Standard Time Difference (STD) |  | - | 09 | 00 (List 1) |
| UTC | $16^{\text {th }}$ |  | 05 | 25 |
| Arc/Time (Long.East) |  | + | 9 | 20 (UTC LEAST) |
| LMT Tokyo | $16^{\text {th }}$ |  | 14 | 45 |

## Example 2

Find the LMT in Vancouver, British Columbia Canada (longitude 123W) if the Standard Time is 0008 hours on $5^{\text {th }}$ March.

Standard Time Vancouver
Standard Time Difference (STD)
UTC
Arc/Time (Long.West)
LMT Vancouver

| $\mathbf{D}$ |  | $\mathbf{H}$ | $\mathbf{M}$ |
| :--- | :--- | :--- | :--- |
| $5^{\text {th }}$ |  | 00 | 08 |
|  | + | 08 | 00 |
| $5^{\text {th }}$ |  | 08 | 08 |
|  | - | 8 | 12 |
| $\mathbf{4}^{\text {th }}$ |  | $\mathbf{2 3}$ | $\mathbf{5 6}$ | (UTC 3) BEST)

## Example 3

Find the Standard Time in Fairbanks, Alaska (longitude 148W) if the LMT is 0614 on $18^{\text {th }}$ February.

|  | D |  | H | M |
| :--- | :--- | :--- | :--- | :--- |
| LMT Fairbanks | $18^{\text {th }}$ |  | 06 | 14 |
| Arc/Time (Long.West) |  | + | 9 | 52 |
| UTC | $18^{\text {th }}$ |  | 16 | 06 |
| STD Fairbanks |  | - | 9 | 00 |
| ST Fairbanks | $\mathbf{1 8}^{\text {th }}$ |  | $\mathbf{0 7}$ | $\mathbf{0 6}$ |

## Example 4

Find the Standard Time in Georgetown, Guyana (08N 060W), when the LMT in Sydney, New South Wales, (longitude $151^{\circ} 20^{\prime} \mathrm{E}$ ) is 1116 hours on $21^{\text {st }}$ January.

|  | $\mathbf{D}$ | $\mathbf{H}$ | $\mathbf{M}$ |
| :--- | :--- | :---: | :--- |
| LMT Sydney | $21^{\text {st }}$ | 11 | 16 |
| Arc/Time (Long.East) |  | -10 | 05 |
| UTC | ${21^{\text {st }}}^{\text {(UTC LEAST) }}$ | 01 | 11 |
| STD Guyana | $\mathbf{2 0}^{\text {th }}$ | $\mathbf{0}$ | 00 |
| ST Georgetown |  | $\mathbf{2 2}$ | $\mathbf{1 1}$ |

## Example 5

An aircraft left position A (longitude 164W) on a westerly heading at 2200 hours LMT on $3^{\text {rd }}$ May. Give the LMT and local date of arrival at position B (longitude 173E) if the flight time was 6 hours.

|  | D |  | H | M |
| :---: | :---: | :---: | :---: | :---: |
| Depart A | $3^{\text {rd }}$ |  | 22 | 00 |
| LMT Arc/Time (Long.West) |  | + | 10 | 56 |
| Depart A | $4^{\text {th }}$ |  | 08 |  |
| Flight Time |  | + | 06 | 00 |
| Arrive B | $4^{\text {th }}$ |  | 14 | 56 |
| Arc/Time (Long.East) |  | + | 11 | 32 |
| Arrive B | $5^{\text {th }}$ |  | 02 | 28 |

## Example 6

An aircraft left position $X$ (longitude 175E) on an easterly heading at 0100 hours LMT on $18^{\text {th }}$ November. Give the LMT and local date of arrival at position Y (longitude 150W) if the flight time was 4 hours.

Depart X
Arc/Time (Long.East)
Depart X
Flight Time
Arrive Y
Arc/Time (Long.West)
Arrive $Y$

| D | $\mathbf{H}$ | $\mathbf{M}$ |  |
| :--- | :--- | :--- | :--- |
| $18^{\text {th }}$ |  | 01 | 00 | LMT $\quad$|  | - | 11 | 40 |
| :--- | :--- | :--- | :--- |
| $17^{\text {th }}$ | 13 | 20 | UTC LEAST |
|  | + | 04 | 00 |
| $17^{\text {th }}$ |  | 17 | 20 |
|  | - | 10 | 00 |
| $17^{\text {th }}$ | $\mathbf{0 7}$ | $\mathbf{2 0}$ |  |

## Summary

LMT is based on the movement of the mean Sun which travels through $15^{\circ}$ of longitude per hour. UTC is the LMT on the Greenwich Meridian.

Difference in LMT at two places is found by finding the difference in their longitudes and converting it into time at the rate of $15^{\circ}$ per hour, except where the shorter longitudinal arc spans the date line in which case work through UTC. Be prepared to calculate the time equivalent of longitude without using the Air Almanac.

Zone Time is in the EASA syllabus. Be prepared to convert longitude into zone number and to convert between Zone Time and UTC.

Standard Time is used to prevent the extreme inconvenience of using LMT on clocks. Standard Time Differences for all countries are found in Lists 1,2 and 3 in the Air Almanac. Conversion of LMT to Standard Time or vice versa is best done through UTC. LMT problems involving an aircraft crossing the International Date Line are also best done through UTC.

## Questions 1

1. Without using the Air Almanac, convert $153^{\circ} 30^{\prime}$ of arc of longitude into time.
a. $\quad 10 \mathrm{hrs} 24 \mathrm{~min}$
b. $\quad 10 \mathrm{hrs} 22 \mathrm{~min}$
c. $\quad 10 \mathrm{hrs} 14 \mathrm{~min}$
d. $\quad 10 \mathrm{hrs} 8 \mathrm{~min}$
2. The definition of Local Mean Time (LMT) is:
a. time based upon the average movement of the Earth around the Sun.
b. when the mean Sun is transiting (crossing) your meridian, it is 1200 hrs LMT.
c. when the mean Sun is transiting (crossing) your anti-meridian, it is 0000hrs LMT (2400 hrs LMT, previous day).
d. all of the above.
3. Local Mean Time (LMT) always changes by a day when crossing:
a. the Greenwich Meridian
b. $\quad 180^{\circ} \mathrm{E} / \mathrm{W}$
c. the International Date Line
d. the Equator
4. Zone Time (ZT) is used:
a. by aircraft on trans-oceanic routes
b. as legal time in all countries
c. by ships at sea
d. in polar regions
5. A ship at longitude 83E observes sunrise at a Zone Time of 0500 ZT on Zone Date 15th May. What is the UTC?
a. $\quad 2300$ UTC 14th May
b. 1100 UTC 15th May
c. 2328 UTC 14th May
d. 1032 UTC 15th May
6. On Mid-summer Day in the Southern hemisphere, the Sun will be overhead:
a. $\quad 661 / 2 \mathrm{~S}$
b. $\quad 231 / 2 \mathrm{~N}$
c. $\quad 231 / 2 \mathrm{~S}$
d. the Equator
7. On Mid-winter Day in the Northern hemisphere, the Sun will be overhead:
a. $\quad 661 / 2 \mathrm{~S}$
b. $\quad 231 / 2 \mathrm{~N}$
c. $\quad 231 / 2 \mathrm{~S}$
d. the Equator

## Questions 2

1. At position 4830N 10315E the time is 1045 LMT on the 17th May. What is the LMT and local date at 4830S 00715E?
2. When LMT at 4600N 00830W is 2300 on the 9th May, what is the LMT and local date at:
a. 4600N 10830W?
b. 4600N 10830E?
3. The time is 1300 UTC on 1st April. Calculate the LMT and local date at:
a. 5430 N 00715 E
b. 1627 S 10743 W
c. 1846 N 16835E
4. At 3000 N 4630 E the time is 0300 LMT. What is the time at 6000 N 01615 W in:
a. UTC?
b. LMT?
5. At 1400 UTC on 6th November what is the Standard Time at the following locations:
a. Iraq
b. Libya
c. Tonga
d. Canada, Labrador
e. New York
f. Ghana
6. In position 4830N 6357W the LMT is 1430 on the 10th February. What is the Standard Time in Hong Kong (2220N 11407E)?
7. An aircraft is to fly from Wellington, New Zealand (4110S 17454E) to Samoa (1350S 17145 W ). The estimated flight time is 6 hours 15 minutes.

Give the Standard Time and date of departure from Wellington if the ETA for Samoa is $\mathbf{2 3 5 0}$ LMT on the 8th September.

## Questions 3

Note: Unless otherwise stated assume that Summer Time is not being observed.

1. Determine the LMT in WELLINGTON, NEW ZEALAND (17446E) at 0730 UTC.
2. LMT in BOSTON, MASSACHUSETTS, USA (07108W) is 1200. What is the UTC?
3. Determine the LMT in HONOLULU, HAWAII, USA (15752W) and the Standard Time in SYDNEY, NEW SOUTH WALES, AUSTRALIA (15113E) for 1335 UTC.
4. An aircraft leaves BRISBANE, QUEENSLAND, AUSTRALIA (15302E) at 2200 UTC on 4th May and lands in MANILA, PHILIPPINE REPUBLIC (12058E) some 6 hours and 45 minutes later. What is the LMT and Standard Time of landing?
5. By how much is LMT, BANGKOK, THAILAND (10100E) later than LMT, SAN FRANCISCO, CALIFORNIA, USA (12225W)?
6. An aircraft left NEW YORK, USA (07500W) on a 5 hour 25 minute scheduled flight to VANCOUVER, BRITISH COLUMBIA, CANADA (12320W). If the ATD was 0915 NEW YORK Summer Time, what will be the ETA (Standard Time) VANCOUVER?
7. Inclusive of flying and refuelling time, the scheduled time from ROME, ITALY (01230E) to HONG KONG (11400E) is 21 hours 40 minutes. If the flight must land at HONG KONG at 1835
Standard Time, at what Standard Time must the flight leave ROME to arrive on schedule?
8. The flight time between SYDNEY, NEW SOUTH WALES, AUSTRALIA and HONOLULU, HAWAII (15752W) is 9 hours 50 minutes. If an aircraft leaves SYDNEY at 0930 Standard Time on the 8th May, what is the LMT and local date of arrival?
9. An aircraft leaves HONOLULU, HAWAII at 2300 Standard Time on the 10th May. If the flight time to SYDNEY, NEW SOUTH WALES, AUSTRALIA is $\mathbf{1 0}$ hours 20 minutes, what is the Standard Time of arrival at SYDNEY?
10. A flight from NEW YORK, USA (07345W) to MADRID, SPAIN (00333W) is planned to leave at 2000 Standard Time on 16th February. If the flight time is 7 hours 45 minutes, give the Standard Time and local date of arrival at MADRID.

Answers 1

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| c | d | b | c | a | c | c |

## Answers 2

## Question 1

|  | D |  | H |
| :--- | :--- | :--- | :--- |
| LMT @ $103^{\circ} 15^{\prime} \mathrm{E}$ | $\mathbf{1 7}$ |  | 10 |
| Arc/Time $103^{\circ} 15^{\prime} \mathrm{E}$ |  | - | 6 |
| UTC | 17 |  | 03 |
| Arc/Time $007^{\circ} 15^{\prime} \mathrm{E}$ |  | + | 0 |
| LMT @ $007^{\circ} 15^{\prime} \mathrm{E}$ | $\mathbf{1 7}$ | $\mathbf{0 4}$ | 53 |

## Question 2

a. Difference in longitude $100^{\circ} 00^{\prime}$

|  | 2300 | LMT 9 May |
| :--- | :--- | :--- | :--- |
| Arc to time | $-\quad \frac{0640}{1620}$ | LMT 9 May at $108^{\circ} 30 \mathrm{~W}$ |

b. Difference in longitude $117^{\circ} 00^{\prime}$

|  | 2300 | LMT | 9 May |
| :--- | :--- | :--- | :--- |
| Arc to time | +34 |  |  |
| Arc to time | 2334 | UTC | 9 May |
|  | $\frac{+714}{\mathbf{0 6 4 8}}$ |  |  |
|  | LMT10 May |  |  |

## Question 3

a.

Arc to time
b.

Arc to time
c.

Arc to time

## Question 4

a.

0300 LMT
$-\frac{0306}{2354}$ UTC
previous day
b.

Arc to time

## Question 5

1400 UTC on 6th November

| a. Iraq +0300 | $=$ | 1700 | ST | 6 Nov |
| :--- | :--- | :--- | :--- | :--- | :--- |
| b. Libya +0100 | $=$ | 1500 | ST | 6 Nov |
| c. Tonga +1300 | $=$ | 0300 | ST | 7 Nov |
| d. Labrador -0400 | $=$ | 1000 | ST | 6 Nov |
| e. New York -0500 | $=$ | 0900 | ST | 6 Nov |
| f. Ghana keeps UTC | $=$ | 1400 | ST | 6 Nov |

## Question 6

| Arc to time | $+\frac{0416}{1846}$ | UTC | 10 Feb |
| :--- | ---: | :--- | :--- |
| Standard Time Correction | $+\frac{0800}{\mathbf{0 2 4 6}}$ | ST | 11 Feb in Hong Kong |


| Question 7 |  |  |  |
| :---: | :---: | :---: | :---: |
| Arrival Samoa | 2350 | LMT | 8 Sep |
| Arc to time | + 1127 |  |  |
| Arrive Samoa | 1117 | UTC | 9 Sep |
| Flight Time | 0615 |  |  |
| Depart Wellington | 0502 | UTC | 9 Sep |
| ST correction | + 1200 |  |  |
| Depart Wellington | 1702 | ST | 9 Sep |

You have won yourself a day!

## Answers 3

## Question 1

$$
\begin{aligned}
& 0730 \text { UTC } \\
+\quad & \frac{1139}{\mathbf{1 9 0 9}} \text { LMT } \quad \begin{array}{l}
\text { Arc to time } \\
\text { Wellington }
\end{array}
\end{aligned}
$$

## Question 2

$$
\begin{aligned}
& 1200 \text { LMT } \\
+ & \frac{0445}{1645} \text { UTC }
\end{aligned}
$$

Boston
Arc to time

## Arc to time

 Hawaiib.

$$
\begin{aligned}
& 1335 \text { UTC } \\
& +\quad 1000 \text { ST } \\
& \hline \mathbf{2 3 3 5} \text { ST }
\end{aligned}
$$

correction
New South Wales

Question 4

| Departure | 2200 | UTC | 4 May |
| :---: | :---: | :---: | :---: |
| Flight time | 0645 |  |  |
| Arrival | 0445 | UTC | 5 May |
|  | 0804 |  | Arc to time |
|  | 1249 | LMT | 5 May |
| Arrival | 0445 | UTC | 5 May |
| + | 0800 |  | ST correction |
| Arrival | 1245 | ST | Philippines |

## Question 5

Bangkok ( $101^{\circ} \mathrm{E}$ ) 0644 Later than UTC
San Fran. ( $122^{\circ} 25^{\prime}$ W) 0810 Earlier than UTC
Total 1454 Bangkok later than San Francisco

Question 6
Departure 0915 New York Summer Time (Daylight Saving Time \{DST\})
ST correction
Departure 1315 UTC
Flight time
Arrival 0525

ST correction
Arrival $\quad 1040$ ST British Columbia 1840 UTC

- 0800


## Question 7

Arrival time

|  | 1835 ST |
| ---: | :--- |
| - | Hong Kong |
|  | $\frac{08}{1035}$ UTC |
|  |  |
| + |  |
| + | $\frac{2140}{1255}$ UTC | Previous day

ST correction
Arrival time Schedule time
Departure 1255 UTC

Previous day
ST correction
Departure
1355 ST
Rome

Question 8
Departure
ST correction Departure
Flight time
Arrival
Arc to time
Arrival
0930 ST
8 May NSW
10
2330 UTC 7 May
0950
0920 UTC 8 May

- 1031

2249 LMT
7 May

Question 9

| Departure | 2300 ST | Hawaii 10 May |
| :---: | :---: | :---: |
| ST correction | + 1000 |  |
| Departure | 0900 UTC | 11 May |
| Flight time | 1020 |  |
| Arrival | 1920 UTC | 11 May |
| ST correction | + 1000 |  |
| Arrival | 0520 ST | 12 May |

Question 10
Departure
2000 ST
$+0500$
ST correction
Departure Flight time Arrival
ST correction Arrival

0100 UTC
0745
0845 UTC $\quad 17 \mathrm{Feb}$
0100
0945 ST Madrid 17 Feb

## Air Almanac Extract

STANDARD TIMES (Corrected to June 1988)
LIST 1 - PLACES FAST ON UTC (mainly those EAST OF GREENWICH) The times given below should be: added to UTC to give Standard Time. Subtracted from Standard Time to give UTC.

| Admiralty Islands | 10 |  | Cyprus, Ercan* | 02 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Afghanistan | 04 | 30 | Larnaca* | 02 |  |
| Albania* | 01 |  | Czechosolovakia* | 01 |  |
| Algeria | 01 |  | Denmark* | 01 |  |
| Amirante Islands | 04 |  | Dijibouti | 03 |  |
| Andaman Islands | 05 | 30 | Egypt, Arab Republic of* | 02 |  |
| Angola | 01 |  | Equatorial Guinea, Republic of | 01 |  |
| Australia |  |  | Ethiopia | 03 |  |
| Australian Capital Territory* | 10 |  | Fiji | 12 |  |
| New South Wales ${ }^{\text {1* }}$ | 10 |  | Finland* | 02 |  |
| Northern Territory | 09 | 30 | France | 01 |  |
| Queensland | 10 |  | Gabon | 01 |  |
| South Australia* | 09 | 30 | Germany, East* | 01 |  |
| Tasmania* | 10 |  | West ${ }^{3 *}$ | 01 |  |
| Victoria* | 10 |  | Gibraltar* | 01 |  |
| Western Australia* | 08 |  | Greece* | 02 |  |
| Austria* | 01 |  | Guam | 10 |  |
| Bahrain | 03 |  | Holland (The Netherlands)* | 01 |  |
| Balearic Islands* | 01 |  | Hong Kong | 08 |  |
| Banaba | 11 | 30 | Hungary* | 01 |  |
| Bangladesh | 06 |  | India | 05 | 30 |
| Belgium* | 01 |  | Indonesia, Republic of Bangka, Billiton, Java, West and Middle Kalimantan, Madura, Sumatra | 07 |  |
| Benin (Dahomey) | 01 |  | Bali, Flores, South and East Kalimantan, Lombok, Sulawesi, Sumba, Sumbawa, Timor | 08 |  |
| Bhutan | 06 |  | Aru, Irian Jaya, Kai, Moluccas, Tanimbar | 09 |  |
| Botswana, Republic of | 02 |  | Iran | 03 | 30 |
| Brunei | 08 |  | Iraq* | 03 |  |
| Bulgaria* | 02 |  | Israel* | 02 |  |
| Burma | 06 | 30 | Italy* | 01 |  |
| Burundi | 02 |  | Japan | 09 |  |
| Cameroon Republic | 01 |  | Jordan* | 02 |  |
| Caroline Islands, Pala Islands | 10 |  | Kampuchea, Democratic | 07 |  |
| Yal Islands, Truk Islands | 10 |  | Kenya | 03 |  |
| Ponnapel | 11 |  | Kiribati Republic ${ }^{4}$ | 12 |  |
| Pingelap Islands, Kusaie | 12 |  | Korea, North | 09 |  |
| Central African Republic | 01 |  | Republic of (South)* | 09 |  |
| Chad | 01 |  | Kuril Islands | 11 |  |
| Chagos Archipelago ${ }^{2}$ | 05 |  | Kuwait | 03 |  |
| Chatham Islands* | 12 | 45 | Laccadive Islands | 05 | 30 |
| China* | 08 |  | Laos | 07 |  |
| Christmas Island, Indian Ocean | 07 |  | Lebanon* | 02 |  |
| Cocos Keeling Islands | 06 | 30 | Lesotho | 02 |  |
| Comro Islands (Comoros) | 03 |  | Libya* | 01 |  |
| Congo Republic | 01 |  | Liechtenstein* | 01 |  |
| Corsica* | 01 |  | Lord Howe Island* | 10 | 30 |
| Crete* | 02 |  | Luxembourg* | 01 |  |

[^0]| Macao | 08 |  | Singapore | 08 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Macias Nguema Biyogo Island (Fernando Poo) | 01 |  | Socotra | 03 |  |
| Madagascar, Democratic Republic of | 03 |  | Solomon Islands | 11 |  |
| Malawi | 02 |  | Somalia Republic | 03 |  |
| Malaysia | 08 |  | South Africa, Republic of | 02 |  |
| Malaya, Sabah, Sarawak | 08 |  | South West Africa (Namibia) | 02 |  |
| Maldives, Republic of The | 05 |  | Spain* | 01 |  |
| Malta* | 01 |  | Spanish Possessions in North Africa |  |  |
| Mariana Islands | 10 |  | (Ceuta, Meilla)* | 01 |  |
| Marshall Islands ${ }^{1}$ | 12 |  | Spitsbergen (Svalbard) | 01 |  |
| Mauritius | 04 |  | Sri Lanka | 05 | 30 |
| Monaco* | 01 |  | Sudan, Republic of | 02 |  |
| Mongolia* | 08 |  | Swaziland | 02 |  |
| Mozambique | 02 |  | Sweden* | 01 |  |
| Namibia (South West Africa) | 02 |  | Switzerland* | 01 |  |
| Nauru | 12 |  | Syria (Syrian Arab Republic)* | 02 |  |
| Nepal | 05 | 45 | Taiwan* | 08 |  |
| Netherlands, The* | 01 |  | Tanzania | 03 |  |
| New Caledonia* | 11 |  | Thailand | 07 |  |
| New Zealand* | 12 |  | Tonga | 13 |  |
| Nicobar Islands | 05 | 30 | Tunisia* | 01 |  |
| Niger | 01 |  | Turkey* | 02 |  |
| Nigeria, Republic of | 01 |  | Tuvalu | 12 |  |
| Norfolk Island | 11 | 30 | Uganda | 03 |  |
| Norway* | 01 |  | Union of Soviet Socialist Republics ${ }^{2 *}$ |  |  |
| Novaya Zemlya | 03 |  | Zone 1 Amderma, Arkhangelsk, Kiev, Leningrad, Moscow, Odessa | 03 |  |
| Okinawa | 09 |  | Zone 2 Baku, Tbilsi, Volgogrnd | 04 |  |
| Oman | 04 |  | Zone 3 Ashkabad, Novvy Port, Sverdlovsk | 05 |  |
| Pagalu (Annobon Islands) | 01 |  | Zone 4 Alma-Ata, Omsk, |  |  |
| Tashkent | 06 |  |  |  |  |
| Pakistan | 05 |  | Zone 5 Krasnoyarsk, Novosibirsk | 07 |  |
| Papua New Guinea | 10 |  | Zone 6 Irkutsk | 08 |  |
| Pescadores Islands | 08 |  | Zone 7 Tiksi, Yakutsk | 09 |  |
| Philippine Republic | 08 |  | Zone 8 Khabarovsk, okhotsk, Vladivostok | 10 |  |
| Poland ${ }^{*}$ | 01 |  | Zone 9 Magnad, Sakhalin I | 11 |  |
| Qatar | 03 |  | Zone 10 Anadyr, Petropaviovsk | 12 |  |
| Reunion | 04 |  | United Arab Emirates | 04 |  |
| Romania* | 02 |  | Vanuatu, Republic of ${ }^{*}$ | 11 |  |
| Rwanda | 02 |  | Vietnam, Socialist Republic of | 07 |  |
| Ryukyu Islands | 09 |  | Wrangell Island | 12 |  |
| Sakhalin | 11 |  | Yemen | 03 |  |
| Santa Cruz Islands | 11 |  | Yugoslavia* | 01 |  |
| Sardinia* | 01 |  | Zaire |  |  |
| Saudi Arabia | 03 |  | Kinshasa, Mbandaka | 01 |  |
| Schouten Islands | 09 |  | Haut-Zaire, Kasai, Kivu, Shaba | 02 |  |
| Seychelles | 04 |  | Zambia, Republic of | 02 |  |
| Sicily* | 01 |  | Zimbabwe | 02 |  |

* Summer time may be kept in these countries.

1 Except the islands of Kwajalein and Eniwctok which keep a time 24 h slow on that ot the rest of the islands.
2 The boundaries between the zones are irregular; listed are towns in each zone.

## STANDARD TIMES (Corrected to June 1988)

## LIST II - PLACES NORMALLY KEEPING UTC

| Ascension Island | Ghana | Irish Republic* | Morocco | Sierra Leone |
| :--- | :--- | :--- | :--- | :--- |
| Bourkina-Faso | Great Britain1 | Ivory Coast | Portugal ${ }^{*}$ | Togo Republic |
| Canary Islands* | Guinea Bissau | Liberia | Principe | Tristan da Cunha |
| Channel Islands1 | Guinea Republic | Madeira* | St. Helena |  |
| Faeroes*, The | Iceland | Mali | Sao Tome |  |
| Gambia | Ireland, Northern1 | Mauritania | Senegal |  |

* Summer time may be kept in these countries.

1 Summer time, one hour in advance of UTC, is kept from March 25d01h to October 28d01hUTC, subject to confirmation.

## LIST III - PLACES SLOW ON UT (WEST OF GREENWICH)

The times given
below should be
subtracted from UTC to give Standard Time. added to Standard Time to give UTC.

| Argentina | 03 |  | Cape Verde Islands | 01 |
| :---: | :---: | :---: | :---: | :---: |
| Austral Islands ${ }^{1}$ | 10 |  | Cayman Islands | 05 |
| Azores* | 01 |  | Chile* | 04 |
|  |  |  | Christmas Island, Pacific Ocean | 10 |
| Bahamas* | 05 |  | Colombia | 05 |
| Barbados | 04 |  | Cook Islands* | 10 |
| Belize | 06 |  | Costa Rica | 06 |
| Bermuda* | 04 |  | Cuba* | 05 |
| Bolivia | 04 |  | Curacao Island | 04 |
| Brazil, eastern ${ }^{2 *}$ | 03 |  |  |  |
| Territory of Acre* | 05 |  | Dominican Republic | 04 |
| western* | 04 |  |  |  |
| British Antarctic Territory ${ }^{3}$ | 03 |  | Easter Island (I. De Pascua)* | 06 |
|  |  |  | Ecuador | 05 |
| Canada |  |  |  |  |
| Alberta* | 07 |  | Falkland Islands* | 04 |
| British Columbia* | 08 |  | Fanning Island | 10 |
| Labrador* | 04 |  | Fernando de Noronha Island* | 02 |
| Manitoba* | 06 |  | French Guiana | 03 |
| New Brunswick* | 04 |  |  |  |
| Newfoundland* | 03 | 30 | Galapagos Islands | 06 |
| Northwest Territories* |  |  | Greenland ${ }^{4}$, Scoresby Sound* | 01 |
| east of long. W. $68{ }^{\circ}$ | 04 |  | Angmagssalik and west coast* | 03 |
| long. W. $68^{\circ}$ to W. $85^{\circ}$ | 05 |  | Thule area | 04 |
| long W. $85^{\circ}$ to W. $102^{\circ}$ | 07 |  | Grenada | 04 |
| west of long. W. $102^{\circ}$ | 07 |  | Guadeloupe | 04 |
| Nova Scotia* | 04 |  | Guatemala | 06 |
| Ontario*, east of long. W. $90^{\circ}$ | 05 |  | Guyana, Republic of | 03 |
| west of long. W. $90^{\circ}$ | 06 |  |  |  |
| Prince Edward Island* | 04 |  | Haiti* | 05 |
| Quebec*, east of long. W. $63^{\circ}$ | 04 |  | Honduras | 06 |
| west of long. W. $63^{\circ}$ | 05 |  |  |  |
| Saskatchewan* |  |  | Jamaica | 05 |
| east of long. W. $106{ }^{\circ}$ | 06 |  | Jan Mayen Island | 01 |
| west of long. W106 ${ }^{\circ}$ | 07 |  | Johnston Island | 10 |
| Yukon* | 08 |  | Juan Fernandez Islands* | 04 |

* Summer time may be kept in these countries.

1 This is the legal standard time, but local mean time is generally used.
2 Including all the coast and Brasilia.
3 Except South Georgia which keeps 02h
4 Danmarkshaven keeps UT

## STANDARD TIMES (Corrected to June 1988)

## LIST III - (Continued)

| Leeward Islands | 04 |  | Indiana ${ }^{3}$ | 05 |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Iowa ${ }^{2}$ | 06 |
| Marquesas Islands | 09 | 30 | Kansas ${ }^{2,3}$ | 06 |
| Martinique | 04 |  | Kentucky ${ }^{2}$, eastern part | 05 |
| Mexico ${ }^{1}$ | 06 |  | western part | 06 |
| Midway Islands | 11 |  | Louisana ${ }^{2}$ | 06 |
|  |  |  | Maine ${ }^{2}$ | 05 |
| Nicaragua | 06 |  | Maryland ${ }^{2}$ | 05 |
| Niue | 11 |  | Massachusetts ${ }^{2}$ | 05 |
|  |  |  | Michigan ${ }^{2,3}$ | 05 |
| Panama, Republic of | 05 |  | Minnesota ${ }^{2}$ | 06 |
| Paraguay* | 04 |  | Mississippi ${ }^{2}$ | 06 |
| Peru | 05 |  | Missouri ${ }^{2}$ | 06 |
| Pitcairn Island | 08 | 30 | Montana ${ }^{2}$ | 07 |
| Puerto Rico | 04 |  | Nebraska ${ }^{2,3}$ | 06 |
|  |  |  | Nevada ${ }^{2}$ | 08 |
| St. Pierre and Miqeulon | 03 |  | New Hampshire ${ }^{2}$ | 05 |
| Salvador, El | 06 |  | New Jersey ${ }^{2}$ | 05 |
| Samoa | 11 |  | New Mexico ${ }^{2}$ | 07 |
| Society Islands | 10 |  | New York ${ }^{2}$ | 05 |
| South Georgia | 02 |  | North Carolina ${ }^{2}$ | 05 |
| Surinam | 03 |  | North Dakota ${ }^{2,3}$ | 06 |
|  |  |  | Ohio ${ }^{2}$ | 05 |
| Tobago | 04 |  | Oklahoma ${ }^{2}$ | 06 |
| Trinidade Island*, South Atlantic | 02 |  | Oregon ${ }^{2,3}$ | 08 |
| Trinidad | 04 |  | Pennsylvania ${ }^{2}$ | 05 |
| Tuamotu Archipelago | 10 |  | Rhode Island ${ }^{2}$ | 05 |
| Tubuai Islands | 10 |  | South Carolina ${ }^{2}$ | 05 |
| Turks and Caicos Islands* | 05 |  | South Dakota ${ }^{2}$, eastern part | 06 |
|  |  |  |  | 07 |
| United States of America |  |  | Tennessee ${ }^{2,3}$ | 06 |
| Alabama ${ }^{2}$ | 06 |  | Texas ${ }^{2,3}$ | 06 |
| Alaska ${ }^{2}$, east of long. W. $169^{\circ} 30^{\prime}$ | 09 |  | Utah ${ }^{2}$ | 07 |
| Aleutian Is., west of W. $169^{\circ} 30^{\prime}$ | 10 |  | Vermont ${ }^{2}$ | 05 |
| Arizona | 07 |  | Virginia ${ }^{2}$ | 05 |
| Arkansas ${ }^{2}$ | 06 |  | Washington ${ }^{2}$ | 08 |
| California ${ }^{2}$ | 08 |  | Washington, D.C. ${ }^{2}$ | 05 |
| Colorado ${ }^{2}$ | 07 |  | West Virginia ${ }^{2}$ | 05 |
| Conneticut ${ }^{2}$ | 05 |  | Wisconsin ${ }^{2}$ | 06 |
| Delaware ${ }^{2}$ | 05 |  | Wyoming ${ }^{2}$ | 07 |
| District of Columbia ${ }^{2}$ | 05 |  | Uruguay* | 03 |
| Florida ${ }^{2,3}$ | 05 |  |  |  |
| Georgia ${ }^{2}$ | 05 |  | Venezuela | 04 |
| Hawaii ${ }^{2}$ | 10 |  | Virgin Islands | 04 |
| Idaho ${ }^{2,3}$ | 07 |  |  |  |
| Illinois ${ }^{2}$ | 06 |  | Windward Islands | 04 |

* Summer time may be kept in these countries.

1 Except the states of Sonora, Sinaloa, Nayarit and the Southern District of Lower California, which keep 07h, and the Northern District of lower California, which keeps 08h.
2 Summer (daylight-saving) time, one hour fast on the time given, is kept in these states from the first Sunday in April to the last Sunday in October, changing at 02h00m local clock time.
3 This applies to the greater portion of the state.

| - | hour | min | - | hour | min | - | hour | min | - | hour | min | - | hour | min | - | hour | min | , | min | sec |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 00 | 60 | 4 | 00 | 120 | 8 | 00 | 180 | 12 | 00 | 240 | 16 | 00 | 300 | 20 | 00 | 0 | 0 | 00 |
| 1 | 0 | 04 | 61 | 4 | 04 | 121 | 8 | 04 | 181 | 12 | 04 | 241 | 16 | 04 | 301 | 20 | 04 | 1 | 0 | 04 |
| 2 | 0 | 08 | 62 | 4 | 08 | 122 | 8 | 08 | 182 | 12 | 08 | 242 | 16 | 08 | 302 | 20 | 08 | 2 | 0 | 08 |
| 3 | 0 | 12 | 63 | 4 | 12 | 123 | 8 | 12 | 183 | 12 | 12 | 243 | 16 | 12 | 303 | 20 | 12 | 3 | 0 | 12 |
| 4 | 0 | 16 | 64 | 4 | 16 | 124 | 8 | 16 | 184 | 12 | 16 | 244 | 16 | 16 | 304 | 20 | 16 | 4 | 0 | 16 |
| 5 | 0 | 20 | 65 | 4 | 20 | 125 | 8 | 20 | 185 | 12 | 20 | 245 | 16 | 20 | 305 | 20 | 20 | 5 | 0 | 20 |
| 6 | 0 | 24 | 66 | 4 | 24 | 126 | 8 | 24 | 186 | 12 | 24 | 246 | 16 | 24 | 306 | 20 | 24 | 6 | 0 | 24 |
| 7 | 0 | 28 | 67 | 4 | 28 | 127 | 8 | 28 | 187 | 12 | 28 | 247 | 16 | 28 | 307 | 20 | 28 | 7 | 0 | 28 |
| 8 | 0 | 32 | 68 | 4 | 32 | 128 | 8 | 32 | 188 | 12 | 32 | 248 | 16 | 32 | 308 | 20 | 32 | 8 | 0 | 32 |
| 9 | 0 | 36 | 69 | 4 | 36 | 129 | 8 | 36 | 189 | 12 | 36 | 249 | 16 | 36 | 309 | 20 | 36 | 9 | 0 | 36 |
| 10 | 0 | 40 | 70 | 4 | 40 | 130 | 8 | 40 | 190 | 12 | 40 | 250 | 16 | 40 | 310 | 20 | 40 | 10 | 0 | 40 |
| 11 | 0 | 44 | 71 | 4 | 44 | 131 | 8 | 44 | 191 | 12 | 44 | 251 | 16 | 44 | 311 | 20 | 44 | 11 | 0 | 44 |
| 12 | 0 | 48 | 72 | 4 | 48 | 132 | 8 | 48 | 192 | 12 | 48 | 252 | 16 | 48 | 312 | 20 | 48 | 12 | 0 | 48 |
| 13 | 0 | 52 | 73 | 4 | 52 | 133 | 8 | 52 | 193 | 12 | 52 | 253 | 16 | 52 | 313 | 20 | 52 | 13 | 0 | 52 |
| 14 | 0 | 56 | 74 | 4 | 56 | 134 | 8 | 56 | 194 | 12 | 56 | 254 | 16 | 56 | 314 | 20 | 56 | 14 | 0 | 56 |
| 15 | 1 | 00 | 75 | 5 | 00 | 135 | 9 | 00 | 195 | 13 | 00 | 255 | 17 | 00 | 315 | 21 | 00 | 15 | 1 | 00 |
| 16 | 1 | 04 | 76 | 5 | 04 | 136 | 9 | 04 | 196 | 13 | 04 | 256 | 17 | 04 | 316 | 21 | 04 | 16 | 1 | 04 |
| 17 | 1 | 08 | 77 | 5 | 08 | 137 | 9 | 08 | 197 | 13 | 08 | 257 | 17 | 08 | 317 | 21 | 08 | 17 | 1 | 08 |
| 18 | 1 | 12 | 78 | 5 | 12 | 138 | 9 | 12 | 198 | 13 | 12 | 258 | 17 | 12 | 318 | 21 | 12 | 18 | 1 | 12 |
| 19 | 1 | 16 | 79 | 5 | 16 | 139 | 9 | 16 | 199 | 13 | 16 | 259 | 17 | 16 | 319 | 21 | 16 | 19 | 1 | 16 |
| 20 | 1 | 20 | 80 | 5 | 20 | 140 | 9 | 20 | 200 | 13 | 20 | 260 | 17 | 20 | 320 | 21 | 20 | 20 | 1 | 20 |
| 21 | 1 | 24 | 81 | 5 | 24 | 141 | 9 | 24 | 201 | 13 | 24 | 261 | 17 | 24 | 321 | 21 | 24 | 21 | 1 | 24 |
| 22 | 1 | 28 | 82 | 5 | 28 | 142 | 9 | 28 | 202 | 13 | 28 | 262 | 17 | 28 | 322 | 21 | 28 | 22 | 1 | 28 |
| 23 | 1 | 32 | 83 | 5 | 32 | 143 | 9 | 32 | 203 | 13 | 32 | 263 | 17 | 32 | 323 | 21 | 32 | 23 | 1 | 32 |
| 24 | 1 | 36 | 84 | 5 | 36 | 144 | 9 | 36 | 204 | 13 | 36 | 264 | 17 | 36 | 324 | 21 | 36 | 24 | 1 | 36 |
| 25 | 1 | 40 | 85 | 5 | 40 | 145 | 9 | 40 | 205 | 13 | 40 | 265 | 17 | 40 | 325 | 21 | 40 | 25 | 1 | 40 |
| 26 | 1 | 44 | 86 | 5 | 44 | 146 | 9 | 44 | 206 | 13 | 44 | 266 | 17 | 44 | 326 | 21 | 44 | 26 | 1 | 44 |
| 27 | 1 | 48 | 87 | 5 | 48 | 147 | 9 | 48 | 207 | 13 | 48 | 267 | 17 | 48 | 327 | 21 | 48 | 27 | 1 | 48 |
| 28 | 1 | 52 | 88 | 5 | 52 | 148 | 9 | 52 | 208 | 13 | 52 | 268 | 17 | 52 | 328 | 21 | 52 | 28 | 1 | 52 |
| 29 | 1 | 56 | 89 | 5 | 56 | 149 | 9 | 56 | 209 | 13 | 56 | 269 | 17 | 56 | 329 | 21 | 56 | 29 | 1 | 56 |
| 30 | 2 | 00 | 90 | 6 | 00 | 150 | 10 | 00 | 210 | 14 | 00 | 270 | 18 | 00 | 330 | 22 | 00 | 30 | 2 | 00 |
| 31 | 2 | 04 | 91 | 6 | 04 | 151 | 10 | 04 | 211 | 14 | 04 | 271 | 18 | 04 | 331 | 22 | 04 | 31 | 2 | 04 |
| 32 | 2 | 08 | 92 | 6 | 08 | 152 | 10 | 08 | 212 | 14 | 08 | 272 | 18 | 08 | 332 | 22 | 08 | 32 | 2 | 08 |
| 33 | 2 | 12 | 93 | 6 | 12 | 153 | 10 | 12 | 213 | 14 | 12 | 273 | 18 | 12 | 333 | 22 | 12 | 33 | 2 | 12 |
| 34 | 2 | 16 | 94 | 6 | 16 | 154 | 10 | 16 | 214 | 14 | 16 | 274 | 18 | 16 | 334 | 22 | 16 | 34 | 2 | 16 |
| 35 | 2 | 20 | 95 | 6 | 20 | 155 | 10 | 20 | 215 | 14 | 20 | 275 | 18 | 20 | 335 | 22 | 20 | 35 | 2 | 20 |
| 36 | 2 | 24 | 96 | 6 | 24 | 156 | 10 | 24 | 216 | 14 | 24 | 276 | 18 | 24 | 336 | 22 | 24 | 36 | 2 | 24 |
| 37 | 2 | 28 | 97 | 6 | 28 | 157 | 10 | 28 | 217 | 14 | 28 | 277 | 18 | 28 | 337 | 22 | 28 | 37 | 2 | 28 |
| 38 | 2 | 32 | 98 | 6 | 32 | 158 | 10 | 32 | 218 | 14 | 32 | 278 | 18 | 32 | 338 | 22 | 32 | 38 | 2 | 32 |
| 39 | 2 | 36 | 99 | 6 | 36 | 159 | 10 | 36 | 219 | 14 | 36 | 279 | 18 | 36 | 339 | 22 | 36 | 39 | 2 | 36 |
| 40 | 2 | 40 | 100 | 6 | 40 | 160 | 10 | 40 | 220 | 14 | 40 | 280 | 18 | 40 | 340 | 22 | 40 | 40 | 2 | 40 |
| 41 | 2 | 44 | 101 | 6 | 44 | 161 | 10 | 44 | 221 | 14 | 44 | 281 | 18 | 44 | 341 | 22 | 44 | 41 | 2 | 44 |
| 42 | 2 | 48 | 102 | 6 | 48 | 162 | 10 | 48 | 222 | 14 | 48 | 282 | 18 | 48 | 342 | 22 | 48 | 42 | 2 | 48 |
| 43 | 2 | 52 | 103 | 6 | 52 | 163 | 10 | 52 | 223 | 14 | 52 | 283 | 18 | 52 | 343 | 22 | 52 | 43 | 2 | 52 |
| 44 | 2 | 56 | 104 | 6 | 56 | 164 | 10 | 56 | 224 | 14 | 56 | 284 | 18 | 56 | 344 | 22 | 56 | 44 | 2 | 56 |
| 45 | 3 | 00 | 105 | 7 | 00 | 165 | 11 | 00 | 225 | 15 | 00 | 285 | 19 | 00 | 345 | 23 | 00 | 45 | 3 | 00 |
| 46 | 3 | 04 | 106 | 7 | 04 | 166 | 11 | 04 | 226 | 15 | 04 | 286 | 19 | 04 | 346 | 23 | 04 | 46 | 3 | 04 |
| 47 | 3 | 08 | 107 | 7 | 08 | 167 | 11 | 08 | 227 | 15 | 08 | 287 | 19 | 08 | 347 | 23 | 08 | 47 | 3 | 08 |
| 48 | 3 | 12 | 108 | 7 | 12 | 168 | 11 | 12 | 228 | 15 | 12 | 288 | 19 | 12 | 348 | 23 | 12 | 48 | 3 | 12 |
| 49 | 3 | 16 | 109 | 7 | 16 | 169 | 11 | 16 | 229 | 15 | 16 | 289 | 19 | 16 | 349 | 23 | 16 | 49 | 3 | 16 |
| 50 | 3 | 20 | 110 | 7 | 20 | 170 | 11 | 20 | 230 | 15 | 20 | 290 | 19 | 20 | 350 | 23 | 20 | 50 | 3 | 20 |
| 51 | 3 | 24 | 111 | 7 | 24 | 171 | 11 | 24 | 231 | 15 | 24 | 291 | 19 | 24 | 351 | 23 | 24 | 51 | 3 | 24 |
| 52 | 3 | 28 | 112 | 7 | 28 | 172 | 11 | 28 | 232 | 15 | 28 | 292 | 19 | 28 | 352 | 23 | 28 | 52 | 3 | 28 |
| 53 | 3 | 32 | 113 | 7 | 32 | 173 | 11 | 32 | 233 | 15 | 32 | 293 | 19 | 32 | 353 | 23 | 32 | 53 | 3 | 32 |
| 54 | 3 | 36 | 114 | 7 | 36 | 174 | 11 | 36 | 234 | 15 | 36 | 294 | 19 | 36 | 354 | 23 | 36 | 54 | 3 | 36 |
| 55 | 3 | 40 | 115 | 7 | 40 | 175 | 11 | 40 | 235 | 15 | 40 | 295 | 19 | 40 | 355 | 23 | 40 | 55 | 3 | 40 |
| 56 | 3 | 44 | 116 | 7 | 44 | 176 | 11 | 44 | 236 | 15 | 44 | 296 | 19 | 44 | 356 | 23 | 44 | 56 | 3 | 44 |
| 57 | 3 | 48 | 117 | 7 | 48 | 177 | 11 | 48 | 237 | 15 | 48 | 297 | 19 | 48 | 357 | 23 | 48 | 57 | 3 | 48 |
| 58 | 3 | 52 | 118 | 7 | 52 | 178 | 11 | 52 | 238 | 15 | 52 | 298 | 19 | 52 | 358 | 23 | 52 | 58 | 3 | 52 |
| 59 | 3 | 56 | 119 | 7 | 56 | 179 | 11 | 56 | 239 | 15 | 56 | 299 | 19 | 56 | 359 | 23 | 56 | 59 | 3 | 56 |

The above table is for converting expressions in arc to their equivalent in time; its main use in this Almanac is for the conversion of longitude
for application to L.M.T. (added if west, subtracted if east) to give UT or vice versa, particularly in the case of sunrise, sunset, etc.
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## Chapter <br> 26 <br> Time (3)

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## Sunrise and Sunset

Local noon occurs at the same Local Mean Time on all meridians ( 1200 hrs LMT). It is also true to say that the Sun rises at the same LMT at all places on the same parallel of latitude and it sets at the same LMT at all places on the same parallel of latitude.

On an imaginary Earth with no tilt and no atmosphere, the Sun would rise at 0600 LMT and set at 1800 LMT (and there would be no seasons). However, the Earth's tilt results in seasonal changes and the declination of the Sun affects the length of night and day, i.e. it changes the times of sunrise and sunset. The effect varies with latitude. The declination of the Sun varies with the calendar. This is illustrated in the next simplified diagrams.

Northern Summer/Southern Winter


Figure 26.1 Sunrise/sunset
This is illustrated for the situation on or about 21 June (Midsummer Day) when the Sun's declination is furthest North, i.e. the Sun is overhead the Tropic of Cancer ( $23^{1} 1^{\circ} \mathrm{N}$ ): the Earth's North Pole is tilted towards the Sun.

Ignoring the effects of atmosphere (see later), half the Earth would be in darkness (night).
An observer at $R$ would rotate eastwards as shown. He would experience a short period of night and a long period of day. Thus, the Sun must rise earlier than 0600 hours and set later than 1800 hours. (In the Northern hemisphere, it is summer).

An observer at $S$ would experience a long period of night and a short period of day. The Sun will rise later than 0600 hours and set earlier than 1800 hrs . It is southern winter.

An observer at E (the Equator) experiences equal periods of night and day with sunrise and sunset at approximately 0600 hours and 1800 hours respectively. At the Spring and Autumn equinoxes the sunrise and the sunset occur approximately at the same time at all latitudes.

An observer at Position X (the Arctic Circle) or further north experiences constant day - this is the 'land of the midnight Sun'. With the effects of the atmosphere included, this occurs at anywhere north of 66 N on June 21.

In contrast, an observer at position Y (the Antarctic Circle or further south), experiences constant night - the Antarctic winter when the Sun never rises.

Northern Winter/Southern Summer


Figure 26.2 Sunrise/sunset 2
A corresponding situation for December $21^{\text {st }}$ when the Northern hemisphere is tilted away from the Sun is given below.

The observer at $\mathbf{R}$ experiences a short period of day and a long period of night. The Sun rises later than 0600 and sets earlier than 1800 hours. It is northern winter.

The observer at S experiences a short night and a long day. The Sun rises earlier than 0600 and sets later than 1800. It is southern summer.

The observer at E (the Equator) again experiences equal periods of day and night
The observer at $X$ experiences constant night as do all observers north of $X$ (Arctic Circle)
The observer at $Y$ experiences constant day as do all observers south of $Y$ (Antarctic Circle)
The length of day and night and the timings of sunrise and sunset vary with latitude and with the declination of the Sun (the calendar date). At the Equator, periods of day and night may be considered to be equal at all times of the year. (Because of atmospheric effects, the length of the day at the Equator is approximately 6 minutes longer than the length of the night - see later).

## Sunrise and Sunset Tables - Methodology

The Air Almanac provides tables from which the LMT of sunrise and sunset can be extracted.
Times are shown for every third day and for various latitudes between 72 N and 60 S .
If the latitude for which the time of sunrise or sunset is required is not one of the listed latitudes, it is necessary to interpolate to the nearest minute of time.

Note: Listed latitudes are $10^{\circ}$ apart near the Equator, $5^{\circ}$ apart in mid latitudes, and $2^{\circ}$ apart in high latitudes

Similarly, if the required date is not one of those listed, it is necessary to interpolate to the nearest minute. Interpolation is only required to the nearest minute.

Sunrise and sunset problems are really ordinary time problems, the starting point of which is the LMT of sunrise or sunset at a given position.

This LMT is extracted from the Air Almanac and is then converted to UTC or Standard Time as required using the methods given in the previous chapter. Examples of these problems are now given.

## Example 1

Give the Standard Time of sunset at Innsbruck, Austria (4715N 01120E) on 5th September.

|  | D | H | M |  |
| :--- | :--- | :--- | :--- | :--- |
| Sunset $47^{\circ} 15^{\prime} \mathrm{N}$ (from AA) | 5th | 18 | 33 | LMT |
| Arc/Time (Long. $\left.11^{\circ} 20^{\prime} \mathrm{E}\right)$ |  |  | - | 45 |
|  |  |  |  |  |
| Sunset Innsbruck |  | 17 | 48 | UTC |
| STD Austria |  | + | 01 | 00 |
|  | 5th | $\mathbf{1 8}$ | $\mathbf{4 8}$ | (List 1) |
|  |  |  |  |  |

## Example 2

What is the Standard Time of sunrise at Keflavik, Iceland ( $64^{\circ} 00^{\prime} \mathrm{N} 22^{\circ} 30^{\prime} \mathrm{W}$ ) on 14 th October? Summer Time is not being kept.

Sunrise $64^{\circ}$ (from AA)

| D | H | M |  |
| :--- | :--- | :--- | :--- |
| 14th | 06 | 45 | LMT |
| + | 01 | 30 |  |
| 14th | 08 | 15 | UTC |
|  | 00 | 00 | (List 2) |
| 14th | $\mathbf{0 8}$ | $\mathbf{1 5}$ | ST |

## Example 3

An aircraft lands at Goose, Labrador, Canada (5320N 06020W) at 1841 Standard Time on 20th September. Is it a day or night landing? (Ignore Summer Time).

The Air Navigation Order defines the period of night flying as from 30 minutes after sunset to 30 minutes before sunrise at the surface. (This is not necessarily the same as when it gets dark).

|  | D | H | M |  |
| :---: | :---: | :---: | :---: | :---: |
| Sunset Goose (from AA) | 20th | 18 | 05 | LMT |
| Arc/Time (Long. 06020West) | $+$ | 04 | 01 |  |
| UTC of Sunset Goose | 20th | 22 | 06 | UTC |
| STD Labrador | - | 04 | 00 | (List 3) |
| Sunset Goose (Labrador ST) | 20th | 18 | 06 | ST |
| Landing Goose | 20th | 18 | 41 | ST |

Landing is 35 minutes after sunset. Night Landing (according to the ANO definition)

## Twilight

Before sunrise, there is a period when it becomes light, and after sunset there is a period when it remains light. These periods are called generally 'twilight'. Although the Sun is below the horizon, the Sun's rays are being refracted in from the atmosphere and result in an extended period of light. It is therefore worthwhile to consider what we call the 'horizon'.

Consider an observer at position Q on the Earth's surface. His 'Sensible Horizon' is the tangent to the Earth's surface (ignoring terrain features). This is the horizon that would be 'sensed' by instruments such as a spirit level.


Figure 26.3 Sensible/visual horizons
Because of atmospheric refraction, the 'visual' horizon is below the 'sensible' horizon by about 34' of arc (2). When the top 'lip' of the Sun passes below the visual horizon, at sunset, the centre of the Sun is another $16^{\prime}$ of arc (the radius of the Sun viewed from Earth below the visual horizon). Thus the centre of the Sun is approximately 50 minutes of arc below the sensible horizon at sunset. As 50' of arc equates to 3 minutes of time, this means that at sunset, the centre of the Sun is already 3 minutes of time below the sensible horizon. An equivalent argument would show that, at sunrise, the centre of the Sun is 3 minutes of time below the sensible horizon. This explains why the length of day at the Equator is approximately 6 minutes longer than the length of night.

Twilight is that period before sunrise and after sunset when refracted light from the Earth's atmosphere gives an amount of illumination. The amount of illumination varies with the Sun's depression below the sensible horizon and with atmospheric conditions.

## Civil Twilight

Civil twilight occurs when the Sun's centre is between $0^{\circ} 50^{\prime}$ and $6^{\circ}$ below the sensible horizon. Illumination is such that it is possible to carry out day time tasks without additional artificial lighting. For example, a daytime visual approach is possible. This is the only twilight period considered in the Air Almanac.

## Nautical Twilight

The centre of the Sun is between $6^{\circ}$ and $12^{\circ}$ below the sensible horizon. (General outlines are discernible and the brighter stars are visible).

## Astronomical Twilight

The centre of the Sun is between $12^{\circ}$ and $18^{\circ}$ below the sensible horizon. (Astronomical twilight is often considered to be 'complete darkness'.)

## Civil Twilight Tables (Air Almanac)

The LMT of civil twilight is given in the Air Almanac on the intervening pages between the sunrise/sunset tables.

The Morning Civil Twilight (MCT) tables give the time when morning twilight starts. MCT ends at sunrise. All times are tabulated in LMT.

The Evening Civil Twilight (ECT) tables give the time when evening twilight ends. ECT starts at sunset. Again, all times are tabulated in LMT.

The EASA definition of 'night' is the 'period between the end of evening civil twilight and the beginning of morning civil twilight, or such other period between sunset and sunrise as may be prescribed by the appropriate authority'. An example of the latter proviso of the EASA definition is the Air Navigation Order which defines the period of night flying as from 30 minutes after sunset to 30 minutes before sunrise at the surface.

## Duration of Twilight Calculations

To calculate the duration of twilight at a given latitude on a given date, extract the time of sunrise or sunset for that date and the time of MCT/ECT on the same date. The difference in the corresponding times gives the duration of twilight.

## Example

What is the duration of evening civil twilight in New York, USA, (41N 074W) on 19 July?
(AA Page 13) Sunset 19 July (41N) $=1929$ LMT (interpolate)
(AA Page 14) ECT 19 July (41N) = 2001 LMT (interpolate)
Duration of twilight $=2001$ LMT-1929 LMT
= 32 minutes

## Extreme Cases of Twilight

Two extreme cases exist (and are frequently examined) as follows:

- Equator
- High latitudes (say above 62N/S)

In high latitudes, the Sun may be above or below the horizon all day, or civil twilight may last all night.

The following symbols in the Air Almanac indicate these conditions:
$\square$ Sun remains continuously above the horizon.

- Sun remains continuously below the horizon.
//// In the period between sunset and sunrise, the Sun is never less than $6^{\circ}$ below the sensible horizon. In plain English, twilight lasts from sunset to sunrise the next morning.


## Twilight at the Equator

It is a romantic, but false idea that after sunset at the Equator, it is dark immediately.
To demonstrate the true situation, consider the path of the Sun as viewed by an observer at the Equator on or about March 21st/Sep 21st (the Equinoxes). Looking to the East, the Sun will rise and travel apparently vertically upwards until it is directly overhead the observer. It will then descend vertically to the western horizon where it will set.

The sunset situation described is illustrated below:


Figure 26.4 Twilight 1
At sunset, at position A, the top edge of the Sun has just passed below the visual horizon. At this position, the centre of the Sun is 50 ' of arc below the sensible horizon. At position $B$, the centre of the Sun is $6^{\circ}$ below the sensible horizon; this is the end of evening civil twilight. The duration of twilight is therefore $5^{\circ} 10^{\prime}$ of angular arc - 21 minutes of time - the minimum period of twilight at the Equator.

Twilight Between the Equator and $60^{\circ} \mathrm{N} / \mathrm{S}$


Figure 26.5 Twilight 2

## Case (i)

Using the previous illustration, we can demonstrate how the duration of twilight varies with latitude. As a baseline, we use the equatorial situation described previously which gave a minimum duration of twilight of 21 minutes. This is case (i).

## Case (ii)

To an observer whose latitude is farther north than the declination of the Sun (in this example, an observer in the Northern hemisphere):

- the Sun will rise in the East
- the Sun will travel westward.
- at 1200 LMT (noon), the Sun will be due south of the observer
- the Sun will set in the West following the general path indicated in case (ii).

The duration of twilight will be longer than 21 minutes.

## Case (iii)

To an observer whose latitude is farther south than the declination of the Sun (in this example, an observer in the Southern hemisphere):

- the Sun will rise in the East
- the Sun will travel westward
- at 1200 LMT (noon), the Sun will be due north of the observer.
- the Sun will set in the West, following the general path indicated in case (iii).

Again, the duration of twilight will be longer than 21 minutes.
In both the above examples, the duration of twilight can be calculated using the Air Almanac.

## Twilight in High Latitudes

In high latitudes, the Sun may remain above the horizon all day. Thus, on or about June 21, the Sun will remain above the horizon all day at latitudes higher than $66^{\circ} \mathrm{N}$. This situation is illustrated below. (Remember, in the Southern hemisphere further south than $66^{\circ} \mathrm{S}$, the Sun will not rise at all.)


Figure 26.6 Twilight 3

The above illustration is based on a $360^{\circ}$ time-lapse photograph taken in Lapland in midsummer. Using the same diagram, we can illustrate what happens earlier or later than June 21 when the Sun is lower in the sky. Similarly, the Sun will be lower in the sky as the latitude of the observer increases.


Figure 26.7 Twilight 4
In Figure 26.7 the Sun sets at position $A$ and rises at position $B$. Between the times of sunset (A) and sunrise (B), the Sun never goes below $6^{\circ}$ below the sensible horizon. Thus it is:

## Twilight between sunset and sunrise

This situation is identified in the Air Almanac with the symbol ////
(Check this in your Air Almanac by looking at the time for evening civil twilight at 64 N on June 28.)

## Effect on Sunrise/Sunset/Twilight of Increase in Altitude

The times published in the Air Almanac are in Local Mean Time (LMT) and are published for sea level at the appropriate position.

- Increase in altitude results in sunrise occurring earlier and sunset later than published. This is due to the increased visual horizon at altitude.
- Increase in altitude results overall in a decrease in the duration of civil twilight. This is because twilight is caused by refraction of the Sun's rays from the atmosphere; the higher the aircraft, the less light is refracted from the (reduced) upper atmosphere. For observers in the Space Shuttle, the duration of twilight is negligible.


## Questions 1

1. On June 21, what is the lowest northerly latitude (listed in the Air Almanac) at which the Sun will be above the horizon for 24 hrs (all day)?
a. 64 N
b. $\quad 66 \mathrm{~N}$
c. 68 N
d. 70N
2. On December 21, what is the lowest northerly latitude (listed in the Air Almanac) at which the Sun never rises (or sets)?
a. $\quad 64 \mathrm{~N}$
b. 66 N
c. 68 N
d. 70 N
3. Civil twilight is:
a. the period between sunset and the end of evening civil twilight (ECT)
b. the period between the start of morning civil twilight (MCT) and sunrise
c. related to the position of the centre of the Sun being $6^{\circ}$ below the sensible horizon
d. a period when it is possible to carry out daylight tasks without artificial lighting
e. used in the EU-OPS definition of day flying/night flying
f. all of the above
4. Nautical twilight and astronomic twilight are the twilight periods that follow civil twilight. Nautical twilight occurs when the Sun is between $\qquad$ and ___ below the sensible horizon:
a. $\quad 0^{\circ} / 6^{\circ}$
b. $\quad 6^{\circ} / 12^{\circ}$
c. $\quad 12^{\circ} / 18^{\circ}$
d. $\quad 18^{\circ} / 24^{\circ}$
5. Between 60N and 60S, the minimum duration of Civil Twilight is?
a. $\quad 21$ minutes
b. $\quad 16$ minutes
c. $\quad 14$ minutes
d. $\quad 30$ minutes
6. Sunrise/sunset twilight tables in the Air Almanac are listed for a sea level situation. An aircraft flying at FL350 would experience sunrise $\qquad$ , sunset $\qquad$ and the length of twilight would $\qquad$

Sunrise
a. later
b. earlier
c. later
d. earlier

Sunset
earlier
later
earlier
later

Duration of Twilight be longer not change not change be shorter

## Questions 2

Answer required to the nearest minute for the date and position given in the question.
All dates refer to the Air Almanac extract.
Consider summer or daylight saving time only when indicated in the question.

1. For position 3000S 12414E on 9th August, what is the LMT of:
a. Sunrise
b. Sunset
c. Beginning of morning of civil twilight
d. End of evening of civil twilight
2. Give the duration of morning civil twilight for all positions in latitude 55N on 28th June.
What is the meaning of the following symbols which appear in the Air Almanac Extract?
i) $\square$
ii) $\quad$
iii) ////
3. What is the UTC of sunset at KARACHI (2454N 06710E) PAKISTAN on 18th August? Give also the duration of evening civil twilight there on the same date.
4. Determine the interval between the end of evening civil twilight on 14th October and the beginning of morning civil twilight on 15th October for any position in latitude 60N.
5. For PERTH (3157S 11557E) WESTERN AUSTRALIA on 3rd January:-
a. What is the LMT of sunrise?
b. What is the UTC of sunset?
c. What is the Standard Time of the end of evening civil twilight?
6. For BUENOS AIRES (3500S 05229W) ARGENTINA:-
a. Give the UTC of sunrise on 9th September
b. Give the Standard Time of sunset on 25th July
7. At 1500 UTC on 4th July, what is the Standard Time at the places listed below?
a. NEW YORK, USA (Daylight Saving Time).
b. CHRISTMAS ISLAND, INDIAN OCEAN.
c. ALGERIA.
d. TONGA ISLAND.
8. What is the UTC and date at the times shown below?
a. $\quad 0400$ Standard Time on 5th November in MADRID (4026N 00332W) SPAIN?
b. $\quad 1215$ LMT on 10th July in WELLINGTON (4100S 17446E) NEW ZEALAND?
c. 2200 UTC on 18th September on the Flight Operations clock at SINGAPORE INTERNATIONAL?
9. What is the difference in hours and minutes between the LMT in KABUL (3430N 07000E) and the LMT in THULE (6930N 05400W) and which place is fast on the other?
10. An aircraft leaves ROME (4150N 01230E) at sunset on 3rd August and after a direct flight lands at NEW YORK (4000N 07500W) USA, at the end of evening civil twilight on the same day. Calculate the flight time.
11. An aircraft leaves TOKYO (3600N 14000E) JAPAN an hour after sunrise on 14th November and lands in HAWAII (2200N 15752W) USA 6 hours 12 minutes later. What is the Standard Time and date of landing in HAWAII?
12. On 29th October an observer in longitude 01815E sees the Sun rise at 0355 UTC. What is his latitude?
13. An observer in SOUTH GEORGIA (6000S 03715W) sees the Sun set at a watch time of 2257 on 19th November. Is the watch fast or slow on UTC and by how much?
14. An aircraft must land at ANCHORAGE (6100N 14900W) ALASKA, USA, at the end of the evening civil twilight on 25th September. The estimated flight time from HONG KONG (2200N 10914E) is 10 hours 45 minutes. What is the latest possible Standard Time and date of departure from HONG KONG?
15. What is the LMT and date at $A(35 N 178 \mathrm{~W})$ when it is 2020 LMT on 28th October at B (70S 176E)?

## Answers 1

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| b | c | f | b | a | d |

## Answers 2

| SR | Sunrise | MCT | Beginning of Morning Civil Twilight |
| :--- | :--- | :--- | :--- |
| SS | Sunset | ECT | End of Evening Civil Twilight |

## Question 1

| SR | 0640 LMT | MCT | 0615 LMT |
| :--- | :--- | :--- | :--- |
| SS | 1732 LMT | ECT | 1757 LMT |

## Question 2

a. SR 28 June @ $55^{\circ} \mathrm{N} \quad 0323$ LMT

MCT
Duration of MCT 58 minutes.
b.
$\qquad$ Sun remains continuously above the horizon.
//// Sun remains continuously below the horizon.
//// Permanent twilight during the time the Sun is below the horizon.

## Question 3

SS 1833 LMT

- $\frac{0429}{1404}$ Arc to time

ECT 1857
SS 1833
Duration of ECT 24 minutes.

Question 4
14 OCTECT 1738 LMT
15 OCTMCT 0556 LMT
Time between $\overline{1218}$

## Question 5

a. SR 0458 LMT
b. SS 1910 LMT

0744 Arc to time
SS 1126 UTC
c. ECT 1938 LMT

0744 Arc to time
$\overline{1154}$ UTC
$+\frac{0800}{1954}$ ST correction
ECT $\overline{1954}$ ST

## Question 6

a. $\quad$ SR @ 35S 0609 LMT

0330 Arc to time
SR 0939 UTC
b. SS

1713 LMT
0330 Arc to time
SS

SS
2043 UTC
$\frac{0300}{1743}$ ST correction

## Question 7

a. 1100 DST 4 July
b. $\quad 2200$ ST 4 July
c. $\quad 1600$ ST 4 July
d. 0400 ST 5 July

## Question 8

a. 0300 UTC 5 November
b. 1215 LMT

1139 Arc to time
0036 UTC 10 July
c. $2200 \quad 18$ September

## Question 9

| Ch.Long. | $124^{\circ} 00^{\prime}$ |
| :--- | :---: |
| Arc to time | $08: 16$ |
| Kabul is 8 Hrs | 16 minutes FAST on Thule |

## Question 10

| SS Rome | 1917 | LMT |
| :---: | :---: | :---: |
|  | 0050 | Arc to time |
| SS | 1827 | UTC |
| ECT NY | 1942 | LMT |
|  | 0500 | Arc to time |
|  | 0042 | UTC |
| Flight time | 1827 | TC to 0042 |

## Question 11

SR Tokyo 0634 LMT 14 November
Take off
0734 LMT
0920 Arc to time
Take off 2214 UTC 13 November
Flight time 0612
Land 0426
UTC 14 November
For ST
1000
Land $\quad \overline{1826}$ ST Hawaii 13 November

## Question 12

| SR | 0355 | UTC |
| :--- | :--- | :--- |
|  | $\frac{0113}{0508}$ | Arc to time |
| LRT |  |  |

Latitude is 30 S

## Question 13

SS 2026 LMT South Georgia
0229 Arc to time
SS 2255 UTC
Observer's watch is 2 minutes FAST on UTC.

## Question 14



## Question 15

2020 LMT

- 1144 Arc to time 176E 0836 UTC 28 October
- 1152 Arc to time 178W

2044 LMT 27 October

## Air Almanac Extract

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## SUNRISE

| Lat. | $\begin{gathered} \text { Dec. } \\ 30 \\ \hline \end{gathered}$ | January |  |  |  |  |  |  |  |  |  | February |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 5 | 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 | 1 | 4 | 7 | 10 | 13 |  | 16 |
|  | $\mathrm{h} \quad \mathrm{m}$ | h m | h m | h m | h m | h m | h m | h m | h m | $\mathrm{h} \quad \mathrm{m}$ | h m | h m | h m | h m | $\mathrm{h} \quad \mathrm{m}$ | h m | h | m |
| N72 | - | $\square$ | $\square$ | $\square$ | - | - | - | - | - | 1151 | 1100 | 1031 | $10 \quad 07$ | 0946 | 0927 | 0908 | 08 | 51 |
| 70 | - | - | - | - | - | $\square$ | 1153 | $11 \quad 09$ | $10 \quad 44$ | $10 \quad 23$ | $10 \quad 05$ | 0948 | 0932 | 0916 | 0901 | $08 \quad 46$ |  | 31 |
| 68 | - | - | $11 \begin{array}{ll}11 & 32\end{array}$ | 1111 | $10 \quad 55$ | $10 \quad 40$ | $10 \quad 25$ | $10 \quad 12$ | $09 \quad 58$ | 0945 | 0932 | 0919 | $09 \quad 07$ | $08 \quad 54$ | $08 \quad 41$ | 28 |  | 16 |
| 66 | $10 \quad 31$ | $10 \quad 26$ | $10 \quad 20$ | $10 \quad 13$ | $10 \quad 05$ | $09 \quad 57$ | $09 \quad 48$ | 0938 | 29 | 0919 | $09 \quad 08$ | $08 \quad 58$ | $08 \quad 47$ | 36 | 26 | 15 | 08 | 04 |
| 64 | 0951 | 0949 | 0945 | 0940 | 0935 | 29 | 22 | 0914 | $09 \quad 06$ | $08 \quad 58$ | $08 \quad 50$ | 41 | 32 | 22 | 13 | $08 \quad 03$ | 07 | 53 |
| 62 | 24 | 22 | 19 | 0916 | 0912 | $09 \quad 07$ | $09 \quad 01$ | $08 \quad 55$ | $08 \quad 49$ | 42 | 34 | 27 | 19 | 10 | $08 \quad 02$ | $07 \quad 53$ |  | 44 |
| N60 | $09 \quad 03$ | $09 \quad 02$ | $09 \quad 00$ | $08 \quad 57$ | $08 \quad 54$ | $08 \quad 50$ | $08 \quad 45$ | 0840 | $08 \quad 34$ | $08 \quad 28$ | $08 \quad 22$ | $08 \quad 15$ | $08 \quad 08$ | $08 \quad 00$ | $07 \quad 53$ | 0745 | 07 | 37 |
| 58 | $08 \quad 46$ | $08 \quad 45$ | $08 \quad 44$ | 42 | 39 | 35 | 31 | 27 | 22 | 17 | 11 | 08 | $07 \quad 58$ | $07 \quad 52$ | 45 | 37 |  | 30 |
| 56 | 32 | 31 | 30 | 28 | 26 | 23 | 19 | 15 | 11 | $08 \quad 06$ | $08 \quad 01$ | $07 \quad 56$ | 50 | 44 | 38 | 31 |  | 24 |
| 54 | 19 | 19 | 18 | 16 | 14 | 12 | 09 | $08 \quad 06$ | $08 \quad 02$ | $07 \quad 57$ | $07 \quad 53$ | 48 | 42 | 37 | 31 | 25 |  | 19 |
| 52 | $08 \quad 08$ | $08 \quad 08$ | $08 \quad 07$ | 0806 | $08 \quad 04$ | $08 \quad 02$ | $08 \quad 00$ | $07 \quad 57$ | 07 | 49 | 45 | 41 | 36 | 31 | 25 | 20 |  | 14 |
| N50 | $07 \quad 58$ | $07 \quad 58$ | $07 \quad 58$ | $07 \quad 57$ | $07 \quad 56$ | $07 \quad 54$ | $07 \quad 51$ | $07 \quad 49$ | 0746 | $07 \quad 42$ | $07 \quad 38$ | $07 \quad 34$ | $07 \quad 30$ | $07 \quad 25$ | $07 \quad 20$ | $07 \quad 15$ | 07 | 10 |
| 45 | 38 | 38 | 38 | 38 | 37 | 35 | 34 | 32 | 29 | 27 | 24 | 20 | 17 | 13 | 09 | $07 \quad 05$ | 07 | 00 |
| 40 | 22 | 22 | 22 | 22 | 21 | 21 | 19 | 18 | 16 | 14 | 12 | $07 \quad 09$ | 0706 | 0703 | 0700 | 0656 | 06 | 52 |
| 35 | 0708 | $07 \quad 08$ | 0708 | 0708 | 0708 | $07 \quad 08$ | $07 \quad 07$ | $07 \quad 06$ | $07 \quad 05$ | 0703 | $07 \quad 01$ | $06 \quad 59$ | $06 \quad 57$ | $06 \quad 54$ | $06 \quad 52$ | 49 |  | 46 |
| 30 | $06 \quad 55$ | $06 \quad 56$ | $06 \quad 57$ | 0657 | $06 \quad 57$ | $06 \quad 57$ | 0656 | $06 \quad 56$ | $06 \quad 55$ | 0654 | $06 \quad 52$ | 51 | 49 | 47 | 45 | 42 |  | 40 |
| N20 | $06 \quad 34$ | 0635 | 0636 | $06 \quad 37$ | $06 \quad 37$ | $06 \quad 38$ | $06 \quad 38$ | $06 \quad 38$ | $06 \quad 38$ | $06 \quad 37$ | $06 \quad 36$ | 0636 | 0635 | 0633 | $06 \quad 32$ | $06 \quad 31$ | 06 | 29 |
| N10 | 0616 | 17 | 18 | 19 | 20 | 21 | 22 | 22 | 22 | 23 | 23 | 22 | 22 | 22 | 21 | 20 |  | 20 |
| 0 | $05 \quad 59$ | $06 \quad 00$ | 06 | 06 | $06 \quad 04$ | $06 \quad 05$ | 0606 | $\begin{array}{ll}06 & 07\end{array}$ | 06 | 0609 | 0610 | 0610 | 0610 | 0611 | 11 | 11 |  | 11 |
| S10 | 41 | $05 \quad 43$ | $05 \quad 45$ | 0546 | $05 \quad 48$ | $05 \quad 49$ | $05 \quad 51$ | $05 \quad 52$ | $05 \quad 54$ | $05 \quad 55$ | $05 \quad 56$ | $05 \quad 57$ | $05 \quad 59$ | $05 \quad 59$ | $06 \quad 00$ | $06 \quad 01$ | 06 | 02 |
| 20 | 23 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 45 | 47 | 0549 | $05 \quad 50$ | 05 | 52 |
| S30 | 0501 | $05 \quad 03$ | 0505 | 0507 | 0510 | 0512 | 0515 | $05 \quad 18$ | $05 \quad 20$ | $05 \quad 23$ | $05 \quad 26$ | $05 \quad 28$ | $05 \quad 31$ | $05 \quad 33$ | 0536 | 0538 | 05 | 41 |
| 35 | $04 \quad 48$ | $04 \quad 50$ | 0452 | $04 \quad 55$ | $04 \quad 58$ | $05 \quad 01$ | 0503 | $05 \quad 07$ | $05 \quad 10$ | 13 | 16 | 19 | 22 | 25 | 28 | 31 |  | 34 |
| 40 | 33 | 35 | 38 | 41 | 44 | $04 \quad 47$ | $04 \quad 50$ | $04 \quad 54$ | $04 \quad 57$ | $05 \quad 01$ | $05 \quad 05$ | $05 \quad 08$ | 12 | 16 | 20 | 23 |  | 27 |
| 45 | 0415 | 04 18 | $04 \quad 21$ | 24 | 27 | 31 | 35 | 39 | 43 | $04 \quad 47$ | 0452 | 0456 | 0500 | 0505 | 0509 | 14 |  | 18 |
| 50 | 0353 | $03 \quad 56$ | $03 \quad 59$ | $04 \quad 03$ | $04 \quad 07$ | 11 | 16 | 20 | 25 | 30 | 36 | 41 | 0446 | 0451 | 0457 | $05 \quad 02$ |  | 07 |
| S52 | $03 \quad 42$ | $03 \quad 45$ | 0349 | $03 \quad 53$ | $03 \quad 57$ | $04 \quad 02$ | $04 \quad 07$ | $04 \quad 12$ | $04 \quad 17$ | $04 \quad 22$ | $04 \quad 28$ | $04 \quad 34$ | $04 \quad 39$ | $04 \quad 45$ | $04 \quad 51$ | $04 \quad 57$ | 05 | 02 |
| 54 | 30 | 33 | 37 | 41 | 46 | $03 \quad 51$ | $03 \quad 56$ | $04 \quad 02$ | $04 \quad 08$ | 14 | 20 | 26 | 32 | 38 | 45 | 51 | 04 | 57 |
| 56 | 16 | 20 | 24 | 28 | 33 | 39 | 45 | $03 \quad 51$ | $03 \quad 57$ | 0404 | 0410 | 17 | 24 | 30 | 37 | 44 |  | 51 |
| 58 | $03 \quad 00$ | $03 \quad 04$ | 0308 | $03 \quad 13$ | 19 | 25 | 31 | 38 | 45 | 0352 | $03 \quad 59$ | $04 \quad 07$ | 14 | 22 | 29 | 37 |  | 44 |
| S60 | $02 \quad 40$ | $02 \quad 44$ | $02 \quad 49$ | $02 \quad 55$ | $03 \quad 01$ | $03 \quad 08$ | $03 \quad 15$ | $03 \quad 23$ | $03 \quad 31$ | $03 \quad 39$ | $03 \quad 47$ | $03 \quad 55$ | $04 \quad 03$ | $04 \quad 12$ | $04 \quad 20$ | $04 \quad 28$ | 04 | 36 |

SUNSET

|  | Dec. | January |  |  |  |  |  |  |  |  |  | February |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 2 | 5 | 8 | 1 | 14 | 17 | 20 | 23 | 26 | 29 | 1 | 4 | 7 | 10 | 13 | 16 |
|  | h m | h m | h m | m | h m | h m | h m | h m | m | h m | h m | h m | h m | h m | m |  | h m |
| N72 | - | - |  | - |  |  |  |  |  | 1236 | 1327 | 1357 | 1422 | 1444 | 1503 | 1522 | 15 |
|  |  |  |  |  |  |  | 1230 | 1314 | 1341 | 1403 | 1422 | 1440 | 1458 | 1514 | 29 | 1544 | 15 |
| 68 |  |  | 12 | 1302 | 1321 | 1338 | 1355 | 1411 | 1426 | 1441 | 1455 | 1509 | 1522 | 36 | 1549 | 1601 | 16 |
| 66 | 1334 | 13 | 1350 | 1400 | 1410 | 1421 | 1432 | 1444 | 1456 | 1507 | 1519 | 30 | 42 | 1553 | 1604 | 15 | 26 |
| 64 | 1414 | 1419 | 1425 | 33 | 1441 | 1449 | 1459 | 1508 | 1518 | 27 | 37 | 1547 | 1557 | $16 \quad 07$ | 17 | 27 | 36 |
| 62 | 1441 | 1445 | 1451 | 1457 | 1504 | 1511 | 1519 | 27 | 35 | 44 | 1552 | 1601 | 1610 | 19 | 28 | 36 | 45 |
| 60 | 1502 | 1506 | 1511 | 1516 | 1522 | 1529 | 1536 | 43 | 1550 | 1558 | 1605 | 1613 | 1621 | 1629 | 1637 | 1645 | $16 \quad 52$ |
| 58 | 19 | 23 | 27 | 32 | 37 | 43 | 1549 | 1556 | 1602 | 1609 | 16 | 23 | 30 | 38 | 45 | 52 | $16 \quad 59$ |
| 56 | 34 | 37 | 41 | 45 | 1550 | 1555 | 1601 | 1607 | 13 | 19 | 26 | 32 | 39 | 5 | 52 | 1658 | 17 |
| 54 | 46 | 1549 | 1553 | 1557 | 1602 | 1606 | 12 | 17 | 22 | 28 | 34 | 40 | 46 | 52 | 1658 | 1704 | 10 |
| 52 | $15 \quad 57$ | 1600 | 1603 | 1607 | 11 | 16 | 21 | 26 | 31 |  | 42 | 47 | 53 | 1658 | 1704 | 09 | 15 |
| 50 | 1607 | 1609 | 1613 | 1616 | 1620 | 1625 | 1629 | 34 | 38 | 1643 | 1648 | 1653 | 1659 | 1704 | 1709 | 1714 | 17 |
| 45 | 27 | 30 | 33 | 36 | 39 | 43 | 1647 | 1650 | 1655 | 1659 | 1703 | 1707 | 1712 | 16 | 20 | 24 | 29 |
| 40 | 44 | 1646 | 1649 | 1651 | 1654 | 1658 | 1701 | 1704 | 1708 | 1711 | 15 | 19 | 22 | 26 | 29 | 33 | 36 |
| 35 | 1658 | 1700 | 1702 | 1705 | 1707 | $17 \quad 10$ | 13 | 16 | 19 | 22 | 25 | 28 | 31 | 35 | 37 | 40 | 43 |
| 30 | 1710 | 12 | 14 | 16 | 19 | 21 | 24 | 27 | 29 | 32 | 34 | 37 | 39 | 2 | 44 | 47 | 1749 |
| 20 | 1731 | 1732 | 1734 | 1736 | 1738 | 1740 | 1742 | 1744 | 1746 | 1748 | 1750 | 1752 | 1754 | 1755 | 1757 | $17 \quad 58$ | 1800 |
| 10 | 1749 | 1751 | 1752 | 1754 | 1755 | 1757 | 1758 | 1800 | 1801 | 1803 | 1804 | 1805 | 1806 | 1807 | 1807 | 18 08 | 09 |
| 0 | 1806 | 1808 | 1809 | 1810 | 1811 | 1813 | $18 \quad 14$ | 15 | 15 | 16 | 17 | 17 | 17 | 8 | 18 | 18 | 18 |
| 10 | 24 | 25 | 26 | 27 | 28 | 29 | 29 | 29 | 30 | 30 | 30 | 30 | 29 | 29 | 28 | 27 | 26 |
| 20 | 1842 | 1843 | 1844 | 1845 | 1845 | 1846 | 1846 | 1846 | 1845 | 1845 | 1844 | 43 | 42 | 41 | 39 | 38 |  |
| 30 | 1904 | 1905 | 1905 | 1906 | 1906 | 1905 | 1905 | 1904 | 1903 | 1902 | 1900 | 1859 | 1857 | 1855 | 1852 | 1850 | 1847 |
| 35 | 17 | 17 | 18 |  | 17 | 17 | 16 | 15 | 13 | 12 | 10 | 1908 | 1905 | 1903 | 1900 | 1857 | $18 \quad 53$ |
| 40 | 32 | 32 | 32 | 32 | 31 | 30 | 29 | 27 | 26 | 23 | 21 | 18 | 15 | 12 | 08 | 1905 | 19 |
| 45 | 1950 | 1950 | 1949 | 1949 | 1948 | 1946 | 1944 | 1942 | 40 | 37 | 34 | 30 | 27 | 23 | 18 | 14 | 09 |
| 50 | 2012 | 2012 | 2011 | 2010 | 2008 | 2006 | 2003 | 2001 | 1957 | 1954 | 50 | 45 | 41 | 36 | 31 | 25 | 20 |
| 52 | 2022 | 2022 | 2021 | 2020 | 2018 | 2015 | 2012 | 2009 | $20 \quad 06$ | 2001 | 1957 | 1952 | 1947 | 1942 | 1936 | 1931 | 1925 |
| 54 | 34 | 34 | 33 | 31 | 29 | 26 | 23 | 19 | 15 | 10 | 2005 | 2000 | 1955 | 49 | 43 | 36 | 30 |
| 56 | 2048 | 2048 | 2046 | 44 | 41 | 38 | 34 | 30 | 25 | 20 | 15 | 09 | 2003 | 1956 | 50 | 43 | 36 |
| 58 | 2105 | 2104 | 2102 | 2059 | 2056 | 2052 | 2047 | 43 | 37 | 31 | 25 | 19 | 12 | 2005 | 1958 | 50 | 43 |
| S60 | 2125 | 2123 | 2120 | 2117 | 21 | 2108 | 2103 | 2058 | $20 \quad 51$ | 2045 | 2038 | 203 | $20 \quad 23$ | 20 | 20.07 | 19 | 19 |

## MORNING CIVIL TWILIGHT



EVENING CIVIL TWILIGHT

|  | Dec. |  | January |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | February |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 2 |  | 5 |  | 8 |  | 11 |  | 14 |  | 17 |  | 20 |  | 23 |  | 26 |  | 29 |  | 1 |  | 4 |  | 7 |  | 10 |  | 13 |  | 16 |  |
|  | h | m | h | m | h | m |  | m |  | m | h | m | h | m | h | m | h | m | h | m | h | m | h |  | h |  | h |  | h | m | h | m | h | m |
| N72 | 13 | 17 | 13 | 27 | 13 | 40 | 13 | 53 | 14 | 07 | 14 | 21 | 14 | 36 | 14 | 50 | 15 | 05 | 15 | 19 | 15 | 34 | 15 | 48 | 16 | 02 | 16 | 16 | 16 | 30 | 16 | 44 | 16 | 58 |
| 70 | 14 | 13 | 14 | 19 | 14 | 27 | 14 | 36 | 14 | 46 | 14 | 57 | 15 | 08 | 15 | 19 |  | 31 | 15 | 43 | 15 | 55 | 16 | 07 |  | 19 |  | 31 |  | 43 | 16 | 55 | 17 | 07 |
| 68 | 14 | 47 | 14 | 52 | 14 | 58 | 15 | 05 | 15 | 13 | 15 | 22 |  | 31 |  | 41 | 15 | 50 | 16 | 01 | 16 | 11 |  | 21 |  | 32 |  | 43 | 16 | 54 | 17 | 04 |  | 15 |
| 66 | 15 | 11 | 15 | 15 | 15 | 21 |  | 27 |  | 34 |  | 41 | 15 | 49 | 15 | 58 | 16 | 06 |  | 15 |  | 24 |  | 34 |  | 43 | 16 | 53 | 17 | 02 |  | 12 |  | 22 |
| 64 |  | 30 |  | 34 |  | 39 |  | 44 | 15 | 50 | 15 | 57 | 16 | 04 | 16 | 12 |  | 19 |  | 27 |  | 36 |  | 44 | 16 | 52 | 17 | 01 |  | 10 |  | 18 |  | 27 |
| 62 |  | 46 | 15 | 49 | 15 | 54 | 15 | 59 | 16 | 04 | 16 | 10 |  | 17 |  | 23 |  | 30 |  | 38 |  | 45 | 16 | 53 | 17 | 01 |  | 08 |  | 16 |  | 24 |  | 32 |
| 60 | 15 | 59 | 16 | 02 | 16 | 06 | 16 | 11 | 16 | 16 | 16 | 22 | 16 | 27 | 16 | 34 | 16 | 40 | 16 | 47 | 16 | 53 | 17 | 00 | 17 | 08 | 17 | 15 | 17 | 22 | 17 | 29 | 17 | 36 |
| 58 | 16 | 11 |  | 14 |  | 17 |  | 22 |  | 26 |  | 31 |  | 37 |  | 42 |  | 48 | 16 | 54 | 17 | 01 |  | 07 |  | 14 |  | 20 |  | 27 |  | 34 |  | 40 |
| 56 |  | 21 |  | 24 |  | 27 |  | 31 |  | 36 |  | 41 |  | 45 |  | 51 | 16 | 56 | 17 | 02 |  | 07 |  | 13 |  | 19 |  | 25 |  | 31 |  | 38 |  | 44 |
| 54 |  | 30 |  | 33 |  | 36 |  | 40 |  | 44 |  | 48 | 16 | 53 | 16 | 58 | 17 | 03 |  | 08 |  | 13 |  | 19 |  | 24 |  | 30 |  | 36 |  | 41 |  | 47 |
| 52 |  | 38 |  | 41 |  | 44 |  | 47 |  | 51 | 16 | 55 | 17 | 00 | 17 | 04 |  | 09 |  | 14 |  | 19 |  | 24 |  | 29 |  | 34 |  | 39 |  | 45 |  | 50 |
| 50 | 16 | 45 | 16 | 48 | 16 | 51 | 16 | 54 | 16 | 58 | 17 | 02 | 17 | 06 | 17 | 10 | 17 | 15 | 17 | 19 | 17 | 24 | 17 | 28 | 17 | 33 | 17 | 38 | 17 | 43 | 17 | 48 | 17 | 53 |
| 45 | 17 | 01 | 17 | 03 | 17 | 06 | 17 | 09 | 17 | 12 |  | 16 |  | 19 |  | 23 |  | 27 |  | 31 |  | 34 |  | 38 |  | 42 |  | 47 |  | 51 | 17 | 55 | 17 | 59 |
| 40 |  | 14 |  | 17 |  | 19 |  | 22 |  | 24 |  | 27 |  | 30 |  | 34 |  | 37 |  | 40 |  | 44 |  | 47 |  | 51 | 17 | 54 | 17 | 57 | 18 | 01 | 18 | 04 |
| 35 |  | 26 |  | 28 |  | 30 |  | 32 |  | 35 |  | 38 |  | 41 |  | 43 |  | 46 |  | 49 | 17 | 52 | 17 | 55 | 17 | 58 | 18 | 01 | 18 | 03 |  | 06 |  | 09 |
| 30 |  | 36 |  | 38 |  | 40 | 17 | 42 | 17 | 45 | 17 | 47 | 17 | 50 | 17 | 52 | 17 | 55 | 17 | 57 | 18 | 00 | 18 | 02 | 18 | 04 |  | 07 |  | 09 |  | 11 |  | 14 |
| 20 | 17 | 55 | 17 | 56 | 17 | 58 | 18 | 00 | 18 | 02 | 18 | 04 | 18 | 06 | 18 | 08 | 18 | 10 | 18 | 11 | 18 | 13 | 18 | 15 | 18 | 16 | 18 | 18 | 18 | 19 | 18 | 21 | 18 | 22 |
| 10 | 18 | 12 | 18 | 13 | 18 | 15 |  | 16 |  | 18 |  | 19 |  | 21 |  | 22 |  | 23 |  | 25 |  | 26 |  | 27 |  | 28 |  | 28 |  | 29 |  | 30 |  | 30 |
| 0 |  | 29 |  | 30 |  | 32 |  | 33 |  | 34 |  | 35 |  | 36 |  | 37 |  | 37 |  | 38 |  | 38 |  | 39 |  | 39 |  | 39 |  | 39 |  | 39 |  | 39 |
| 10 | 18 | 47 | 18 | 48 | 18 | 49 | 18 | 50 | 18 | 51 | 18 | 51 | 18 | 52 | 18 | 52 | 18 | 52 | 18 | 52 | 18 | 52 | 18 | 52 | 18 | 51 | 18 | 51 | 18 | 50 | 18 | 49 |  | 48 |
| 20 | 19 | 07 | 19 | 08 | 19 | 09 | 19 | 09 | 19 | 09 | 19 | 10 | 19 | 10 | 19 | 09 | 19 | 09 | 19 | 08 | 19 | 07 | 19 | 06 | 19 | 05 | 19 | 04 | 19 | 02 | 19 | 00 | 18 | 59 |
| 30 | 19 | 32 | 19 | 32 | 19 | 33 | 19 | 33 | 19 | 33 | 19 | 32 | 19 | 31 | 19 | 30 | 19 | 29 | 19 | 28 | 19 | 26 | 19 | 24 | 19 | 22 | 19 | 20 | 19 | 17 | 19 | 15 | 19 | 12 |
| 35 | 19 | 46 | 19 | 47 | 19 | 47 | 19 | 47 | 19 | 47 | 19 | 46 | 19 | 45 |  | 43 |  | 42 |  | 40 |  | 38 |  | 35 |  | 33 |  | 30 |  | 27 |  | 23 |  | 20 |
| 40 | 20 | 05 | 20 | 05 | 20 | 05 | 20 | 04 | 20 | 03 | 20 | 02 | 20 | 00 | 19 | 59 | 19 | 56 | 19 | 54 | 19 | 51 | 19 | 48 |  | 45 |  | 41 |  | 37 |  | 33 |  | 29 |
| 45 |  | 27 |  | 27 |  | 26 |  | 25 |  | 24 |  | 22 |  | 20 | 20 | 17 | 20 | 14 | 20 | 11 | 20 | 07 | 20 | 04 | 19 | 59 | 19 | 55 | 19 | 50 | 19 | 46 |  | 41 |
| 50 | 20 | 56 | 20 | 55 | 20 | 54 | 20 | 53 | 20 | 51 | 20 | 48 |  | 45 |  | 41 |  | 37 |  | 33 |  | 28 |  | 23 | 20 | 18 | 20 | 12 | 20 | 07 | 20 | 01 | 19 | 55 |
| 52 | 21 | 11 | 21 | 10 | 21 | 08 | 21 | 06 | 21 | 04 | 21 | 00 | 20 | 57 | 20 | 53 | 20 | 48 | 20 | 43 | 20 | 38 | 20 | 33 | 20 | 27 | 20 | 21 | 20 | 15 | 20 | 08 | 20 | 02 |
| 54 |  | 28 |  | 27 |  | 25 |  | 22 |  | 19 |  | 15 | 21 | 11 | 21 | 06 | 21 | 01 | 20 | 56 | 20 | 50 |  | 43 |  | 37 |  | 30 |  | 23 |  | 16 |  | 09 |
| 56 | 21 | 50 | 21 | 48 | 21 | 45 | 21 | 42 | 21 | 38 |  | 33 |  | 28 |  | 22 |  | 16 | 21 | 10 | 21 | 03 | 20 | 56 | 20 | 49 |  | 41 |  | 33 |  | 26 |  | 18 |
| 58 | 22 | 18 | 22 | 16 | 22 | 12 | 22 | 07 | 22 | 01 | 21 | 55 | 21 | 49 | 21 | 42 |  | 34 |  | 26 |  | 19 | 21 | 10 | 21 | 02 | 20 | 54 |  | 45 |  | 36 |  | 28 |
| S60 | 23 | 04 | 22 | 58 | 22 | 51 | 22 | 43 | 22 | 34 | 22 | 25 | 22 | 16 | 22 | 06 | 21 | 57 | 21 | 47 | 21 | 38 | 21 | 28 | 21 | 18 | 21 | 08 | 20 | 58 | 20 | 49 | 20 | 39 |

## SUNRISE

| Lat | February |  |  |  |  | March |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \hline \mathrm{Apr} \\ 2 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 16 | 19 | 22 | 25 | 28 | 3 | 6 | 9 | 12 | 15 | 18 | 21 |  | 24 |  | 27 |  | 30 |  |  |  |
|  | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h | m | h | m | h | m | h | m | h | m |
| N 72 | 0851 | 0833 | 0817 | 0801 | 0745 | 0729 | 0713 | 0658 | 0642 | 0627 | 0611 | 05 | 56 | 05 | 41 | 05 | 25 | 05 | 09 | 04 | 53 |
| 70 | 31 | 17 | 0802 | 0748 | 34 | 20 | 06 | 53 | 39 | 25 | 11 |  | 57 |  | 43 |  | 29 |  | 15 | 05 | 01 |
| 68 | 16 | 0803 | 0751 | 38 | 26 | 13 | 0701 | 48 | 36 | 23 | 11 |  | 58 |  | 45 |  | 33 |  | 20 |  | 07 |
| 66 | 0804 | 0753 | 41 | 30 | 19 | 07 | 0656 | 45 | 33 | 22 | 10 | 05 | 59 |  | 47 |  | 36 |  | 24 |  | 13 |
| 64 | 0753 | 43 | 33 | 23 | 13 | 0702 | 52 | 42 | 31 | 21 | 10 | 06 | 00 |  | 49 |  | 38 |  | 28 |  | 17 |
| 62 | 44 | 35 | 26 | 17 | 08 | 0658 | 48 | 39 | 29 | 20 | 10 |  | 00 |  | 50 |  | 41 |  | 31 |  | 21 |
| N 60 | 0737 | 0728 | 0720 | 0711 | 0703 | 0654 | 0645 | 0636 | 0627 | 0618 | 0609 | 06 | 00 | 05 | 51 | 05 | 42 | 05 | 33 | 05 | 24 |
| 58 | 30 | 22 | 15 | 07 | 0659 | 51 | 43 | 34 | 26 | 17 | 09 |  | 01 |  | 52 |  | 44 |  | 35 |  | 27 |
| 56 | 24 | 17 | 10 | 0703 | 55 | 48 | 40 | 32 | 25 | 17 | 09 |  | 01 |  | 53 |  | 45 |  | 37 |  | 29 |
| 54 | 19 | 12 | 06 | 0659 | 52 | 45 | 38 | 31 | 24 | 16 | 09 |  | 01 |  | 54 |  | 46 |  | 39 |  | 32 |
| 52 | 14 | 08 | 0702 | 56 | 49 | 43 | 36 | 29 | 22 | 15 | 09 |  | 02 |  | 55 |  | 48 |  | 41 |  | 34 |
| N 50 | 0710 | 0704 | 0658 | 0653 | 0647 | 0640 | 0634 | 0628 | 0621 | 0615 | 0608 | 06 | 02 | 05 | 55 | 05 | 49 | 05 | 42 | 05 | 36 |
| 45 | 0700 | 0656 | 51 | 46 | 41 | 35 | 30 | 25 | 19 | 14 | 08 |  | 02 |  | 57 |  | 51 |  | 46 |  | 40 |
| 40 | 0652 | 48 | 44 | 40 | 36 | 31 | 27 | 22 | 19 | 12 | 08 |  | 03 |  | 58 |  | 53 |  | 48 |  | 43 |
| 35 | 46 | 42 | 39 | 35 | 31 | 28 | 24 | 20 | 16 | 11 | 07 |  | 03 | 05 | 59 |  | 55 |  | 51 |  | 46 |
| 30 | 40 | 37 | 34 | 31 | 28 | 24 | 21 | 18 | 14 | 11 | 07 |  | 030 | 06 | 00 |  | 56 |  | 53 |  | 49 |
| N 20 | 0629 | 0627 | 0625 | 0623 | 0621 | 0619 | 0616 | 0614 | 0611 | 0609 | 0606 | 06 | 04 | 06 | 01 | 05 | 58 | 05 | 56 | 05 | 53 |
| N 10 | 20 | 19 | 17 | 16 | 15 | 13 | 12 | 11 | 09 | 07 | 06 |  | 04 |  | 02 | 06 | 01 | 05 | 59 | 05 | 57 |
|  | 11 | 11 | 10 | 10 | 09 | 09 | 08 | 07 | 07 | 06 | 05 |  | 04 |  | 03 |  | 02 | 06 | 01 | 06 | 01 |
| S 10 | 0602 | 6002 | 0603 | 0603 | 0603 | 0604 | 0604 | 04 | 04 | 04 | 04 |  | 04 |  | 04 |  | 04 |  | 04 |  | 04 |
| 20 | 0552 | 0553 | 0555 | 0556 | 0557 | 0558 | 0559 | 0600 | 0601 | 02 | 03 |  | 04 |  | 05 |  | 06 |  | 06 |  | 07 |
| S 30 | 0541 | 0543 | 0545 | 0548 | 0550 | 0552 | 0554 | 0556 | 0558 | 0600 | 0602 | 06 | 04 | 06 | 05 | 06 | 07 | 06 | 09 | 06 | 11 |
| 35 | 34 | 37 | 40 | 43 | 46 | 48 | 51 | 53 | 56 | 0559 | 01 |  | 04 |  | 06 |  | 08 |  | 11 |  | 13 |
| 40 | 27 | 30 | 34 | 37 | 41 | 44 | 47 | 51 | 54 | 57 | 0600 |  | 03 |  | 06 |  | 09 |  | 12 |  | 16 |
| 45 | 18 | 22 | 27 | 31 | 35 | 39 | 43 | 47 | 51 | 55 | 0559 |  | 03 |  | 07 |  | 11 |  | 14 |  | 18 |
| 50 | 07 | 13 | 18 | 23 | 28 | 33 | 38 | 43 | 48 | 53 | 58 |  | 03 |  | 07 |  | 12 |  | 17 |  | 21 |
| S 52 | 0502 | 0508 | 0514 | 0519 | 0525 | 0531 | 0536 | 0541 | 0547 | 0552 | 0557 | 06 | 02 | 06 | 08 | 06 | 13 | 06 | 18 | 06 | 23 |
| 54 | 0457 | 0503 | 09 | 15 | 21 | 27 | 33 | 39 | 45 | 51 | 57 |  | 02 |  | 08 |  | 13 |  | 19 |  | 25 |
| 56 | 51 | 0458 | 0504 | 11 | 18 | 24 | 31 | 37 | 43 | 50 | 56 |  | 02 |  | 08 |  | 14 |  | 20 |  | 26 |
| 58 | 44 | 51 | 0459 | 06 | 13 | 20 | 27 | 34 | 41 | 48 | 55 |  | 02 |  | 08 |  | 15 |  | 22 |  | 28 |
| 60 | 0436 | 0445 | 0453 | 0501 | 0508 | 0516 | 0524 | 0531 | 0539 | 0546 | 0554 | 06 | 01 | 06 | 09 | 06 | 6 | 06 |  | 06 | 30 |

## SUNSET

| -at. | February |  |  |  |  |  |  |  |  |  | March |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \hline \text { Apr } \\ 2 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 16 |  | 19 |  | 22 |  | 25 |  | 28 |  | 3 |  | 6 |  | 9 |  | 12 |  | 15 |  | 18 |  | 21 |  | 24 |  | 27 |  | 30 |  |  |  |
| - | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N 72 | 15 | 39 | 15 | 56 | 16 | 12 | 16 | 28 | 16 | 43 | 16 | 57 | 17 | 12 | 17 | 26 | 17 | 40 | 17 | 54 | 18 | 08 | 18 | 22 | 18 | 35 | 18 | 49 | 19 | 03 | 19 | 17 |
| 70 | 15 | 59 | 16 | 13 |  | 27 |  | 40 | 16 | 53 | 17 | 06 |  | 19 |  | 31 |  | 44 |  | 56 |  | 08 |  | 20 |  | 32 |  | 45 | 18 | 57 |  | 09 |
| 68 | 16 | 14 |  | 26 |  | 38 |  | 50 | 17 | 01 |  | 13 |  | 24 |  | 35 |  | 46 |  | 57 |  | 08 |  | 19 |  | 30 |  | 41 |  | 52 | 19 | 03 |
| 66 |  | 26 |  | 37 |  | 47 | 16 | 58 |  | 08 |  | 19 |  | 29 |  | 39 |  | 49 | 17 | 59 |  | 08 |  | 18 |  | 28 |  | 38 |  | 47 | 18 | 57 |
| 64 |  | 36 |  | 46 | 16 | 56 | 17 | 05 |  | 14 |  | 23 |  | 33 |  | 42 |  | 51 | 18 | 00 |  | 09 |  | 17 |  | 26 |  | 35 |  | 44 |  | 53 |
| 62 |  | 45 | 16 | 54 | 17 | 02 |  | 11 |  | 19 |  | 28 |  | 36 |  | 44 |  | 52 |  | 01 |  | 09 |  | 17 |  | 25 |  | 33 |  | 41 |  | 49 |
| N 60 | 16 | 52 | 17 | 00 | 17 | 08 | 17 | 16 | 17 | 24 | 17 | 31 | 17 | 39 | 17 | 46 | 17 | 54 | 17 | 01 | 18 | 09 | 18 | 16 | 18 | 23 | 18 | 30 | 18 | 38 | 18 | 45 |
| 58 | 16 | 59 |  | 06 |  | 13 |  | 20 |  | 27 |  | 34 |  | 41 |  | 48 |  | 55 |  | 02 |  | 09 |  | 15 |  | 22 |  | 29 |  | 35 |  | 42 |
| 56 | 17 | 05 |  | 12 |  | 18 |  | 25 |  | 31 |  | 37 |  | 44 |  | 50 |  | 56 |  | 03 |  | 09 |  | 15 |  | 21 |  | 27 |  | 33 |  | 39 |
| 54 |  | 10 |  | 16 |  | 22 |  | 28 |  | 34 |  | 40 |  | 46 |  | 52 |  | 57 |  | 03 |  | 09 |  | 14 |  | 20 |  | 26 |  | 31 |  | 37 |
| 52 |  | 15 |  | 21 |  | 26 |  | 32 |  | 37 |  | 42 |  | 48 |  | 53 | 17 | 58 |  | 04 |  | 09 |  | 14 |  | 19 |  | 24 |  | 30 |  | 35 |
| N 50 | 17 | 19 | 17 | 24 | 17 | 29 | 17 | 35 | 17 | 40 | 17 | 45 | 17 | 50 | 17 | 55 | 18 | 00 | 18 | 04 | 18 | 09 | 18 | 14 | 18 | 19 | 18 | 23 | 18 | 28 | 18 | 33 |
| 45 |  | 29 |  | 33 |  | 37 |  | 41 |  | 45 |  | 49 |  | 54 | 17 | 58 |  | 01 |  | 05 |  | 09 |  | 13 |  | 17 |  | 21 |  | 25 |  | 28 |
| 40 |  | 36 |  | 40 |  | 43 |  | 47 |  | 50 |  | 53 | 17 | 57 | 18 | 00 |  | 03 |  | 06 |  | 09 |  | 12 |  | 16 |  | 19 |  | 22 |  | 25 |
| 35 |  | 43 |  | 46 |  | 49 |  | 52 |  | 54 | 17 | 57 | 18 | 00 |  | 02 |  | 05 |  | 07 |  | 10 |  | 12 |  | 14 |  | 17 |  | 19 |  | 22 |
| 30 | 17 | 49 | 17 | 52 | 17 | 54 | 17 | 56 | 17 | 58 | 18 | 00 |  | 02 |  | 04 |  | 06 |  | 08 |  | 10 |  | 12 |  | 14 |  | 15 |  | 17 |  | 19 |
| N 20 | 18 | 00 | 18 | 01 | 18 | 02 | 08 | 03 | 18 | 05 | 18 | 06 | 18 | 07 | 18 | 08 | 18 | 09 | 18 | 10 | 18 | 10 | 18 | 11 | 18 | 12 | 18 | 13 | 18 | 14 | 18 | 14 |
| N 10 |  | 09 |  | 09 |  | 10 |  | 10 |  | 10 |  | 11 |  | 11 |  | 11 |  | 11 |  | 11 |  | 11 |  | 11 |  | 11 |  | 11 |  | 11 |  | 10 |
| 0 |  | 18 |  | 17 |  | 17 |  | 17 |  | 16 |  | 15 |  | 15 |  | 14 |  | 13 |  | 12 |  | 11 |  | 11 |  | 10 |  | 09 |  | 08 |  | 07 |
| S 10 |  | 26 |  | 25 |  | 24 |  | 23 |  | 22 |  | 20 |  | 19 |  | 17 |  | 16 |  | 14 |  | 12 |  | 10 |  | 09 |  | 07 |  | 05 |  | 03 |
| 20 |  | 36 |  | 34 |  | 32 |  | 30 |  | 28 |  | 26 |  | 23 |  | 21 |  | 18 |  | 16 |  | 13 |  | 10 |  | 08 |  | 05 |  | 03 | 18 | 00 |
| S 30 | 18 | 47 | 18 | 44 | 18 | 41 | 18 | 38 | 18 | 35 | 18 | 32 | 18 | 28 | 18 | 25 | 18 | 21 | 18 | 18 | 18 | 14 | 18 | 10 | 18 | 07 | 18 | 03 | 18 | 00 | 17 | 56 |
| 35 | 18 | 53 |  | 50 |  | 47 |  | 43 |  | 39 |  | 35 |  | 31 |  | 27 |  | 23 |  | 19 |  | 15 |  | 11 |  | 06 |  | 02 | 17 | 58 |  | 54 |
| 40 | 19 | 01 | 18 | 57 | 18 | 53 |  | 48 |  | 44 |  | 39 |  | 35 |  | 30 |  | 25 |  | 50 |  | 16 |  | 11 |  | 06 |  | 01 |  | 56 |  | 51 |
| 45 |  | 09 | 19 | 05 | 19 | 00 | 18 | 55 |  | 49 |  | 44 |  | 39 |  | 33 |  | 28 |  | 22 |  | 17 |  | 11 |  | 05 | 18 | 00 |  | 54 |  | 49 |
| 50 |  | 20 |  | 14 |  | 08 | 19 | 02 |  | 56 |  | 50 |  | 44 |  | 37 |  | 31 |  | 24 |  | 18 |  | 11 |  | 05 | 17 | 58 |  | 52 |  | 45 |
| S 52 | 19 | 25 | 19 | 18 | 19 | 12 | 19 | 16 | 18 | 56 | 18 | 53 | 18 | 46 | 18 | 39 | 18 | 32 | 18 | 25 | 18 | 18 | 18 | 11 | 18 | 04 | 17 | 57 | 17 | 51 | 17 | 44 |
| 54 |  | 30 |  | 23 |  | 16 |  | 10 | 19 | 03 |  | 55 |  | 48 |  | 41 |  | 34 |  | 26 |  | 19 |  | 11 |  | 04 |  | 57 |  | 49 |  | 42 |
| 56 |  | 36 |  | 29 |  | 21 |  | 14 |  | 06 | 18 | 59 |  | 51 |  | 43 |  | 35 |  | 27 |  | 19 |  | 12 |  | 04 |  | 56 |  | 48 |  | 40 |
| 58 |  | 43 |  | 35 |  | 27 |  | 19 |  | 11 | 19 | 02 |  | 54 |  | 46 |  | 37 |  | 29 |  | 20 |  | 12 |  | 03 |  | 55 |  | 46 |  | 38 |
| S 60 | 19 | 50 | 19 | 42 | 19 | 33 | 19 | 24 | 19 | 15 | 19 | 06 | 18 | 57 | 18 | 48 | 18 | 39 | 18 | 30 | 18 | 21 | 18 | 12 | 18 | 03 | 17 | 54 | 17 | 45 | 17 | 36 |


| -at. | February |  |  |  |  | March |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \hline \text { Apr } \\ 2 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 16 | 19 | 22 | 25 | 28 | 3 |  | 6 |  | 9 |  | 12 |  | 15 |  | 18 |  | 21 |  | 24 |  | 27 |  | 30 |  |  |  |
|  | h m | h m | h m | h m | h m | h | m | h | m | h | m |  | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N 72 | 0732 | 0718 | 0704 | 0650 | 0635 | 06 | 20 | 06 | 06 | 05 | 51 | 05 | 35 | 05 | 20 | 05 | 04 | 04 | 48 | 04 | 31 | 04 | 14 | 03 | 57 | 03 | 38 |
| 70 | 23 | 10 | 0658 | 45 | 32 |  | 19 |  | 05 |  | 52 |  | 38 |  | 24 |  | 10 | 04 | 56 |  | 41 |  | 26 | 04 | 11 | 03 | 55 |
| 68 | 15 | 0704 | 52 | 41 | 29 |  | 17 |  | 05 |  | 53 |  | 41 |  | 28 |  | 15 | 05 | 02 |  | 49 |  | 36 |  | 22 | 04 | 08 |
| 66 | 08 | 0658 | 48 | 37 | 27 |  | 16 |  | 05 |  | 54 |  | 42 |  | 31 |  | 19 |  | 08 | 04 | 56 |  | 44 |  | 31 |  | 19 |
| 64 | 0702 | 53 | 44 | 34 | 25 |  | 15 |  | 05 |  | 54 |  | 44 |  | 33 |  | 23 |  | 12 | 05 | 01 |  | 50 |  | 39 |  | 27 |
| 62 | 0657 | 49 | 41 | 32 | 23 |  | 16 |  | 04 |  | 55 |  | 45 |  | 36 |  | 26 |  | 16 |  | 06 | 04 | 56 |  | 45 |  | 35 |
| N 60 | 0653 | 0645 | 0639 | 0629 | 0621 | 06 | 13 | 06 | 04 | 05 | 55 | 05 | 46 | 05 | 37 | 05 | 28 | 05 | 19 | 05 | 10 | 05 | 00 | 04 | 51 | 04 | 41 |
| 58 | 49 | 42 | 35 | 27 | 19 |  | 12 |  | 04 |  | 55 |  | 47 |  | 39 |  | 30 |  | 22 |  | 13 |  | 40 | 40 | 56 |  | 47 |
| 56 | 45 | 39 | 32 | 25 | 18 |  | 10 |  | 03 |  | 55 |  | 48 |  | 40 |  | 32 |  | 24 |  | 16 |  | 08 | 05 | 00 |  | 51 |
| 54 | 42 | 36 | 30 | 23 | 17 |  | 10 |  | 03 |  | 55 |  | 48 |  | 41 |  | 33 |  | 26 |  | 18 |  | 11 |  | 03 |  | 56 |
| 52 | 39 | 34 | 28 | 22 | 15 |  | 09 |  | 02 |  | 56 |  | 49 |  | 42 |  | 35 |  | 28 |  | 21 |  | 14 |  | 07 | 04 | 59 |
| N 50 | 0636 | 0631 | 0625 | 0620 | 0614 | 06 | 08 | 06 | 02 | 05 | 56 | 05 | 49 | 05 | 43 | 05 | 36 | 05 | 30 | 05 | 23 | 05 | 16 | 05 | 10 | 05 | 03 |
| 45 | 30 | 25 | 21 | 16 | 11 |  | 06 | 06 | 01 |  | 55 |  | 50 |  | 44 |  | 39 |  | 33 |  | 27 |  | 22 |  | 16 |  | 10 |
| 40 | 24 | 21 | 17 | 13 | 08 |  | 04 | 05 | 59 |  | 55 |  | 50 |  | 46 |  | 41 |  | 36 |  | 31 |  | 26 |  | 21 |  | 16 |
| 35 | 20 | 16 | 13 | 10 | 06 |  | 02 |  | 58 |  | 54 |  | 50 |  | 46 |  | 42 |  | 39 |  | 34 |  | 29 |  | 25 |  | 21 |
| 30 | 15 | 12 | 1 | 07 | 0604 | 06 | 00 |  | 57 |  | 54 |  | 50 |  | 47 |  | 43 |  | 39 |  | 36 |  | 32 |  | 28 |  | 25 |
| N 20 | 0606 | 0605 | 0603 | 0601 | 0559 | 05 | 56 | 05 | 54 | 05 | 52 | 05 | 49 | 05 | 47 | 05 | 44 | 05 | 42 | 05 | 39 | 05 | 39 | 05 | 34 | 05 | 31 |
| N 10 | 0558 | 0557 | 0556 | 0555 | 54 |  | 53 |  | 51 |  | 50 |  | 48 |  | 46 |  | 45 |  | 43 |  | 41 |  | 40 |  | 39 |  | 36 |
|  | 50 | 4 | 49 | 49 | 48 |  | 48 |  | 47 |  | 47 |  | 46 |  | 45 |  | 44 |  | 43 |  | 42 |  | 42 |  | 41 |  | 40 |
| S 10 | 40 | 41 | 41 | 42 | 42 |  | 42 |  | 43 |  | 43 |  | 43 |  | 43 |  | 43 |  | 43 |  | 43 |  | 43 |  | 43 |  | 43 |
| 20 | 29 | 31 | 32 | 34 | 35 |  | 36 |  | 37 |  | 38 |  | 39 |  | 40 |  | 41 |  | 42 |  | 43 |  | 44 |  | 44 |  | 45 |
| S 30 | 0516 | 0518 | 0521 | 0523 | 0526 | 05 | 28 | 05 | 30 | 05 | 32 | 05 | 34 | 05 | 36 | 05 | 38 | 05 | 40 | 05 | 42 | 05 | 44 | 05 | 45 | 05 | 47 |
| 35 | 0508 | 11 | 14 | 17 | 20 |  | 23 |  | 25 |  | 28 |  | 31 |  | 33 |  | 36 |  | 38 |  | 41 |  | 43 |  | 45 |  | 48 |
| 40 | 0458 | 0502 | 0506 | 09 | 13 |  | 16 |  | 20 |  | 23 |  | 26 |  | 30 |  | 33 |  | 36 |  | 39 |  | 42 |  | 45 |  | 48 |
| 45 | 46 | 0451 | 0456 | 0500 | 0505 |  | 09 |  | 13 |  | 18 |  | 22 |  | 26 |  | 30 |  | 34 |  | 38 |  | 41 |  | 45 |  | 49 |
| 50 | 32 | 38 | 43 | 0449 | 0454 | 05 | 00 |  | 05 |  | 10 |  | 15 |  | 21 |  | 25 |  | 30 |  | 35 |  | 40 |  | 44 |  | 49 |
| S 52 | 0425 | 0431 | 0438 | 0444 | 0450 | 04 | 55 | 05 | 01 | 05 | 07 | 05 | 1 | 05 | 18 | 05 | 23 | 05 | 29 | 05 | 34 | 05 | 39 | 05 | 44 | 05 | 49 |
| 54 | 17 | 24 | 31 | 38 | 44 |  | 51 | 04 | 57 | 05 | 03 |  | 09 |  | 15 |  | 21 |  | 27 |  | 32 |  | 28 |  | 44 |  | 49 |
| 56 | 0409 | 16 | 24 | 31 | 38 |  | 45 |  | 52 | 04 | 59 |  | 05 |  | 12 |  | 18 |  | 25 |  | 31 |  | 37 |  | 43 |  | 49 |
| 58 | 0359 | $04 \quad 07$ | 15 | 23 | 31 |  | 39 |  | 47 |  | 54 | 05 | 01 |  | 08 |  | 15 |  | 22 |  | 29 |  | 36 |  | 42 |  | 49 |
| S 60 | 0347 | 0356 | 0406 | 0415 | 0423 | 04 | 32 | 04 | 40 | 04 | 48 | 04 | 57 | 05 | 04 | 05 | 12 | 05 | 20 | 05 | 27 | 05 | 34 | 05 | 42 | 05 | 49 |

EVENING CIVIL TWILIGHT

| -at. | February |  |  |  |  |  |  |  |  |  | March |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \mathrm{Apr} \\ 2 \\ \hline \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 16 |  | 19 |  |  | 22 | 25 |  | 28 |  | 3 |  | 6 |  | 9 |  | 12 |  | 15 |  | 18 |  | 21 |  | 24 |  | 27 |  | 30 |  |  |  |
|  | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | ¢ m |
| N 72 | 16 | 58 | 17 | 12 | 17 | 26 | 17 | 39 | 17 | 53 | 18 | 06 | 18 | 20 | 18 | 34 | 18 | 48 | 1 | 02 | 19 | 16 | 19 | 30 | 19 | 45 | 20 | 01 | 20 | 17 | 20 | 34 |
| 70 | 17 | 07 |  | 20 |  | 32 |  | 44 |  | 56 |  | 08 |  | 20 |  | 32 |  | 44 | 18 | 57 |  | 09 |  | 22 |  | 35 | 19 | 48 | 20 | 02 |  | 16 |
| 68 |  | 15 |  | 26 |  | 37 |  | 47 | 17 | 58 |  | 09 |  | 20 |  | 31 |  | 42 |  | 53 |  | 04 |  | 15 |  | 27 |  | 38 | 19 | 50 | 20 | 03 |
| 66 |  | 22 |  | 31 |  | 41 |  | 51 | 18 | 00 |  | 10 |  | 20 |  | 30 |  | 40 |  | 50 | 19 | 00 |  | 10 |  | 20 |  | 30 |  | 41 | 19 | 52 |
| 64 |  | 27 |  | 36 |  | 45 |  | 54 |  | 02 |  | 11 |  | 20 |  | 29 |  | 38 |  | 47 | 18 | 45 |  | 05 |  | 14 |  | 24 |  | 33 |  | 43 |
| 62 |  | 32 |  | 40 |  | 48 |  | 56 |  | 04 |  | 12 |  | 20 |  | 28 |  | 36 |  | 45 |  | 53 | 19 | 01 |  | 09 |  | 18 |  | 26 |  | 35 |
| N 60 | 17 | 36 | 17 | 44 | 17 | 51 | 17 | 59 | 18 | 06 | 18 | 13 | 18 | 21 | 18 | 28 | 18 | 35 | 18 | 43 | 18 | 50 | 18 | 58 | 19 | 05 | 19 | 13 | 19 | 21 | 19 | 29 |
| 58 |  | 40 |  | 47 |  | 54 | 18 | 01 |  | 07 |  | 14 |  | 21 |  | 28 |  | 34 |  | 41 |  | 48 |  | 55 | 19 | 02 |  | 09 |  | 16 |  | 23 |
| 56 |  | 44 |  | 50 |  | 56 |  | 02 |  | 09 |  | 15 |  | 21 |  | 27 |  | 33 |  | 40 |  | 46 |  | 52 | 18 | 58 |  | 05 |  | 11 |  | 17 |
| 54 |  | 47 |  | 53 | 17 | 58 |  | 04 |  | 10 |  | 16 |  | 21 |  | 27 |  | 33 |  | 38 |  | 44 |  | 50 |  | 56 | 19 | 01 |  | 07 |  | 13 |
| 52 |  | 50 |  | 55 | 18 | 01 |  | 06 |  | 11 |  | 16 |  | 22 |  | 27 |  | 32 |  | 37 |  | 43 |  | 48 |  | 53 | 18 | 58 |  | 04 |  | 09 |
| N 50 | 17 | 53 | 17 | 58 | 18 | 02 | 18 | 07 | 18 | 12 | 18 | 17 | 18 | 22 | 18 | 27 | 18 | 31 | 18 | 36 | 18 | 41 | 18 | 46 | 18 | 51 | 18 | 56 | 19 | 01 | 19 | 06 |
| 45 | 17 | 59 | 18 | 03 |  | 07 |  | 11 |  | 15 |  | 19 |  | 23 |  | 27 |  | 31 |  | 35 |  | 38 |  | 42 |  | 46 |  | 50 | 18 | 54 | 18 | 58 |
| 40 | 18 | 04 |  | 08 |  | 11 |  | 14 |  | 17 |  | 21 |  | 24 |  | 27 |  | 31 |  | 33 |  | 36 |  | 40 |  | 43 |  | 46 |  | 49 |  | 52 |
| 35 |  | 09 |  | 12 |  | 15 |  | 17 |  | 20 |  | 22 |  | 25 |  | 27 |  | 31 |  | 32 |  | 35 |  | 37 |  | 40 |  | 42 |  | 45 |  | 47 |
| 30 |  | 14 |  | 16 |  | 18 |  | 20 |  | 22 |  | 24 |  | 26 |  | 28 |  | 30 |  | 32 |  | 34 |  | 36 |  | 37 |  | 39 |  | 41 |  | 43 |
| N 20 | 18 | 22 | 18 | 23 | 18 | 25 | 18 | 26 | 18 | 27 | 18 | 27 | 18 | 29 | 18 | 30 | 18 | 31 | 18 | 32 | 18 | 32 | 18 | 33 | 18 | 34 | 18 | 35 | 18 | 36 | 18 | 37 |
| N 10 |  | 30 |  | 31 |  | 31 |  | 31 |  | 32 |  | 32 |  | 32 |  | 32 |  | 32 |  | 32 |  | 32 |  | 32 |  | 32 |  | 32 |  | 32 |  | 32 |
|  |  | 39 |  | 38 |  | 38 |  | 37 |  | 37 |  | 36 |  | 36 |  | 35 |  | 34 |  | 33 |  | 32 |  | 31 |  | 30 |  | 30 |  | 29 |  | 28 |
| S 10 |  | 48 |  | 47 |  | 46 |  | 44 |  | 43 |  | 41 |  | 40 |  | 38 |  | 37 |  | 35 |  | 33 |  | 32 |  | 30 |  | 28 |  | 26 |  | 25 |
| 20 | 18 | 59 | 18 | 57 | 18 | 55 | 18 | 52 |  | 50 |  | 48 |  | 45 |  | 43 |  | 40 |  | 38 |  | 35 |  | 32 |  | 30 |  | 27 |  | 25 |  | 22 |
| S 30 | 19 | 12 | 19 | 09 | 19 | 06 | 19 | 03 | 18 | 59 | 18 | 56 | 18 | 52 | 18 | 49 | 18 | 45 | 18 | 42 | 18 | 28 | 18 | 34 | 18 | 31 | 18 | 27 | 18 | 23 | 18 | 20 |
| 35 |  | 20 |  | 16 |  | 13 |  | 09 | 19 | 05 | 19 | 01 | 18 | 57 |  | 53 |  | 49 |  | 44 |  | 40 |  | 36 |  | 32 |  | 27 |  | 23 |  | 19 |
| 40 |  | 29 |  | 25 |  | 21 |  | 16 |  | 12 |  | 04 | 19 | 02 | 18 | 57 |  | 52 |  | 48 |  | 43 |  | 38 |  | 33 |  | 28 |  | 23 |  | 18 |
| 45 |  | 41 |  | 36 |  | 30 |  | 25 |  | 20 |  | 14 |  | 08 | 19 | 03 | 18 | 57 |  | 51 |  | 46 |  | 40 |  | 34 |  | 29 |  | 23 |  | 18 |
| 50 | 19 | 55 |  | 49 |  | 42 |  | 36 |  | 30 |  | 23 |  | 19 |  | 10 | 19 | 03 |  | 57 |  | 50 |  | 43 |  | 37 |  | 30 |  | 24 |  | 17 |
| S 52 | 20 | 02 | 19 | 55 | 19 | 48 | 19 | 41 | 19 | 34 | 19 | 27 | 19 | 20 | 19 | 13 | 19 | 06 | 18 | 59 | 18 | 52 | 18 | 45 | 18 | 28 | 18 | 31 | 18 | 24 | 18 | 17 |
| 54 |  | 09 | 20 | 02 | 19 | 55 |  | 47 |  | 40 |  | 32 |  | 24 |  | 17 |  | 09 | 19 | 02 |  | 54 |  | 47 |  | 39 |  | 32 |  | 24 |  | 17 |
| 56 |  | 18 |  | 10 | 20 | 02 | 19 | 45 |  | 46 |  | 39 |  | 29 |  | 21 |  | 13 |  | 05 | 18 | 57 |  | 49 |  | 41 |  | 33 |  | 25 |  | 17 |
| 58 |  | 28 |  | 19 |  | 10 | 20 | 01 | 19 | 52 |  | 44 |  | 35 |  | 26 |  | 17 |  | 08 | 19 | 00 |  | 51 |  | 42 |  | 34 |  | 26 |  | 17 |
| S 60 | 20 | 39 | 20 | 29 | 20 | 19 | 20 | 10 | 20 | 00 | 19 | 50 | 19 | 41 | 19 | 31 | 19 | 22 | 19 | 12 | 19 | 03 | 18 | 54 | 18 | 44 | 18 | 35 | 18 | 26 | 18 | 17 |


| -at. | 2 |  | 5 |  | 8 |  | April |  |  |  |  |  | 20 |  | 23 |  | 26 |  | 29 |  | 2 |  | 5 |  | May |  |  |  | 14 |  | 17 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 11 |  |  |  | 14 |  | 17 | 8 | 8 |  |  | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |
|  | h | m |  |  | h | m | h | m | h | m | h | m | h | m |  |  | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N 72 | 04 | 53 | 04 | 28 | 04 | 21 | 04 | 05 | 03 | 48 | 03 | 30 | 03 | 12 | 02 | 54 | 02 | 34 | 02 | 12 | 01 | 48 | 01 | 19 | 00 | 30 | $\square$ |  | $\square$ |  | $\square$ |  |
| 70 | 05 | 01 |  | 47 |  | 33 |  | 18 | 04 | 03 | 0 | 49 |  | 33 | 03 | 18 | 03 | 02 | 02 | 45 | 02 | 28 | 02 | 10 | 01 | 50 | 01 | 27 | 00 | 59 | $\square$ |  |
| 68 |  | 07 | 04 | 55 |  | 42 |  | 29 |  | 16 | 04 | 03 | 03 | 50 |  | 36 |  | 23 | 03 | 09 | 02 | 55 | 02 | 41 | 02 | 26 | 02 | 11 | 01 | 55 | 01 | 38 |
| 66 |  | 13 | 05 | 01 |  | 49 |  | 38 |  | 26 |  | 14 | 04 | 03 | 03 | 51 |  | 39 |  | 27 | 03 | 16 | 03 | 04 | 02 | 52 | 02 | 40 | 02 | 28 | 02 | 16 |
| 64 |  | 17 |  | 06 | 04 | 56 |  | 45 |  | 35 |  | 24 |  | 13 | 04 | 03 | 03 | 52 |  | 42 |  | 32 |  | 22 | 03 | 12 | 03 | 02 | 02 | 52 | 02 | 42 |
| 62 |  | 21 |  | 11 | 05 | 01 |  | 52 |  | 42 |  | 32 |  | 23 |  | 13 | 04 | 04 | 03 | 54 |  | 45 |  | 36 |  | 27 |  | 19 | 03 | 11 | 03 | 02 |
| N 60 | 05 | 24 | 05 | 15 | 05 | 06 | 04 | 57 | 04 | 48 | 04 | 19 | 04 | 30 | 04 | 22 | 04 | 13 | 04 | 05 | 03 | 57 | 03 | 49 | 03 | 41 | 03 | 33 | 03 | 26 | 03 | 19 |
| 58 |  | 27 |  | 18 |  | 10 | 05 | 02 |  | 53 |  | 45 |  | 37 |  | 29 |  | 22 |  | 14 | 04 |  | 03 | 59 | 03 | 52 |  | 45 |  | 39 |  | 33 |
| 56 |  | 29 |  | 22 |  | 14 |  | 06 | 04 | 58 |  | 51 |  | 43 |  | 36 |  | 29 |  | 22 |  | 15 | 04 | 08 | 04 | 02 | 03 | 56 | 03 | 50 |  | 44 |
| 54 |  | 32 |  | 24 |  | 17 |  | 10 | 05 | 03 | 04 | 56 |  | 49 |  | 42 |  | 35 |  | 29 |  | 23 |  | 17 |  | 11 | 04 | 05 | 04 | 00 | 03 | 55 |
| 52 |  | 34 |  | 27 |  | 20 |  | 13 |  | 07 | 05 | 00 |  | 54 |  | 47 |  | 41 |  | 35 |  | 29 |  | 24 |  | 18 |  | 13 |  | 08 | 04 | 04 |
| N 50 | 05 | 36 | 05 | 29 | 05 | 23 | 05 | 17 | 05 | 10 | 05 | 04 | 04 | 58 | 04 | 52 | 04 | 47 | 04 | 41 | 04 | 36 | 04 | 30 | 04 | 25 | 04 | 21 | 04 | 16 | 04 | 12 |
| 45 |  | 40 |  | 34 |  | 29 |  | 23 |  | 18 |  | 13 | 05 | 08 | 05 | 03 | 04 | 58 | 04 | 53 | 04 | 49 |  | 44 |  | 40 |  | 36 |  | 33 |  | 29 |
| 40 |  | 43 |  | 39 |  | 34 |  | 29 |  | 25 |  | 20 |  | 16 |  | 11 | 05 | 07 | 05 | 03 | 05 | 00 | 04 | 56 | 04 | 52 | 04 | 49 |  | 46 |  | 43 |
| 35 |  | 46 |  | 42 |  | 39 |  | 34 |  | 30 |  | 26 |  | 22 |  | 19 |  | 15 |  | 12 |  | 09 | 05 | 06 | 05 | 03 | 05 | 00 | 04 | 58 | 04 | 55 |
| 30 |  | 49 |  | 45 |  | 42 |  | 38 |  | 35 |  | 32 |  | 28 |  | 25 |  | 22 |  | 19 |  | 17 |  | 14 |  | 12 |  | 10 | 05 | 08 | 05 | 06 |
| N 20 | 05 | 53 | 05 | 51 | 05 | 48 | 05 | 46 | 05 | 43 | 05 | 41 | 05 | 38 | 05 | 36 | 05 | 34 | 05 | 32 | 05 | 30 | 05 | 29 | 05 | 27 | 05 | 26 | 05 | 25 | 05 | 23 |
| N 10 | 05 | 57 | 05 | 55 |  | 54 |  | 52 |  | 50 |  | 49 |  | 47 |  | 46 |  | 45 |  | 44 |  | 43 |  | 42 |  | 41 |  | 10 |  | 39 |  | 39 |
|  | 06 | 01 | 06 | 00 | 05 | 59 | 05 | 58 | 05 | 57 | 05 | 56 | 05 | 56 | 05 | 55 | 05 | 55 | 05 | 54 | 05 | 54 | 05 | 53 | 05 | 53 | 05 | 53 | 05 | 53 | 05 | 53 |
| S 10 |  | 04 |  | 04 | 06 | 04 | 06 | 04 | 06 | 04 | 06 | 04 | 06 | 0 | 06 | 04 | 06 | 04 | 06 | 04 | 06 | 05 | 06 | 05 | 06 | 05 | 06 | 06 | 06 | 06 | 06 | 07 |
| 20 |  | 07 |  | 08 |  | 09 |  | 09 |  | 10 |  | 11 |  | 12 |  | 13 |  | 14 |  | 15 |  | 16 |  | 17 |  | 18 |  | 19 |  | 20 |  | 22 |
| S 30 | 06 | 11 | 06 | 13 | 06 | 14 | 06 | 16 | 06 | 18 | 06 | 20 | 06 | 22 | 06 | 24 | 06 | 25 | 06 | 27 | 06 | 29 | 06 | 31 | 06 | 33 | 06 | 35 | 06 | 37 | 06 | 39 |
| 35 |  | 13 |  | 16 |  | 18 |  | 20 |  | 23 |  | 25 |  | 27 |  | 30 |  | 32 |  | 34 |  | 37 |  | 39 |  | 41 |  | 44 |  | 46 | 06 | 48 |
| 40 |  | 16 |  | 19 |  | 22 |  | 25 |  | 28 |  | 30 |  | 33 |  | 36 |  | 39 |  | 24 |  | 25 |  | 48 | 06 | 51 | 06 | 54 | 06 | 57 | 07 | 00 |
| 45 |  | 18 |  | 22 |  | 26 |  | 29 |  | 33 |  | 37 |  | 41 |  | 44 |  | 48 | 06 | 52 | 06 | 55 | 06 | 59 | 07 | 02 | 07 | 06 | 06 | 09 |  | 13 |
| 50 |  | 21 |  | 26 |  | 31 |  | 35 |  | 40 |  | 45 |  | 49 |  | 54 | 06 | 58 | 07 | 03 | 07 | 07 | 07 | 12 |  | 16 |  | 20 |  | 25 |  | 29 |
| S 52 | 06 | 23 | 06 | 28 | 06 | 33 | 06 | 38 | 06 | 43 | 06 | 48 | 06 | 53 | 06 | 58 | 07 | 06 | 07 | 08 | 07 | 13 | 07 | 18 | 07 | 23 | 07 | 27 | 07 | 32 | 07 | 36 |
| 54 |  | 25 |  | 30 |  | 36 |  | 41 |  | 46 |  | 52 | 06 | 57 | 07 | 03 |  | 08 |  | 14 |  | 19 |  | 24 |  | 29 |  | 35 |  | 40 |  | 44 |
| 56 |  | 26 |  | 32 |  | 28 |  | 44 |  | 50 | 06 | 56 | 07 | 02 |  | 08 |  | 14 |  | 20 |  | 26 |  | 32 |  | 37 |  | 43 |  | 48 | 07 | 54 |
| 58 |  | 28 |  | 35 |  | 41 |  | 48 |  | 54 | 07 | 01 |  | 08 |  | 14 |  | 21 |  | 27 |  | 33 |  | 40 |  | 46 | 07 | 52 | 07 | 58 | 08 | 04 |
| S 60 | 06 | 30 | 06 | 37 | 06 | 45 | 06 | 52 | 06 | 59 | 07 | 06 | 07 | 13 | 07 | 21 | 07 | 28 | 07 | 35 | 07 | 24 | 07 | 49 | 07 | 56 | 08 | 03 | 08 | 10 | 08 | 16 |

SUNSET

| at. | April |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | May |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 |  | 5 |  | 8 |  | 11 |  | 14 |  | 17 |  | 20 |  | 23 |  | 26 |  | 29 |  | 2 |  | 5 |  | 8 |  | 11 |  | 14 |  | 17 |  |
|  | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N72 | 19 | 17 | 19 | 32 | 19 | 46 | 20 | 01 | 20 | 17 | 20 | 33 | 20 | 50 | 21 | 08 | 21 | 27 | 21 | 49 | 22 | 13 | 22 | 44 | 23 | 52 | $\square$ |  | $\square$ |  | $\square$ |  |
| 70 |  | 09 |  | 22 |  | 35 | 19 | 48 | 20 | 01 |  | 14 |  | 28 | 20 | 43 | 20 | 58 | 21 | 14 | 21 | 31 | 21 | 49 | 22 | 09 | 22 | 33 | 23 | 04 | $\square$ |  |
| 68 | 19 | 03 |  | 14 |  | 25 |  | 36 | 19 | 48 | 20 | 00 | 20 | 12 |  | 24 |  | 36 | 20 | 49 | 21 | 03 | 21 | 16 | 21 | 31 | 21 | 46 | 22 | 02 | 22 | 20 |
| 66 | 28 | 57 |  | 07 |  | 17 |  | 27 |  | 38 | 19 | 48 | 19 | 58 | 20 | 09 |  | 19 |  | 30 | 20 | 34 | 20 | 53 | 21 | 04 | 21 | 16 | 21 | 28 | 21 | 40 |
| 64 |  | 53 | 19 | 02 |  | 11 |  | 20 |  | 29 |  | 38 |  | 47 | 19 | 56 | 20 | 06 |  | 15 |  | 25 |  | 34 | 20 | 44 | 20 | 54 | 21 | 03 | 21 | 13 |
| 62 |  | 49 | 18 | 57 |  | 05 |  | 13 |  | 21 |  | 29 |  | 38 |  | 46 | 19 | 54 | 2 | 03 | 20 | 11 |  | 19 |  | 28 |  | 36 | 20 | 44 | 20 | 52 |
| N 60 | 18 | 45 | 18 | 52 | 19 | 00 | 19 | 07 | 19 | 14 | 19 | 22 | 19 | 29 | 19 | 37 | 19 | 44 | 19 | 52 | 19 | 59 | 20 | 07 | 20 | 14 | 20 | 21 | 20 | 28 | 20 | 35 |
| 58 |  | 42 |  | 49 | 18 | 55 | 19 | 02 |  | 09 |  | 15 |  | 22 |  | 29 |  | 36 |  | 42 |  | 49 | 19 | 56 | 20 | 02 | 20 | 09 |  | 15 |  | 21 |
| 56 |  | 39 |  | 45 |  | 51 | 18 | 58 | 19 | 04 |  | 10 |  | 16 |  | 22 |  | 28 |  | 34 |  | 40 |  | 46 | 19 | 52 | 19 | 58 | 20 | 04 | 20 | 09 |
| 54 |  | 37 |  | 42 |  | 48 |  | 54 | 18 | 59 |  | 05 |  | 10 |  | 16 |  | 22 |  | 27 |  | 33 |  | 38 |  | 43 |  | 49 | 19 | 54 | 19 | 59 |
| 52 |  | 35 |  | 40 |  | 45 |  | 50 |  | 55 | 19 | 00 |  | 05 |  | 11 |  | 16 |  | 21 |  | 26 |  | 31 |  | 36 |  | 40 |  | 45 |  | 50 |
| N 50 | 18 | 33 | 18 |  | 18 | 42 | 18 | 47 | 18 | 51 | 18 | 56 | 19 | 01 | 19 | 06 | 19 | 10 | 19 | 15 | 19 | 19 | 19 | 24 | 19 | 29 | 19 | 33 | 19 | 37 | 19 | 41 |
| 45 |  | 28 |  | 32 |  | 36 |  | 40 |  | 44 |  | 47 | 18 | 51 | 18 | 55 | 18 | 59 | 19 | 02 | 19 | 06 | 19 | 10 |  | 13 |  | 17 |  | 21 |  | 24 |
| 40 |  | 25 |  | 28 |  | 31 |  | 34 |  | 37 |  | 40 |  | 43 |  | 46 |  | 49 | 18 | 52 | 18 | 55 | 18 | 58 | 19 | 01 | 19 | 04 | 19 | 07 | 19 | 10 |
| 35 |  | 22 |  | 24 |  | 27 |  | 29 |  | 31 |  | 34 |  | 36 |  | 38 |  | 41 |  | 43 |  | 46 |  | 48 | 18 | 51 | 18 | 53 | 18 | 55 | 18 | 58 |
| 30 |  | 19 |  | 21 |  | 23 |  | 24 |  | 26 |  | 28 |  | 30 |  | 32 |  | 34 |  | 36 |  | 38 |  | 40 |  | 42 |  | 43 |  | 45 |  | 47 |
| N 20 | 18 | 14 | 18 | 15 | 18 | 16 | 18 | 17 | 18 | 18 | 18 | 19 | 18 | 20 | 18 | 21 | 18 | 22 | 18 | 23 | 18 | 24 | 18 | 25 | 18 | 26 | 18 | 27 | 18 | 28 | 18 | 30 |
| N 10 |  | 10 |  | 10 |  | 10 |  | 10 |  | 10 |  | 11 |  | 11 |  | 11 |  | 11 |  | 11 |  | 12 |  | 12 |  | 13 |  | 13 |  | 14 |  | 14 |
|  |  | 07 |  | 06 |  | 05 | 18 | 14 | 18 | 04 | 18 | 03 | 18 | 02 | 18 | 02 | 18 | 01 | 18 | 01 | 18 | 00 | 18 | 00 | 18 | 00 | 18 | 00 | 18 | 00 | 18 | 00 |
| S 10 |  | 03 | 18 | 02 | 18 | 00 | 17 | 59 | 17 | 57 | 17 | 56 | 17 | 54 | 17 | 53 | 17 | 52 | 17 | 50 | 17 | 49 | 17 | 48 | 17 | 48 | 17 | 47 | 17 | 46 | 17 | 46 |
| 20 | 18 | 00 | 17 | 58 | 18 | 55 |  | 53 |  | 50 |  | 48 |  | 46 |  | 43 |  | 41 |  | 40 |  | 38 |  | 36 |  | 35 |  | 33 |  | 32 |  | 31 |
| S 30 | 17 | 56 | 17 | 53 | 17 | 49 | 17 | 46 | 17 | 42 | 17 | 39 | 17 | 36 | 17 | 33 | 17 | 30 | 17 | 27 | 17 | 24 | 17 | 22 | 17 | 20 | 17 | 18 | 17 | 16 | 17 | 14 |
| 35 |  | 54 |  | 50 |  | 46 |  | 42 |  | 38 |  | 34 |  | 30 |  | 27 |  | 23 |  | 20 |  | 17 |  | 14 |  | 11 | 17 | 08 | 17 | 06 | 17 | 04 |
| 40 |  | 51 |  | 47 |  | 42 |  | 37 |  | 33 |  | 28 |  | 24 |  | 20 |  | 16 |  | 12 | 17 | 08 | 17 | 5 | 17 | 01 | 16 | 58 | 16 | 56 | 16 | 53 |
| 45 |  | 49 |  | 43 |  | 38 |  | 32 |  | 27 |  | 22 |  | 17 |  | 12 | 17 | 07 | 17 | 02 | 16 | 58 | 16 | 54 | 16 | 50 |  | 46 |  | 43 |  | 40 |
| 50 |  | 45 |  | 39 |  | 32 |  | 26 |  | 20 |  | 14 |  | 08 | 17 | 02 | 16 | 57 | 16 | 51 |  | 46 |  | 41 |  | 36 |  | 32 |  | 27 |  | 23 |
| S 52 | 17 | 44 | 17 | 37 | 17 | 30 | 17 | 23 | 17 | 17 | 17 | 10 | 17 | 04 | 16 | 58 | 16 | 52 | 16 | 46 | 16 | 40 | 16 | 35 | 16 | 30 | 16 | 25 | 16 | 20 | 16 | 16 |
| 54 |  | 42 |  | 35 |  | 27 |  | 20 |  | 13 |  | 06 | 17 | 00 |  | 53 |  | 47 |  | 40 |  | 34 |  | 28 |  | 23 |  | 17 |  | 12 | 16 | 08 |
| 56 |  | 40 |  | 32 |  | 25 |  | 17 |  | 10 | 17 | 02 | 16 | 55 |  | 48 |  | 41 |  | 34 |  | 27 |  | 21 |  | 15 |  | 09 | 16 | 04 | 15 | 58 |
| 58 |  | 38 |  | 30 |  | 22 |  | 13 |  | 05 | 16 | 57 |  | 50 |  | 42 |  | 34 |  | 27 |  | 20 |  | 13 | 16 | 06 | 16 | 00 | 15 | 54 |  | 48 |
| S 60 |  | 36 | 17 | 27 | 17 | 18 | 17 |  | 17 | 01 | 16 |  | 16 | 仡 | 16 |  |  | 7 | 16 | 19 |  |  | 16 | 03 | 15 | 56 | 15 | 49 | 15 | 42 | 15 | 36 |



EVENING CIVIL TWILIGHT


SUNRISE

| Lat. | May |  |  |  |  |  |  |  | June |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \text { July } \\ 1 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 17 | 20 | 23 |  | 26 |  | 29 |  | 1 |  | 4 |  | 7 |  | 10 |  | 13 |  | 16 |  | 19 |  | 22 |  | 25 |  | 28 |  |  |  |
|  | h m | h m | h | m | h | m | h | m | h | m | h | m | n | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N72 | $\square$ | $\square$ | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  |
| 70 | $\square$ | $\square$ | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  |
| 68 | 0138 | 0120 | 00 | 58 | 00 | 25 | $\square$ |  | $\square$ | $\square$ | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  |
| 66 | 0216 | 0204 | 01 | 52 | 01 | 39 | 01 | 27 | 01 | 114 | 01 | 01 | 00 | 47 | 00 | 31 | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | 00 | 13 |
| 64 | 0242 | 33 | 0 | 24 | 02 | 15 | 02 | 07 | 01 | 159 | 01 | 52 | 01 | 45 | 01 | 40 | 01 | 36 | 01 | 33 | 01 | 31 | 01 | 31 | 01 | 32 | 01 | 35 | 01 | 40 |
| 62 | 0302 | 0255 | 02 | 47 | 02 | 40 |  | 34 | 02 | 228 | 02 | 23 | 02 | 18 | 02 | 14 | 02 | 12 | 02 | 10 | 02 | 09 | 02 | 09 | 02 | 10 | 02 | 13 | 02 | 16 |
| N 60 | 0319 | 0312 | 03 | 06 | 03 | 00 | 02 | 55 | 02 | 250 | 02 | 23 | 02 | 42 | 02 | 39 | 02 | 37 | 02 | 36 | 02 | 36 | 02 | 36 | 02 | 37 | 02 | 39 | 02 | 42 |
| 58 | 33 | 27 |  | 21 |  | 16 | 03 | 12 | 03 | 308 | 03 | 04 | 03 | 01 | 02 | 59 | 02 | 57 | 02 | 56 | 02 | 56 | 02 | 56 | 02 | 57 | 02 | 59 | 03 | 01 |
| 56 | 44 | 39 |  | 34 |  | 30 |  | 26 |  | 23 |  | 20 |  | 17 | 03 | 15 | 03 | 14 | 03 | 13 | 03 | 13 | 03 | 13 | 03 | 14 | 03 | 16 |  | 18 |
| 54 | 0355 | 0350 |  | 46 |  | 42 |  | 28 |  | 35 |  | 33 |  | 30 |  | 29 |  | 28 |  | 27 |  | 27 |  | 27 |  | 28 |  | 30 |  | 32 |
| 52 | 0404 | 0400 | 03 | 56 | 03 | 25 |  | 49 |  | 46 |  | 44 |  | 42 |  | 41 |  | 40 |  | 39 |  | 39 |  | 40 |  | 14 |  | 42 |  | 44 |
| N 50 | 0412 | 0408 | 04 | 05 | 04 | 02 | 03 | 59 | 03 | 356 | 03 | 54 | 03 | 52 | 03 | 24 | 03 | 50 | 03 | 50 | 03 | 50 | 03 | 51 | 03 | 51 | 03 | 53 | 03 | 54 |
| 45 | 29 | 26 |  | 23 |  | 21 | 04 | 19 | 04 | 417 | 04 | 15 | 04 | 14 | 04 | 13 | 04 | 13 | 04 | 13 | 04 | 13 | 04 | 13 | 04 | 14 | 04 | 15 | 04 | 17 |
| 40 | 43 | 41 |  | 39 |  | 37 |  | 35 |  | 33 |  | 32 |  | 31 |  | 31 |  | 30 |  | 30 |  | 31 |  | 31 |  | 32 |  | 33 |  | 34 |
| 35 | 0455 | 0453 | 04 | 51 | 04 | 50 | 04 | 48 |  | 47 |  | 46 |  | 46 |  | 45 |  | 45 |  | 45 |  | 46 |  | 46 | 04 | 47 | 04 | 48 | 04 | 49 |
| 30 | 0506 | 0504 | 05 | 03 | 05 | 01 | 05 | 00 | 04 | 459 | 04 | 59 | 04 | 58 | 04 | 58 | 04 | 58 | 04 | 58 | 04 | 59 | 04 | 59 | 05 | 00 | 05 | 01 | 05 | 02 |
| N 20 | 0523 | 0522 | 05 | 22 | 05 | 21 | 05 | 20 | 05 | 520 | 05 | 20 | 05 | 20 | 05 | 20 | 05 | 20 | 05 | 20 | 05 | 21 | 05 | 22 | 05 | 22 | 05 | 23 | 05 | 24 |
| N 10 | 39 | 38 |  | 38 |  | 38 |  | 38 |  | 38 |  | 38 |  | 38 |  | 38 |  | 39 |  | 39 |  | 40 |  | 40 |  | 41 |  | 42 | 05 | 43 |
| 0 | 0553 | 0553 | 05 | 53 | 05 | 53 | 05 | 54 | 05 | 554 | 05 | 55 | 05 | 55 | 05 | 56 | 05 | 56 | 05 | 57 | 05 | 57 | 05 | 58 | 05 | 59 | 05 | 59 | 06 | 00 |
| S 10 | 0607 | 0607 | 06 | 08 | 06 | 09 | 06 | 10 | 06 | 610 | 06 | 11 | 06 | 12 | 06 | 13 | 06 | 13 | 06 | 14 | 06 | 15 | 06 | 16 | 06 | 16 | 06 | 17 |  | 17 |
| 20 | 22 | 23 |  | 24 |  | 25 |  | 26 |  | 28 |  | 29 |  | 30 |  | 31 |  | 32 |  | 33 |  | 33 |  | 34 |  | 35 |  | 35 |  | 35 |
| S 30 | 0639 | 0640 | 06 | 42 | 06 | 44 | 06 | 46 | 06 | 647 | 06 | 49 | 06 | 50 | 6 | 52 | 06 | 53 | 06 | 54 | 06 | 55 | 06 | 55 | 06 | 56 | 06 | 56 | 06 | 56 |
| 35 | 0648 | 0651 | 06 | 53 | 06 | 55 | 06 | 57 | 06 | 659 | 07 | 01 | 07 | 02 | 07 | 04 | 07 | 05 | 07 | 06 | 07 | 07 | 07 | 08 | 07 | 08 | 07 | 09 | 07 | 08 |
| 40 | 0700 | 0708 | 07 | 05 | 07 | 07 | 07 | 10 | 07 | 712 |  | 14 |  | 16 |  | 17 |  | 19 |  | 20 |  | 21 |  | 22 |  | 22 |  | 23 |  | 23 |
| 45 | 13 | 16 |  | 19 |  | 22 |  | 25 |  | 27 |  | 30 |  | 32 |  | 34 |  | 36 |  | 37 |  | 38 | 07 | 39 | 07 | 39 | 07 | 39 |  | 39 |
| 50 | 29 | 33 |  | 36 |  | 40 |  | 43 |  | 46 |  | 49 | 07 | 52 | 07 | 54 | 07 | 56 | 07 | 58 | 07 | 59 | 08 | 00 | 08 | 00 | 08 | 00 | 07 | 59 |
| S 52 | 0736 | 0740 | 07 | 45 | 07 | 48 | 07 | 52 | 07 | 755 | 07 | 59 | 08 | 01 | 08 | 04 | 08 | 06 | 08 | 08 | 08 | 09 | 08 | 10 | 08 | 10 | 08 | 10 | 08 | 09 |
| 54 | 44 | 49 | 07 | 54 | 07 | 58 | 08 | 02 | 08 | 806 | 08 | 09 |  | 12 |  | 15 |  | 17 |  | 19 |  | 20 |  | 21 |  | 21 |  | 21 |  | 20 |
| 56 | 0754 | $07 \quad 59$ | 08 | 04 | 08 | 08 |  | 13 |  | 17 |  | 21 |  | 24 |  | 27 |  | 29 |  | 31 |  | 33 |  | 33 |  | 34 |  | 33 |  | 33 |
| 58 | 0804 | 0810 |  | 16 |  | 21 |  | 26 |  | 30 |  | 34 |  | 38 |  | 41 | 08 | 44 | 08 | 46 | 08 | 47 | 08 | 48 | 08 | 48 | 08 | 48 | 08 | 47 |
| S 60 | 0816 | $08 \quad 23$ | 08 | 29 | 08 | 35 | 08 | 40 | 08 | 846 | 08 | 50 | 08 | 54 | 08 | 58 | 09 | 01 | 09 | 03 | 09 | 05 | 09 | 06 | 09 | 06 | 09 | 05 | 09 | 04 |

SUNSET

| Lat. | May |  |  |  |  | June |  |  |  |  |  |  |  |  |  |  | July 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 17 | 20 | 23 | 26 | 29 | 1 | 4 | 7 | 10 | 13 | 16 |  | 19 | 22 | 25 | 28 |  |  |
| 。 | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h | m | h m | h m | h m | h | m |
| N 72 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ | $\square$ |  |
| 70 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ | $\square$ |  |
| 68 | 2220 | $22 \quad 39$ | 2303 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ | $\square$ |  |
| 66 | 2140 | $21 \quad 52$ | $22 \quad 05$ | $22 \quad 18$ | 2231 | 2245 | $22 \quad 59$ | 2315 | 23 33 | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ | 23 | 44 |
| 64 | 2113 | 22 | 2132 | 2141 | $21 \quad 50$ | 2158 | 2206 | $22 \quad 14$ | $22 \quad 20$ | $22 \quad 25$ | $22 \quad 29$ | 22 | 32 | 2232 | 2232 | $22 \quad 29$ | 22 | 26 |
| 62 | 2052 | 2100 | 2128 | $21 \quad 15$ | 22 | 29 | 2135 | 2140 | 2145 | 2149 | 2151 | 21 | 53 | 2154 | $21 \quad 54$ | 2153 | 21 | 50 |
| N 60 | 2035 | 5042 | 2049 | $20 \quad 55$ | 2101 | 2107 | $\mid 2112$ | 2116 | $21 \quad 20$ | $21 \quad 23$ | $21 \quad 25$ | 21 | 27 | 2128 | 2128 | 2127 | 21 | 25 |
| 58 | 21 | 27 | 33 | 39 | 2044 | 2049 | $20 \quad 53$ | $20 \quad 57$ | 2100 | 2103 | 2105 | 21 | 06 | 2107 | 2107 | $21 \quad 07$ | 21 | 05 |
| 56 | 2009 | 15 | 20 | 25 | 29 | 34 | 38 | 41 | 2044 | 2046 | 2048 | 20 | 50 | 2050 | $20 \quad 24$ | $20 \quad 50$ | 20 | 49 |
| 54 | 1959 | $20 \quad 04$ | $20 \quad 08$ | 13 | 17 | 21 | 24 | 27 | 30 | 32 | 34 |  | 35 | 36 | 36 | 36 |  | 35 |
| 52 | 50 | 1954 | 1958 | $20 \quad 02$ | 2006 | 10 | 13 | 16 | 18 | 20 | 22 |  | 23 | 24 | 24 | 24 |  | 23 |
| N 50 | 1941 | 1945 | 1949 | 1953 | 1957 | $20 \quad 00$ | $20 \quad 03$ | $20 \quad 05$ | $20 \quad 08$ | $20 \quad 10$ | 2011 | 20 | 12 | 2013 | $20 \quad 13$ | $20 \quad 13$ | 20 | 12 |
| 45 | 24 | 27 | 30 | 33 | 36 | 1939 | 1941 | 1944 | 1946 | 1947 | 1949 | 19 | 50 | 1950 | 1951 | 1951 | 19 | 50 |
| 40 | 1910 | 12 | 15 | 18 | 20 | 22 | 24 | 26 | 28 | 29 | 31 |  | 32 | 32 | 33 | 33 |  | 33 |
| 35 | 1858 | 1900 | 1902 | 1904 | 1906 | 1908 | 1910 | 1912 | 13 | 15 | 16 |  | 17 | 17 | 18 | 18 |  | 18 |
| 30 | 47 | 1849 | 1851 | 1853 | 1855 | 1856 | 1858 | $18 \quad 59$ | 1900 | 1902 | 1903 | 19 | 03 | 1904 | 1905 | 1905 | 19 | 05 |
| N 20 | 1830 | 1831 | 1832 | 1833 | 1834 | 1836 | 1837 | $18 \quad 38$ | 1839 | 1840 | 1841 | 18 | 41 | 1842 | 1843 | 1843 | 18 | 43 |
| N 10 | 14 | 15 | 16 | 16 | 17 | 18 | 19 | 19 | 20 | 21 | 22 |  | 22 | 23 | 24 | 24 |  | 25 |
| 0 | 1800 | 1800 | 1800 | 1801 | 1801 | 1801 | $18 \quad 02$ | 1802 | 1803 | $18 \quad 03$ | 1804 | 18 | 05 | 1805 | 1806 | $18 \quad 07$ | 18 | 07 |
| S 10 | 1746 | 1745 | 1745 | 1745 | 1745 | 1745 | 1745 | 1746 | 1746 | 1746 | $17 \quad 47$ | 17 | 47 | 1748 | 1749 | 1749 | 17 | 50 |
| 20 | 31 | 30 | 29 | 28 | 28 | 28 | 28 | 28 | 28 | 28 | 28 |  | 29 | 30 | 30 | 31 |  | 32 |
| S 30 | 1714 | $17 \quad 12$ | 1711 | $17 \quad 10$ | $17 \quad 09$ | $17 \quad 08$ | $17 \quad 07$ | $17 \quad 07$ | $17 \quad 07$ | $17 \quad 07$ | $17 \quad 07$ | 17 | 08 | 1708 | $17 \quad 09$ | 1710 | 17 | 11 |
| 35 | 1704 | 1702 | 1700 | $16 \quad 59$ | 1657 | 1656 | 1656 | 1655 | 1655 | 1655 | 1655 | 16 | 55 | 1656 | 1657 | 1658 | 16 | 59 |
| 40 | 1653 | 1650 | 1648 | 46 | 45 | 43 | 42 | 41 | 41 | 41 | 41 |  | 41 | 42 | 42 | 44 |  | 45 |
| 45 | 40 | 37 | 34 | 32 | 30 | 28 | 27 | 25 | 25 | 24 | 24 |  | 24 | 25 | 26 | 27 |  | 28 |
| 50 | 23 | 20 | 17 | 14 | 11 | 09 | 1607 | 1605 | 1604 | 1604 | $16 \quad 03$ | 16 | 03 | 1604 | 1605 | 1606 | 16 | 08 |
| S 52 | 1616 | 1612 | 1608 | 1605 | 1602 | 1600 | $15 \quad 57$ | $15 \quad 56$ | 1555 | $15 \quad 54$ | $15 \quad 53$ | 15 | 53 | 1554 | 1555 | 1556 | 15 | 58 |
| 54 | 1608 | 1603 | $15 \quad 59$ | $15 \quad 56$ | $15 \quad 52$ | 1549 | 47 | 45 | 44 | 43 | 42 |  | 42 | 43 | 44 | 45 |  | 47 |
| 56 | 1558 | $15 \quad 53$ | 49 | 45 | 41 | 38 | 35 | 33 | 31 | 30 | 30 |  | 30 | 30 | 31 | 33 |  | 35 |
| 58 | 48 | 42 | 3 | 33 | 29 | 25 | 22 | 19 | 17 | $15 \quad 16$ | $15 \quad 15$ | 15 | 15 | 1516 | $15 \quad 17$ | 18 |  | 21 |
| S 60 | 1536 | $15 \quad 29$ | $15 \quad 24$ | $15 \quad 18$ | $15 \quad 14$ | $15 \quad 09$ | 1506 | $15 \quad 03$ | 1500 | 1459 | 1458 | 14 | 57 | 1458 | $14 \quad 59$ | 1501 | 15 | 04 |


| Lat. | May |  |  |  |  | June |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \text { July } \\ 1 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 17 | 20 | 23 | 26 | 29 | , | 4 | 7 | 10 |  | 13 | 16 |  |  | 9 |  | 22 |  | 25 |  | 28 |  |  |
|  | h m | m | h m | m | m | m | m | m | m | h | m | h | m | h | m | h |  | h | m | h | m | h | m |
| N 72 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  |
| 70 | // /I | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | - |  | $\square$ |  | $\square$ |  |
| 68 | /1 II | // II | // II | // II | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  |
| 66 | /1 /1 | II II | /I II | II II | /1 II | /I II | /I II | /I II | /I II | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | // | /I |
| 64 | 0057 | 0023 | // II | /1 /1 | // II | /1 II | /I II | /I II | /I II | // | II | II | /I | // | // | // | // | // | 1 | // | /I | // | II |
| 62 | 0145 | 0131 | 0117 | 0101 | 0024 | 0010 | /1 II | /I II | /I II | // | $1 /$ | // | $1 /$ | // | // | // | /I | /I | II | /I | $1 /$ | // | $1 /$ |
| N 60 | 0214 | 0205 | 0155 | 0145 | 0136 | 0127 | 0119 | 0111 | 0103 | 00 | 057 | 00 | 52 | 00 | 49 | 00 | 49 | 00 | 51 | 00 | 55 | 01 | 02 |
| 58 | 36 | 29 | 0221 | 0214 | 02 07\| | 0201 | 0155 | 0150 | 0146 | 01 | 143 | 01 | 14 | 01 | 40 | 01 | 40 | 01 | 42 | 01 | 44 | 01 | 48 |
| 56 | 0254 | 0247 | 41 | 35 | 30 | 25 | 0221 | 0217 | 0214 | 02 | 212 | 02 | 11 | 02 | 10 | 02 | 10 | 02 | 12 | 02 | 14 | 02 | 16 |
| 54 | 0309 | 0303 | 0258 | 0253 | 0248 | 0244 | 41 | 38 | 36 |  | 34 |  | 33 |  | 32 |  | 33 |  | 34 |  | 36 |  | 38 |
| 52 | 21 | 16 | 0312 | 0307 | 0303 | 0300 | 0257 | 0255 | 0253 | 02 | 251 | 02 | 51 | 02 | 50 | 02 | 51 | 05 | 52 | 02 | 53 | 02 | 56 |
| N 50 | 0332 | 0328 | 0324 | 0320 | 0317 | 0313 | 0311 | 0309 | 0307 | 03 | 306 | 03 | 06 | 03 | 06 | 03 | 06 | 03 | 07 | 03 | 08 | 03 | 10 |
| 45 | 0355 | 0351 | 0348 | 0345 | 0343 | 0334 | 0339 | 37 | 36 |  | 35 |  | 35 |  | 35 |  | 36 |  | 37 | 03 | 38 | 3 | 39 |
| 40 | 0413 | 0410 | 0407 | 0405 | 0403 | 0401 | 0400 | 0359 | 0358 | 03 | 358 | 03 | 58 | 03 | 58 | 03 | 58 | 03 | 59 | 04 | 00 | 04 | 02 |
| 35 | 27 | 25 | 13 | 21 | 19 | 18 | 17 | 0416 | 0416 | 04 | 416 | 04 | 16 | 04 | 16 | 04 | 17 | 04 | 17 |  | 18 |  | 20 |
| 30 | 0439 | 38 | 36 | 35 | 33 | 32 | 32 | 31 | 31 |  | 31 |  | 31 |  | 31 |  | 32 |  | 33 |  | 34 |  | 35 |
| N 20 | 0500 | 0459 | 0458 | 0457 | 10456 | 0456 | 0455 | 0455 | 0455 | 04 | 455 | 04 | 56 | 04 | 56 | 04 | 57 | 04 | 58 | 04 | 59 | 04 | 59 |
| N 10 | 16 | 0516 | 0515 | 0515 | 0515 | 0515 | 0515 | 0515 | 0515 | 05 | 516 | 05 | 16 | 05 | 17 | 05 | 17 | 05 | 18 | 05 | 19 | 05 | 20 |
| 0 | 31 | 31 | 31 | 31 | 31 | 32 | 32 | 33 | 33 |  | 34 |  | 34 |  | 35 |  | 36 |  | 36 |  | 37 |  | 38 |
| S 10 | 44 | 45 | 0546 | 0546 | 0547 | 0548 | 0548 | 0549 | 0550 | 05 | 551 | 05 | 51 | 05 | 52 | 05 | 53 | 05 | 53 | 05 | 54 | 05 | 54 |
| 20 | 0558 | 0559 | 0601 | 0602 | 0603 | 0604 | 0605 | 0606 | 0607 | 06 | 608 | 06 | 09 | 06 | 09 | 06 | 10 | 06 | 11 | 06 | 11 | 06 | 11 |
| S 30 | 0613 | 0615 | 0617 | 0618 | 0620 | 0621 | 0623 | 0624 | 0625 | 06 | 626 | 06 | 27 | 06 | 28 | 06 | 29 | 06 | 30 | 06 | 30 | 06 | 30 |
| 35 | 21 | 23 | 25 | 27 | 29 | 31 | 33 | 34 | 36 |  | 37 |  | 38 |  | 39 |  | 40 |  | 40 |  | 40 |  | 41 |
| 40 | 30 | 33 | 35 | 58 | 40 | 42 | 44 | 46 | 0647 | 06 | 648 | 06 | 50 | 06 | 51 | 06 | 51 | 06 | 52 | 06 | 52 | 06 | 52 |
| 45 | 41 | 44 | 0647 | 0649 | 0652 | 0654 | 0657 | 0659 | 0700 | 07 | 702 | 07 | 03 | 07 | 04 | 07 | 05 | 07 | 05 | 07 | 06 | 07 | 05 |
| 50 | 53 | 0656 | 0700 | 0703 | 0706 | 0709 | 0711 | 0714 | 16 |  | 18 |  | 19 |  | 20 |  | 21 |  | 21 |  | 21 |  | 21 |
| S 52 | 0658 | 0702 | 0706 | 0709 | 0713 | 0716 | 0719 | 0721 | $07 \quad 23$ | 07 | 725 | 07 | 27 | 07 | 28 | 07 | 29 | 07 | 29 | 07 | 29 | 07 | 29 |
| 54 | 0704 | 08 | 13 | 16 | 20 | 23 | 26 | 29 | 31 |  | 33 |  | 35 |  | 36 |  | 37 |  | 37 |  | 37 |  | 37 |
| 56 | 11 | 15 | 20 | 24 | 28 | 31 | 35 | 37 | 40 |  | 42 |  | 44 |  | 45 |  | 46 |  | 46 |  | 46 |  | 46 |
| 58 | 18 | 23 | 28 | 32 | 37 | 41 | 44 | 47 | $07 \quad 50$ | 07 | 752 | 07 | 54 | 07 | 55 | 07 | 56 | 07 | 57 | 07 | 56 | 07 | 56 |
| S 60 | 0726 | $07 \quad 32$ | 0737 | 0742 | 0747 | 0751 | 0755 | 0758 | 0801 | 08 | 804 | 08 | 06 | 08 | 07 | 08 | 08 | 08 | 08 | 08 | 08 | 08 | 07 |

EVENING CIVIL TWILIGHT

| Lat. | May |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | July 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 17 | 20 | 23 | 26 | 29 | 1 | 4 | 7 | 10 | 13 | 16 |  | 19 | 22 | 25 | 28 |  |  |
| 。 | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h | m | h m | h m | h m | h | m |
| N 72 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ | $\square$ |  |
| 70 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ | $\square$ |  |
| 68 | // // | // /1 | // // | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ | $\square$ |  |
| 66 | // /1 | // /1 | /1 /1 | // /1 | // /1 | // /1 | // /1 | // /1 | // /1 | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ | // | // |
| 64 | 2305 | // // | /1 /1 | /1 /1 | // /1 | // /1 | // /1 | // /1 | // /1 | // /1 | // /1 | // | // | // /1 | // /1 | // /1 | // | // |
| 62 | 2212 | $22 \quad 26$ | 2241 | 2258 | $23 \quad 20$ | // /1 | // /1 | // /1 | // /1 | // /1 | // /1 | // | // | // /1 | // /1 | $1 / 1 /$ | // | // |
| N 60 | 2141 | 2151 | 2201 | 2211 | $22 \quad 21$ | 2231 | 2240 | 2249 | 2257 | 2304 | 2310 | 23 | 13 | 2314 | 2313 | 2309 | 23 | 03 |
| 58 | 18 | 26 | 2134 | 2142 | 2149 | 2156 | 2202 | 2208 | $22 \quad 13$ | 2217 | $22 \quad 20$ | 22 | 22 | $22 \quad 23$ | $22 \quad 23$ | 2221 | 22 | 19 |
| 56 | 2100 | 2107 | $21 \quad 14$ | 20 | 26 | 32 | 2137 | 2141 | 2145 | 2148 | 2151 | 21 | 52 | 2153 | $21 \quad 53$ | 2152 | 21 | 50 |
| 54 | 2045 | 2051 | 2057 | $21 \quad 02$ | 2107 | 2112 | 17 | 20 | 24 | 26 | 29 |  | 30 | 31 | 31 | 30 |  | 29 |
| 52 | 32 | 38 | 43 | 2047 | $20 \quad 52$ | 2056 | 2100 | 2103 | 2106 | 2109 | 2111 | 21 | 12 | 2113 | $21 \quad 13$ | $21 \quad 12$ | 21 | 11 |
| N 50 | 2021 | $20 \quad 26$ | $20 \quad 30$ | 2035 | $20 \quad 39$ | 2043 | 2046 | $20 \quad 49$ | $20 \quad 52$ | $20 \quad 54$ | 2056 | 20 | 57 | 2058 | $20 \quad 58$ | $20 \quad 57$ | 20 | 57 |
| 45 | 1958 | 2002 | 2006 | 2009 | 2012 | 2015 | 2018 | 2021 | 23 | 25 | 26 |  | 27 | 28 | 28 | 28 |  | 28 |
| 40 | 41 | 1944 | 1946 | 1949 | 1952 | 1954 | 1957 | 1959 | $20 \quad 01$ | $20 \quad 02$ | 2003 | 20 | 04 | 2005 | $20 \quad 06$ | 2006 | 20 | 05 |
| 35 | 26 | 28 | 31 | 33 | 35 | 38 | 39 | 41 | 1943 | 1944 | 1945 | 19 | 46 | 1947 | 1947 | 1948 | 19 | 47 |
| 30 | 1913 | 1916 | 1918 | 1920 | 1921 | 23 | 25 | 26 | 28 | 29 | 30 |  | 31 | 32 | 32 | 32 |  | 32 |
| N 20 | 1853 | 1855 | 1856 | $18 \quad 57$ | 1859 | 1900 | 1901 | 1902 | $19 \quad 03$ | 1904 | 1905 | 19 | 06 | 1907 | $19 \quad 07$ | 1908 | 19 | 08 |
| N 10 | 37 | 37 | 38 | 39 | 40 | 1841 | 1841 | 1842 | 1843 | 1844 | 1845 | 18 | 45 | 1846 | 1847 | 1847 | 18 | 48 |
| 0 | 22 | 22 | 22 | 23 | 23 | 24 | 24 | 25 | 25 | 26 | 27 |  | 27 | 28 | 29 | 29 |  | 30 |
| S 10 | 1808 | 1808 | 1807 | 1807 | $18 \quad 07$ | $18 \quad 08$ | 1808 | 1808 | 1809 | 1809 | 1810 | 18 | 10 | 1811 | 1811 | 1812 | 18 | 13 |
| 20 | 1754 | 1753 | 1753 | $17 \quad 52$ | 1752 | 1751 | 1751 | 1751 | 1752 | 1752 | $17 \quad 52$ | 17 | 53 | 1753 | 1754 | 1755 | 17 | 56 |
| S 30 | 1739 | 1738 | $17 \quad 37$ | $17 \quad 35$ | 1735 | 1734 | $17 \quad 33$ | $17 \quad 33$ | $17 \quad 33$ | $17 \quad 33$ | $17 \quad 34$ | 17 | 34 | $17 \quad 35$ | 1735 | $17 \quad 36$ | 17 | 37 |
| 35 | 31 | 29 | 28 | 26 | 25 | 24 | 24 | 23 | 23 | 23 | 23 |  | 23 | 24 | 25 | 26 |  | 27 |
| 40 | 22 | 20 | 18 | 16 | 15 | 13 | 12 | $17 \quad 12$ | 1711 | 1711 | 1711 | 17 | 12 | 1712 | 1713 | 14 |  | 15 |
| 45 | 12 | $17 \quad 09$ | 1706 | 1704 | 1702 | 1701 | 1700 | $16 \quad 59$ | 1658 | 1658 | $16 \quad 58$ | 16 | 58 | 1659 | 1659 | 1701 | 17 | 02 |
| 50 | 1700 | 1656 | 1653 | 1651 | 1648 | 1646 | 1645 | 43 | 42 | 42 | 42 |  | 42 | 43 | 44 | 1645 | 16 | 46 |
| S 52 | 1654 | 1650 | 1647 | 1644 | 1642 | 1639 | 1638 | 1636 | 1635 | 1635 | 1634 | 16 | 34 | 1635 | 1636 | 1637 | 16 | 39 |
| 54 | 48 | 44 | 40 | 37 | 34 | 32 | 30 | 28 | 27 | 26 | 26 |  | 26 | 27 | 28 | 29 |  | 31 |
| 56 | 41 | 37 | 33 | 30 | 26 | 24 | 21 | 20 | 18 | 17 | 17 |  | 17 | 18 | 19 | 20 |  | 22 |
| 58 | 34 | 29 | 25 | 21 | 17 | 14 | 12 | $16 \quad 10$ | $16 \quad 08$ | $16 \quad 07$ | $16 \quad 07$ | 16 | 07 | 1607 | $16 \quad 09$ | 1610 |  | 12 |
| S 60 | 1626 | $16 \quad 20$ | 1616 | 1611 | $16 \quad 07$ | $16 \quad 04$ | $16 \quad 01$ | $15 \quad 59$ | $15 \quad 57$ | $15 \quad 56$ | $15 \quad 55$ | 15 | 55 | $15 \quad 56$ | $15 \quad 57$ | $15 \quad 58$ | 16 | 01 |

## SUNRISE

| Lat. | $\begin{gathered} \text { June } \\ 28 \\ \hline \end{gathered}$ | 1 | 4 | 7 | 10 | 13 |  | $\begin{gathered} \hline \text { July } \\ 16 \end{gathered}$ |  | 19 |  | 22 |  | 25 |  | 28 |  | 31 |  | 3 |  |  |  |  |  |  |  | 15 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | h m | h m | h m | m | m | h m | m | h | m | h m | m | h | m | h | m | h | m | h |  | h | m |  | h m | h | m | h | m | h | m |
| N 72 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ | $\square$ | 00 | 049 | 01 | 30 | 015 | 58 | 02 | 21 |
| 70 | - | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | $\square$ |  | 00 | 21 | 01 | 111 | 01 | 139 | 02 | 201 |  | 20 | 023 | 37 | 02 | 54 |
| 68 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ |  | 003 | 39 | 01 | 10 | 01 | 32 | 01 | 50 | 02 | 07 | 02 | 222 | 02 | 237 | 02 | 51 | 030 | 04 | 03 | 17 |
| 66 | $\square$ | 0013 | 0038 | 0055 | 0110 | 012 | 24 | 01 | 37 | 015 | 50 | 02 | 03 | 02 | 15 | 02 | 27 | 02 | 39 | 02 | 251 | O3 | 303 | 03 | 14 |  | 25 |  | 36 |
| 64 | 0135 | 0140 | 0146 | 0152 | 0200 | 02 | 08 | 02 | 17 | 022 | 26 |  | 35 | 02 | 44 | 02 | 54 | 03 | 30 | 03 | 313 |  | 22 |  | 32 |  | 41 | 03 | 50 |
| 62 | 0213 | 0216 | 0220 | 0225 | 31 |  | 37 | 02 | 44 | 025 | 51 | 02 | 58 | 03 | 06 | 03 | 14 |  | 22 |  | 30 |  | 38 |  | 46 | 035 | 54 | 04 | 02 |
| N 60 | 0239 | 0242 | 0245 | 0249 | 0254 | 025 | 59 | 03 | 04 | 03 | 10 | 03 | 17 | 03 | 23 | 03 | 30 | 03 | 37 | 03 | 344 | 03 | 351 | 03 | 58 | 040 | 06 | 04 | 13 |
| 58 | 0259 | 0301 | 0304 | 0308 | 0312 | 031 | 16 |  | 21 |  | 26 |  | 32 |  | 38 |  | 44 | 03 | 35 |  | 356 | 04 | 402 | 04 |  |  | 15 |  | 22 |
| 56 | 0316 | 18 | 20 | 23 | 27 |  | 31 |  | 35 |  | 40 |  | 45 | 03 | 50 | 03 | 55 | 04 | 01 |  | 406 |  | 12 |  | 18 |  | 24 |  | 30 |
| 54 | 30 | 32 | 34 | 37 | 40 |  | 44 |  | 48 | 035 | 52 | 03 | 56 | 04 | 01 | 04 | 06 |  | 11 |  | 16 |  | 21 |  | 26 |  | 31 |  | 37 |
| 52 | 42 | 44 | 46 | 49 | 0352 | 035 | 55 | 03 | 58 | 04 | 02 | 04 | 06 |  | 10 |  | 15 |  | 19 |  | 24 |  | 29 |  | 33 |  | 38 |  | 43 |
| N 50 | 0353 | 0354 | 0357 | 0359 | 0402 | 04 | 05 | 04 | 08 | 04 | 11 | 04 | 15 | 04 | 19 | 04 | 23 | 04 | 27 | 04 | 431 | 04 | 453 | 04 | 40 | 04 | 4 | 04 | 49 |
| 45 | 0415 | 0417 | 0418 | 0420 | 23 |  | 25 |  | 28 |  | 31 |  | 34 |  | 37 |  | 40 |  | 43 |  | 47 | 04 | 450 | 04 | 54 | 045 | 57 | 05 | 01 |
| 40 | 33 | 34 | 36 | 38 | 40 |  | 42 |  | 44 |  | 46 | 04 | 49 | 04 | 51 | 04 | 54 | 04 | 45 | 04 | 459 |  | 502 | 05 |  | 050 | 08 |  | 11 |
| 35 | 0448 | 0449 | 10451 | 0452 | 0454 | 045 | 56 | 04 | 57 | 04 | 59 | 05 | 01 | 05 | 04 | 05 | 06 | 05 | 08 |  | 510 |  | 13 |  | 15 |  | 17 |  | 19 |
| 30 | 0501 | 0502 | 105 03 | 105 05 | 0506 | 05 | 08 | 05 | 09 | 05 | 11 |  | 13 |  | 14 |  | 16 |  | 18 |  | 20 |  | 22 |  | 23 |  | 25 |  | 27 |
| N 20 | 0523 | 0524 | 05 25 | 0526 | 0527 | 05 | 28 | 05 | 29 | 05 | 30 | 05 | 31 | 05 | 33 | 05 | 34 | 05 | 35 | 05 | 536 | 05 | 537 | 05 | 38 |  | 39 | 05 | 40 |
| N 10 | 42 | 0543 | 0543 | 0544 | 0545 | 05 | 46 | 05 | 46 | 05 | 47 | 05 | 48 | 05 | 48 | 05 | 49 | 05 | 49 |  | 550 |  | 550 | 05 | 50 | 055 | 5 | 05 | 51 |
| 0 | 0559 | 0600 | 0601 | 0601 | 0602 | 06 | 02 | 06 | 02 | 06 | 02 | 06 | 03 | 06 | 03 | 06 | 03 | 06 | 03 | 06 | 603 | 06 | 602 | 06 | 02 | 060 | 02 | 06 | 01 |
| S 10 | 0617 | 17 | 18 | 18 | 18 |  | 18 |  | 18 |  | 18 |  | 18 |  | 17 |  | 17 |  | 16 |  | 15 |  | 15 |  | 14 |  | 12 |  | 11 |
| 20 | 35 | 35 | 36 | 36 | 36 |  | 35 |  | 35 |  | 34 |  | 34 |  | 33 |  | 32 |  | 30 |  | 29 |  | 27 |  | 26 |  | 24 |  | 22 |
| S 30 | 0656 | 10656 | 10656 | 10656 | 0656 | 065 | 55 | 06 | 54 | 06 | 53 | 06 | 52 | 106 | 50 | 106 | 48 | 106 | 46 | 06 | 644 |  | 642 | 106 | 40 | 1063 | 37 | 106 | 34 |
| 35 | 0709 | 0708 | 07 08 | 0708 | 0707 | 07 | 06 | 07 | 05 | 07 | 04 | 07 | 02 | 07 | 00 | 06 | 58 | 06 | 56 | 06 | 653 |  | 650 |  | 47 |  | 44 |  | 41 |
| 40 | 23 | 23 | 22 | 22 | 21 |  | 19 |  | 18 |  | 16 |  | 14 |  | 12 | 07 |  |  | 06 |  |  |  |  | 06 |  |  |  |  | 49 |
| 45 | 0739 | 39 | 39 | 38 | 36 |  | 35 |  | 33 |  | 31 |  | 28 |  | 25 |  | 22 |  | 19 |  | 15 |  | 11 |  |  | 070 | 03 | 06 | 58 |
| 50 | 0800 | 0759 | 0759 | 0757 | 0756 | 075 | 54 | 07 | 51 |  | 48 |  | 45 |  | 42 |  | 38 |  | 34 |  | 29 |  | 25 |  | 20 |  | 14 | 07 | 09 |
| S 52 | 0810 | 0809 | 0808 | 0807 | 0805 | 08 | 03 | 08 | 00 | 07 | 57 | 07 | 53 | 07 | 49 | 07 | 45 | 07 | 41 | 07 | 736 | 07 | 731 |  | 25 |  | 20 | 07 | 14 |
| 54 | 21 | 20 | 19 | 17 | 15 |  | 13 |  | 09 | 08 | 06 | 08 | 02 | 07 | 58 |  | 53 |  | 48 |  | 43 |  | 38 |  | 32 |  | 26 |  | 20 |
| 56 | 33 | 33 | 31 | 29 | 27 |  | 24 |  | 20 |  | 16 |  | 12 | 08 | 07 | 08 | 02 | 07 | 57 |  | 751 |  | 45 |  | 39 |  | 33 |  | 26 |
| 58 | 0848 | 0847 | 0845 | 43 | 40 |  | 37 |  | 33 |  | 28 |  | 24 |  | 18 |  | 13 | 08 | 07 | 08 | 801 |  | 754 |  | 47 |  | 40 |  | 33 |
| S 60 | 0905 | 0904 | 0902 | 0859 | 0856 | 08 | 52 | 08 | 48 | 08 | 43 | 08 | 37 | 08 | 31 | 08 | 25 | 08 | 8 18 | 08 | 811 | 108 | 804 | 07 | 56 | 074 | 49 | 7 | 41 |

SUNSET

| Lat. | June 28 | 1 | 4 | 7 | 10 13 13 $\begin{array}{ccc}\text { July } \\ 16\end{array}$ |  |  | 19 | 22 | 25 | 28 | 31 | August |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | 3 |  |  |  |  | 6 | 9 | 12 |  | 15 |
|  | h m | h m | h m | h m | h m | h m | h m |  | h m | h m | h m | h m | h m | h m | h m | h m | h m | h | m |
| N 72 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | 2308 | 2233 | 2206 | 21 | 43 |
| 70 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | 2329 | 2252 | 2227 | 2206 | 2146 | 2129 | 21 | 12 |
| 68 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | 2322 | 2256 | 2235 | 2218 | 2201 | 2146 | 2131 | 2117 | 2103 | 20 | 49 |
| 66 | $\square$ | 2344 | 2325 | 2310 | 2256 | 2243 | 2231 | 2218 | 2206 | 2154 | 2142 | 2130 | 2118 | 2107 | 2055 | 2043 |  | 31 |
| 64 | 2229 | 2226 | 2221 | 2215 | 2208 | 2201 | 2153 | 2144 | 2136 | 26 | 2117 | 2107 | 2057 | 2047 | 37 | 27 |  | 17 |
| 62 | 2153 | 2150 | 2147 | 2143 | 2138 | 2133 | 27 | 20 | 2113 | 2105 | 2057 | 2049 | 41 | 32 | 23 | 14 | 20 | 05 |
| N 60 | 2127 | 2125 | 2123 | 2120 | 2116 | 2111 | 2106 | 2101 | 2055 | 2048 | 2041 | 2034 | 2027 | 2019 | 2011 | 2003 | 19 | 55 |
| 58 | 2107 | 2105 | 2103 | 2101 | 2058 | 2054 | 2050 | 2045 | 39 | 34 | 28 | 21 | 15 | 2008 | 2001 | 1953 |  | 46 |
| 56 | 2050 | 2049 | 2047 | 2045 | 43 | 39 | 36 | 31 | 27 | 22 | 16 | 10 | 2004 | 1958 | 1952 | 45 |  | 38 |
| 54 | 36 | 35 | 34 | 32 | 30 | 27 | 23 | 20 | 15 | 11 | 2006 | 2001 | 1955 | 50 | 44 | 38 |  | 31 |
| 52 | 24 | 23 | 22 | 20 | 18 | 16 | 13 | 09 | 2006 | 2002 | 1957 | 1952 | 47 | 42 | 37 | 31 |  | 25 |
| N 50 | 2013 | $20 \quad 12$ | 2011 | 2010 | $20 \quad 08$ | 2006 | 2003 | 2000 | 1957 | 1953 | 1949 | 1945 | 1940 | 1935 | 1930 | 1925 | 9 | 19 |
| 45 | 1951 | 1950 | 1950 | 1949 | 1947 | 1946 | 1943 | 1941 | 38 | 35 | 32 | 29 | 25 | 21 | 17 | 12 | 19 | 07 |
| 40 | 33 | 33 | 32 | 32 | 30 | 29 | 27 | 26 | 23 | 21 | 18 | 15 | 12 | 1909 | 1905 | 1901 | 18 | 57 |
| 35 | 18 | 18 | 18 | 17 | 16 | 15 | 14 | 13 | 11 | 1909 | 1907 | 1904 | 1902 | 1859 | 1856 | 1852 |  | 49 |
| 30 | 1905 | 1905 | 1905 | 1905 | 1904 | 1903 | 1902 | 1901 | 1900 | 1858 | 1856 | 1854 | 1852 | 50 | 47 | 44 |  | 42 |
| N 20 | 1843 | 1843 | 1843 | 1843 | 1843 | 1843 | 1843 | 1842 | 1841 | 1840 | 1839 | 1838 | 1836 | 1835 | 1833 | 1831 | 18 | 29 |
| N 10 | 24 | 25 | 25 | 25 | 26 | 26 | 26 | 25 | 25 | 24 | 24 | 23 | 22 | 21 | 20 | 19 |  | 18 |
| 0 | 1807 | 1807 | 1808 | 1808 | 1809 | 1809 | 1810 | 1810 | 1810 | 1810 | 1810 | 1810 | 1810 | 1809 | 1809 | 1808 | 8 | 08 |
| S 10 | 1749 | $17 \quad 50$ | 1751 | $17 \quad 52$ | $17 \quad 52$ | 1753 | 1754 | 1754 | 1755 | 1756 | 1756 | 1756 | $17 \quad 57$ | 1757 | 1758 | $17 \quad 58$ | 17 | 58 |
| 20 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 43 | 44 | 45 | 45 | 46 |  | 47 |
| S 30 | 1710 | $17 \quad 11$ | $17 \quad 12$ | 1714 | 1715 | 1716 | 1718 | 1720 | 1721 | 1723 | 1725 | 1727 | 1729 | 1730 | 1732 | 1734 | 17 | 35 |
| 35 | 1658 | 1659 | 1700 | 1702 | 1703 | 1705 | 1707 | 1709 | 1711 | 13 | 15 | 17 | 20 | 22 | 24 | 26 |  | 28 |
| 40 | 44 | 45 | 1646 | 1648 | 1650 | 1652 | 1654 | 1657 | 1659 | 1702 | 1704 | $17 \quad 07$ | 1710 | 12 | 15 | 18 |  | 21 |
| 45 | 27 | 28 | 30 | 32 | 34 | 37 | 39 | 42 | 45 | 1648 | 1651 | 1655 | 1658 | 1701 | 1705 | 1708 |  | 12 |
| 50 | 1606 | 1608 | 10 | 13 | 15 | 18 | 21 | 25 | 28 | 32 | 36 | 40 | 44 | 1648 | 1652 | 1656 | 17 | 01 |
| S 52 | 1556 | 1558 | 1600 | 1603 | 1606 | 1609 | 1612 | 1616 | 1620 | 1624 | 1628 | 1633 | 1637 | 1642 | 1646 | 1651 | 16 | 56 |
| 54 | 45 | 47 | 1550 | 1552 | 1556 | 1559 | 1603 | 1607 | 11 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |  | 50 |
| 56 | 33 | 35 | 38 | 41 | 44 | 48 | $15 \quad 52$ | 1556 | 1601 | 1606 | 11 | 16 | 22 | 27 | 33 | 38 |  | 44 |
| 58 | 18 | 21 | 23 | 27 | 31 | 35 | 39 | 44 | 1550 | 1555 | 1601 | 1607 | 12 | 18 | 25 | 31 |  | 37 |
| S 60 | 1501 | 1504 | 1507 | 1510 | 1515 | 1520 | 1525 | 1530 | 1536 | 1542 | 1549 | 1555 | 1602 | 1609 | 1615 | 1622 | 16 | 29 |



EVENING CIVIL TWILIGHT

| Lat. |  |  |  |  |  |  |  |  |  |  |  |  |  |  | August |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 28 | 1 | 4 | 7 | 10 | 13 |  |  | 19 | 22 | 25 |  | 28 | 31 |  | 3 |  | 6 | 9 | 12 |  | 15 |
|  |  | h m | h m | h m | h m | h m | h | m | h m | h m | h m |  |  | h m |  | m | m | h m | h m | h m | h | m |
| N 72 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ |  | $\square$ |  |  | II II | II II | II | II |
| 70 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | $\square$ | $\square$ |  |  | // II |  | /I II |  | // II | // II | /1 /1 |  | 51 |
| 68 | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  |  | // II | // II |  |  | /1 II |  |  | $1 /$ | // I/ | 2332 | 2250 |  | 23 |
| 66 | $\square$ | // II | // II | // II | // II | // II | /I | II | // II | // II | // II |  | /1 II | // II |  | 2315 | 15 | 2246 | 2223 | 2204 | 21 | 47 |
| 64 | // II | // II | // II | // II | // II | // II | /I | II | // II | // II | 2324 |  | 2257 | 2237 |  |  | 20 | 2204 | 2149 | 2135 |  | 21 |
| 62 | /I II | // II | /1 II | // II | // II | $23 \quad 36$ | 23 | 12 | 2255 | 2240 | 2226 |  | 2213 | 2201 |  | 2148 | 48 | 2136 | 25 | 2113 | 21 | 02 |
| N 60 | 2309 | 2303 | 2257 | 2249 | 2241 | 2232 | 22 | 23 | $22 \quad 14$ | 2204 | 2155 |  | 2145 | 2135 |  | 2125 | 25 | 2116 | 2106 | 2056 | 0 | 46 |
| 58 | 2221 | 2219 | 2215 | 2211 | 2205 | 2200 | 21 | 53 | 2146 | 2139 | 31 |  | 23 | 2115 |  | 2107 | 07 | 2059 |  | 42 |  | 33 |
| 56 | 2152 | 2150 | 2148 | 2144 | 2141 | 2136 |  | 31 | 25 | 19 | 2113 |  | 2106 | 2059 |  |  | 52 | 45 | 37 | 29 |  | 21 |
| 54 | 30 | 29 | 27 | 24 | 21 | 17 | 21 | 13 | 2108 | 2103 | 2058 |  | 2052 | 46 |  |  | 39 | 33 | 26 | 19 |  | 11 |
| 52 | 2112 | 2111 | 2110 | 2108 | 2105 | 2202 | 20 | 58 | 2054 | 2049 | 45 |  | 39 | 34 |  |  | 28 | 22 | 16 | 10 | 0 | 03 |
| N 50 | 2057 | 2057 | 2055 | 2053 | 2051 | 2048 | 20 | 45 | 2042 | 2038 | 2033 |  | 2029 | 2024 |  |  | 18 | 2013 | 2007 | 2001 | 9 | 55 |
| 45 | 28 | 28 | 27 | 25 | 24 | 22 | 20 | 19 | 2017 | 2013 | 2010 |  | 2006 | 2002 |  | 1958 | 58 | 1954 | 1949 | 1944 |  | 39 |
| 40 | 2006 | 2005 | 2005 | 2004 | 2003 | 2001 | 19 | 59 | 1957 | 1955 | 1952 |  | 949 | 1946 |  |  | 42 | 39 | 35 | 31 |  | 26 |
| 35 | 1948 |  |  | 1947 | 1946 | 1944 |  | 43 |  |  | 37 |  | 35 | 32 |  |  | 29 | 26 | 23 | 19 |  | 16 |
| 30 | 32 | 32 | 32 | 32 | 31 | 30 |  | 29 | 28 | 26 | 25 |  | 23 | 20 |  |  | 18 |  |  |  | 9 | 07 |
| N 20 | 1908 | 1908 | 1908 | 1908 | 1908 | 1907 | 19 | 07 | 1900 | 1905 | 1904 |  | 903 | 0901 |  |  | 00 | 1858 | 1856 | 1854 | 8 | 52 |
| $\text { N } 10$ |  |  |  | 1848 | 1848 |  |  | 48 |  |  |  |  |  |  |  |  |  | 43 |  | 41 |  | 40 |
| 0 | 29 | 30 | 30 | 31 | 31 | 31 |  | 32 | 32 | 32 | 32 |  | 32 | 32 |  |  | 31 | 31 | 30 | 30 |  | 29 |
| S 10 | 1812 | 1813 | 1814 | 1814 | $\begin{array}{ll}18 & 15\end{array}$ | 16 |  | 16 | 17 | 17 | 18 |  | 18 | 19 |  |  | 19 | 19 | 19 | 19 |  | 19 |
| 20 | 1755 | 1756 | 1757 | 1758 | 1759 | 1800 | 18 | 01 | 1802 | 1803 | 1804 |  | 805 | 1806 |  |  | 07 | 1807 | 1808 | 1809 |  | 10 |
| S 30 | 1736 | 1737 |  | 1740 | 1741 | 1742 | 17 | 44 | 1745 |  | 1749 |  | 175 | 1755 |  |  | 45 | 1755 |  | 1758 | 18 | 00 |
| 35 | 26 | 27 | 28 | 30 | 31 | 33 |  | 35 | 36 | 38 | 40 |  | 42 | 44 |  |  | 46 | 48 | 50 | 52 | 17 | 54 |
| 40 | 14 | 15 | 17 | 18 | 20 | 22 |  | 24 | 26 | 29 | 31 |  | 33 | 36 |  |  | 38 | 41 | 43 | 46 |  | 49 |
| 45 | 1701 | 1702 | 1704 | 1705 | 1708 | 1710 | 17 | 12 | 15 | 17 | 20 |  | 23 | 26 |  |  | 29 | 32 | 36 | 39 |  | 42 |
| 50 | 1645 | 1646 | 1648 | 1650 | 1653 | 1655 | 16 | 58 | 1701 | 1704 | 08 |  | 11 | 15 |  |  | 19 | 23 | 26 | 30 |  | 34 |
| S 52 | 1637 | 1639 | 1641 | 1643 | 1646 | 1649 | 16 | 52 | 1655 | 1659 | 1702 |  | 706 | 1710 |  | 1714 | 14 | 1718 | 1722 | 1727 | 17 | 31 |
| 54 | 29 | 31 | 33 | 35 | 38 | 41 |  | 45 | 48 | 52 | 4656 |  | 1700 | 1704 |  |  | 09 | 13 | 18 | 23 |  | 27 |
| 56 | 20 | 22 | 24 | 27 | 30 | 33 |  | 37 | 41 | 45 | 49 |  | 1654 | 1658 |  |  |  | 08 | 13 | 18 |  | 23 |
| 58 | 1610 | 12 | 14 | 17 | 21 | 24 |  | 28 | 32 | 37 | 42 |  | 46 | 52 |  |  | 57 | 1702 | 08 | 13 |  | 19 |
| S 60 | 1558 | 1601 | 1603 | 1606 | 1610 | 1614 | 16 | 18 | 1623 | 1628 | 1633 |  | 638 | 1644 |  | 1650 | 50 | 1656 | 1702 | 1708 | 17 | 14 |



SUNSET

| Lat. | August |  |  |  |  |  | September |  |  |  |  |  |  |  |  |  | $\begin{gathered} \text { Oct } \\ 2 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 15 | 18 | 21 | 24 | 27 | 30 | 2 | 5 | 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 |  |  |
| 。 | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h | m |
| N 72 | 2143 | 2122 | 2103 | 2044 | 2027 | 2010 | 1953 | 1936 | 1920 | 1904 | 1849 | 1833 | 1818 | 1802 | 1747 | 1731 | 17 | 16 |
| 70 | 2112 | 2055 | 2040 | 24 | 2009 | 1954 | 40 | 26 | 11 | 1857 | 43 | 29 | 42 | 01 | 47 | 34 |  | 20 |
| 68 | 2049 | 35 | 22 | 2009 | 1955 | 42 | 30 | 17 | 1904 | 51 | 38 | 26 | 13 | 01 | 48 | 35 |  | 23 |
| 66 | 31 | 20 | 2008 | 1956 | 44 | 33 | 21 | 09 | 1858 | 46 | 35 | 23 | 11 | 1800 | 48 | 37 |  | 26 |
| 64 | 17 | 2006 | 1956 | 46 | 35 | 25 | 14 | 1903 | 53 | 42 | 31 | 21 | 10 | 1759 | 49 | 38 |  | 28 |
| 62 | 2005 | 1956 | 46 | 37 | 27 | 17 | 08 | 1858 | 48 | 38 | 29 | 19 | 09 | 59 | 49 | 39 |  | 30 |
| N 60 | 1955 | 1946 | 1938 | 1929 | 1920 | 1911 | 1902 | 1853 | 1844 | 1835 | 1826 | 1817 | 1808 | 1758 | 1747 | 1740 | 17 | 31 |
| 58 | 46 | 38 | 30 | 22 | 14 | 06 | 1857 | 49 | 41 | 32 | 24 | 15 | 07 | 58 | 50 | 41 |  | 33 |
| 56 | 38 | 31 | 24 | 16 | 09 | 1901 | 53 | 45 | 38 | 30 | 22 | 14 | 06 | 58 | 50 | 42 |  | 34 |
| 54 | 31 | 25 | 18 | 11 | 1904 | 1857 | 49 | 42 | 35 | 27 | 20 | 12 | 05 | 58 | 50 | 43 |  | 35 |
| 52 | 35 | 19 | 12 | 06 | 1859 | 53 | 46 | 39 | 32 | 25 | 18 | 11 | 04 | 57 | 50 | 43 |  | 36 |
| N 50 | 1919 | 1914 | 1908 | 1902 | 1856 | 1849 | 1843 | 1837 | 1830 | 1824 | $18 \quad 17$ | 1810 | 1804 | 1857 | 1751 | 1744 | 17 | 38 |
| 45 | 1907 | 1903 | 1858 | 1852 | 47 | 42 | 36 | 31 | 25 | 20 | 14 | 08 | 03 | 57 | 51 | 46 |  | 40 |
| 40 | 1857 | 1853 | 49 | 45 | 40 | 36 | 31 | 26 | 21 | 16 | 11 | 06 | 01 | 56 | 51 | 47 |  | 42 |
| 35 | 49 | 46 | 42 | 38 | 34 | 30 | 26 | 22 | 18 | 13 | 09 | 05 | 01 | 56 | 52 | 48 |  | 43 |
| 30 | 42 | 39 | 35 | 32 | 27 | 25 | 22 | 18 | 15 | 11 | 07 | 04 | 1800 | 56 | 52 | 49 |  | 45 |
| N 20 | 1829 | 1827 | 1825 | 1822 | 1820 | 1817 | 1815 | 1812 | 1810 | 1807 | 1804 | 1801 | 1759 | 1756 | 1753 | 1750 | 17 | 48 |
| N 10 | 18 | 17 | 15 | 14 | 12 | 10 | 09 | 07 | 05 | 03 | 1801 | 1800 | 58 | 56 | 54 | 52 |  | 50 |
| 0 | 1808 | 1807 | 1806 | 1806 | 1805 | 1804 | 1803 | 1802 | 1801 | 1800 | 1759 | 1758 | 57 | 56 | 55 | 54 |  | 53 |
| S 10 | 1758 | 1758 | 1758 | $17 \quad 58$ | 1758 | 1758 | 1758 | $17 \quad 57$ | $17 \quad 57$ | $17 \quad 57$ | 57 | 56 | 56 | 56 | 56 | 55 |  | 55 |
| 20 | 47 | 48 | 49 | 50 | 50 | 51 | 52 | 52 | 53 | 54 | 54 | 55 | 55 | 56 | 57 | 1757 | 17 | 58 |
| S 30 | 1735 | 1737 | $17 \quad 39$ | 1740 | 1742 | 1744 | 1745 | 1747 | 1748 | 1750 | $17 \quad 52$ | 1753 | 1755 | 1756 | 1758 | 1800 | 18 | 02 |
| 35 | 28 | 31 | 33 | 35 | 37 | 39 | 45 | 44 | 46 | 48 | 50 | 52 | 55 | 57 | 1759 | 01 |  | 04 |
| 40 | 21 | 32 | 26 | 29 | 32 | 35 | 37 | 40 | 43 | 46 | 49 | 51 | 54 | 57 | 1800 | 03 |  | 06 |
| 45 | 12 | 15 | 19 | 22 | 26 | 29 | 33 | 36 | 40 | 43 | 47 | 50 | 54 | 58 | 01 | 05 |  | 09 |
| 50 | 1701 | 05 | 09 | 14 | 18 | 23 | 27 | 31 | 36 | 40 | 45 | 49 | 54 | 58 | 03 | 07 |  | 12 |
| S 52 | 1656 | 1700 | 1705 | 1710 | 1715 | 1720 | 1724 | 1729 | $17 \quad 34$ | 1739 | 1744 | 1749 | 1753 | 1758 | 1803 | 1808 | 18 | 13 |
| 54 | 50 | 1655 | 1701 | 06 | 11 | 16 | 21 | 27 | 32 | 37 | 43 | 48 | 53 | 59 | 04 | 10 |  | 15 |
| 56 | 44 | 50 | 1655 | 1701 | 07 | 13 | 18 | 24 | 30 | 36 | 41 | 47 | 53 | 59 | 05 | 11 |  | 17 |
| 58 | 37 | 43 | 50 | 1656 | $17 \quad 02$ | 09 | 15 | 21 | 27 | 34 | 40 | 46 | 53 | 1759 | 06 | 12 |  | 19 |
| S 60 | 1629 | 1636 | 1643 | 1650 | 1657 | 1704 | 1711 | 1718 | 1725 | $17 \quad 32$ | 1739 | 1746 | $17 \quad 53$ | 1800 | 1807 | 1814 | 18 | 21 |

## MORNING CIVIL TWILIGHT



## EVENING CIVIL TWILIGHT

| Lat. | August |  |  |  |  |  |  | September |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \text { Oct } \\ 2 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 15 | 18 | 21 | 24 | 27 | 30 |  | 2 |  | 5 |  | 8 |  | 11 |  | 14 |  | 17 |  | 20 |  | 23 |  | 26 |  | 29 |  |  |  |
|  | h m | h m | h m | h m | h m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N 72 | // /1 | /1 /I | /1 /I | 2245 | 2211 | 21 | 44 | 21 | 20 | 20 | 58 | 20 | 38 | 20 | 19 | 20 | 01 | 19 | 44 | 19 | 27 | 19 | 10 | 18 | 54 | 18 | 38 | 18 | 23 |
| 70 | 2351 | 2250 | $22 \quad 19$ | 2154 | 2132 | 21 | 12 | 20 | 54 |  | 36 |  | 19 | 20 | 03 | 19 | 47 |  | 32 |  | 17 | 19 | 03 |  | 48 |  | 34 |  | 20 |
| 68 | 2223 | 2201 | 2141 | 23 | 2106 | 20 | 50 |  | 34 |  | 19 | 20 | 04 | 19 | 50 |  | 36 |  | 23 |  | 0 | 18 | 56 |  | 43 |  | 31 |  | 18 |
| 66 | 2147 | 2131 | $21 \quad 15$ | 2100 | $20 \quad 46$ |  | 32 |  | 19 | 20 | 06 | 19 | 53 |  | 40 |  | 27 |  | 15 | 19 | 03 |  | 51 |  | 39 |  | 28 |  | 16 |
| 64 | 21 | 2108 | 2055 | 2043 | 30 |  | 18 | 20 | 06 | 19 | 55 |  | 43 |  | 32 |  | 20 |  | 09 | 18 | 58 |  | 47 |  | 36 |  | 25 |  | 15 |
| 62 | 2102 | 2050 | 39 | 28 | 17 | 20 | 07 | 19 | 56 |  | 45 |  | 34 |  | 24 |  | 14 | 19 | 04 |  | 53 |  | 43 |  | 33 |  | 23 |  | 14 |
| N 60 | 2046 | 2036 | $20 \quad 26$ | 2016 | 2007 | 19 | 57 | 19 | 47 | 19 | 37 | 9 | 28 | 19 | 18 | 19 | 08 | 18 | 59 | 18 | 50 | 18 | 40 | 18 | 31 | 18 | 22 | 18 | 13 |
| 58 | 33 | 24 | 15 | 2006 | 1957 |  | 48 |  | 39 |  | 30 |  | 22 |  | 13 | 19 | 04 |  | 55 |  | 46 |  | 38 |  | 29 |  | 20 |  | 12 |
| 56 | 21 | 13 | $20 \quad 05$ | 1957 | 49 |  | 41 |  | 32 |  | 24 |  | 16 |  | 08 | 18 | 59 |  | 51 |  | 43 |  | 35 |  | 27 |  | 19 |  | 11 |
| 54 | 11 | $20 \quad 04$ | 1957 | 49 | 42 |  | 34 |  | 26 |  | 19 |  | 11 |  | 03 |  | 56 |  | 48 |  | 40 |  | 33 |  | 25 |  | 18 |  | 10 |
| 52 | 2003 | 1956 | 49 | 42 | 35 |  | 28 |  | 21 |  | 14 |  | 07 | 19 | 00 |  | 52 |  | 45 |  | 38 |  | 31 |  | 24 |  | 17 |  | 10 |
| N 50 | 1955 | 1949 | 1943 | 1936 | 1930 | 19 | 23 | 19 | 16 | 19 | 10 | 19 | 03 | 18 | 56 | 18 | 50 | 18 | 43 | 18 | 36 | 18 | 29 | 18 | 23 | 18 | 16 | 18 | 10 |
| 45 | 39 | 34 | 29 | 23 | 18 |  | 12 | 19 | 07 | 19 | 01 | 18 | 55 |  | 49 |  | 43 |  | 38 |  | 32 |  | 26 |  | 20 |  | 15 |  | 09 |
| 40 | 26 | 22 | 18 | 13 | 08 | 19 | 03 | 18 | 59 | 18 | 54 |  | 49 |  | 44 |  | 39 |  | 34 |  | 28 |  | 23 |  | 18 |  | 13 |  | 09 |
| 35 | 16 | 12 | 08 | 1904 | 1900 | 18 | 56 |  | 52 |  | 48 |  | 43 |  | 39 |  | 35 |  | 30 |  | 26 |  | 21 |  | 17 |  | 13 |  | 09 |
| 30 | 1907 | 1904 | 1900 | 1857 | 1853 |  | 50 |  | 46 |  | 43 |  | 39 |  | 35 |  | 31 |  | 27 |  | 24 |  | 20 |  | 16 |  | 12 |  | 09 |
| N 20 | 1852 | 1850 | 1847 | 1845 | 1843 | 18 | 40 | 18 | 37 | 18 | 34 | 18 | 32 | 18 | 29 | 18 | 26 | 18 | 23 | 18 | 21 | 18 | 18 | 18 | 15 | 18 | 12 | 18 | 10 |
| N 10 | 40 | 38 | 37 | 35 | 33 |  | 32 |  | 30 |  | 28 |  | 26 |  | 24 |  | 22 |  | 21 |  | 19 |  | 17 |  | 15 |  | 13 |  | 11 |
| 0 | 29 | 29 | 28 | 27 | 26 |  | 25 |  | 24 |  | 23 |  | 22 |  | 21 |  | 20 |  | 19 |  | 18 |  | 16 |  | 15 |  | 14 |  | 13 |
| S 10 | 19 | 19 | 19 | 19 | 19 |  | 19 |  | 19 |  | 18 |  | 18 |  | 18 |  | 18 |  | 17 |  | 17 |  | 17 |  | 17 |  | 17 |  | 16 |
| 20 | 10 | 11 | 11 | 12 | 13 |  | 13 |  | 14 |  | 14 |  | 15 |  | 16 |  | 16 |  | 17 |  | 18 |  | 18 |  | 19 |  | 20 |  | 20 |
| S 30 | 1800 | 1802 | 1803 | 1805 | 1806 | 18 | 08 | 18 | 09 | 18 | 11 | 18 | 12 | 18 | 14 | 18 | 16 | 18 | 17 | 18 | 19 | 18 | 20 | 18 | 22 | 18 | 24 | 18 | 26 |
| 35 | 1754 | 1757 | $17 \quad 57$ | 1801 | 1803 |  | 05 |  | 07 |  | 09 |  | 11 |  | 13 |  | 16 |  | 18 |  | 20 |  | 22 |  | 24 |  | 27 |  | 29 |
| 40 | 49 | 51 | 54 | $17 \quad 57$ | $17 \quad 59$ | 18 | 02 |  | 05 |  | 07 |  | 10 |  | 13 |  | 16 |  | 18 |  | 21 |  | 24 |  | 27 |  | 30 |  | 33 |
| 45 | 42 | 45 | 49 | 52 | 55 | 17 | 59 | 18 | 02 |  | 06 |  | 09 |  | 13 |  | 16 |  | 20 |  | 23 |  | 27 |  | 31 |  | 34 |  | 38 |
| 50 | 34 | 39 | 43 | 47 | 51 |  | 55 | 17 | 59 |  | 04 |  | 08 |  | 12 |  | 17 |  | 21 |  | 26 |  | 31 |  | 35 |  | 40 |  | 45 |
| S 52 | 1731 | $17 \quad 36$ | 1740 | 1745 | 1749 | 17 | 54 | 17 | 58 | 18 | 03 | 18 | 08 | 18 | 12 | 18 | 17 | 18 | 22 | 18 | 27 | 18 | 32 | 18 | 37 | 18 | 42 | 18 | 47 |
| 54 | 27 | 32 | 37 | 42 | 47 |  | 52 |  | 57 |  | 02 |  | 07 |  | 13 |  | 18 |  | 23 |  | 29 |  | 34 |  | 40 |  | 45 |  | 51 |
| 56 | 23 | 29 | 34 | 39 | 45 |  | 50 |  | 56 |  | 01 |  | 07 |  | 13 |  | 18 |  | 24 |  | 30 |  | 36 |  | 42 |  | 48 |  | 55 |
| 58 | 19 | 25 | 31 | 37 | 42 |  | 48 |  | 54 |  | 01 |  | 07 |  | 13 |  | 19 |  | 26 |  | 32 |  | 39 |  | 45 |  | 52 | 18 | 59 |
| S 60 | 1714 | 1721 | $17 \quad 27$ | 1733 | 1740 | 17 | 46 | 17 | 53 | 18 | 00 | 18 | 06 | 18 | 13 | 18 | 20 | 18 | 27 | 18 | 34 | 18 | 41 | 18 | 49 | 18 | 56 | 19 | 04 |

SUNRISE

| Lat | October |  |  |  |  |  |  |  |  |  |  |  |  |  |  | November |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 5 | 8 | 11 | 14 |  | 17 | 20 |  | 23 |  | 26 |  | 29 |  | 1 |  | 4 |  | 7 |  | 10 |  | 13 |  | 16 |  | 19 |  |
|  | h m | h m | h m | h m | h | m | h m | h m | m |  |  |  | m | h |  |  | m |  | h m | h | m | h | m | h | m | h | m | h | m |
| N72 | 0621 | 0635 | 0649 | 0703 | 07 | 17 | 0732 | 074 | 47 | 080 | 03 | 08 | 19 | 08 |  |  | 854 |  | 0913 | 09 | 934 | 09 | 58 | 10 | 27 | 11 | 14 | - |  |
| 70 | 18 | 30 | 42 | 0654 |  | 07 | 20 |  | 33 | 074 | 46 | 08 | 00 |  | 14 |  | 28 |  | 0844 | 08 | 859 | 09 | 16 | 09 | 34 | 09 | 53 | 10 | 14 |
| 68 | 15 | 26 | 36 | 47 | 065 | 58 | 10 |  | 21 |  | 33 | 07 | 44 | 07 |  |  | 809 |  | 21 |  | 34 | 08 | 48 | 09 | 01 | 09 | 15 | 09 | 30 |
| 66 | 12 | 22 | 32 | 41 |  | 51 | 0701 |  | 11 |  | 22 |  | 32 |  | 43 | 07 | 753 |  | 0804 |  | 15 |  | 27 | 08 | 38 | 08 | 49 | 09 | 01 |
| 64 | 10 | 19 | 28 | 36 |  | 45 | 0654 | 070 | 03 |  | 13 |  | 22 |  | 31 |  | 41 |  | 0750 | 08 | 80 | 08 | 10 |  | 19 |  | 29 | 08 | 39 |
| 62 | 08 | 16 | 24 | 32 |  | 40 | 48 | 065 | 57 |  | 05 |  | 13 |  | 22 |  | 30 |  | 39 | 07 | 747 | 07 | 56 | 08 | 04 | 08 | 13 |  | 21 |
| N60 | 0607 | 0614 | 0621 | 0628 | 06 | 36 | 0643 | 065 | 50 | 065 | 58 | 07 | 06 | 07 |  |  | 721 |  | 0729 | 07 | 736 | 07 | 44 | 07 | 52 | 07 | 59 | 08 | 07 |
| 58 | 05 | 12 | 18 | 25 |  | 32 | 38 |  | 45 |  | 52 | 065 | 59 |  | 06 |  | 13 |  | 20 |  | 27 |  | 34 |  | 41 |  | 48 | 07 | 54 |
| 56 | 04 | 10 | 16 | 22 |  | 28 | 34 |  | 41 |  | 47 |  | 53 |  |  |  | 06 |  | 12 |  | 19 |  | 25 |  | 31 |  | 37 |  | 43 |
| 54 | 03 | 08 | 14 | 19 |  | 25 | 31 |  | 36 |  | 42 |  | 48 | 06 |  |  | 700 |  | 0705 |  | 11 |  | 17 |  | 23 |  | 29 |  | 34 |
| 52 | 02 | 07 | 12 | 17 |  | 22 | 27 |  | 33 |  | 38 |  | 43 |  | 49 |  | 654 |  | 0659 | 07 | 705 |  | 10 |  | 16 |  | 21 |  | 26 |
| N50 | 0601 | 0605 | 0610 | 0615 | 06 | 19 | 0624 | 062 | 29 | 063 | 34 | 06 | 39 | 06 |  |  | 649 |  | 0654 | 06 | 65 | 07 | 04 | 07 | 09 | 07 | 14 | 07 | 18 |
| 45 | 0558 | 02 | 06 | 10 |  | 14 | 18 |  | 22 |  | 26 |  | 30 |  | 34 |  | 38 |  | 42 |  | 46 | 06 | 50 | 06 | 54 | 06 | 58 | 07 | 02 |
| 40 | 57 | 10600 | 0603 | 06 |  | 09 | 12 |  | 15 |  | 18 |  | 22 |  | 25 |  | 28 |  | 32 |  | 35 |  | 39 |  | 42 |  | 46 | 06 | 49 |
| 35 | 55 | 0557 | 0559 | 10602 |  | 04 | 07 |  | 10 |  | 12 |  | 15 |  | 18 |  | 1 |  | 23 |  | 26 |  | 29 |  | 32 |  | 35 |  | 38 |
| 30 | 54 | 55 | 57 | 10559 | 06 | 01 | 10603 | 060 | 05 | 106 0 | 07 | 06 | 09 |  | 11 |  | 13 |  | 16 |  | 18 |  | 20 |  | 23 |  | 25 |  | 28 |
| N20 | 0551 | 0552 | 0552 | 10553 | 055 | 54 | 0555 | 055 | 56 | 105 | 57 | 05 | 59 | 06 | 00 |  | 601 |  | 0603 | 06 | 604 | 06 | 06 | 06 | 07 | 06 | 09 | 06 | 11 |
| N10 | 49 | 48 | 48 | 48 |  | 48 |  |  |  |  |  |  | 49 | 05 |  |  | 550 |  | 5551 | 05 | 55 | 05 | 53 | 05 | 54 | 05 | 55 | 05 | 56 |
| , | 46 | 45 | 44 | 44 |  | 43 | 42 |  | 42 |  | 41 |  | 41 |  | 40 |  | 40 |  | 40 |  | 40 |  | 40 |  | 41 |  | 41 |  | 42 |
| S10 | 44 | 42 | 40 | 39 |  | 37 | 36 |  | 34 |  | 33 |  | 32 |  | 31 |  | 30 |  | 29 |  | 29 |  | 28 |  | 28 |  | 28 |  | 28 |
| 20 | 41 | 38 | 36 | 33 |  | 31 | 29 |  | 26 |  | 24 |  | 22 |  | 20 |  | 19 |  | 17 |  | 16 | 05 | 15 | 05 | 14 | 05 | 13 | 05 | 12 |
| S30 | 0538 | 1534 | 0531 | 0527 | 05 | 24 | 0520 | 051 |  |  |  | 05 |  | 05 |  |  | 506 |  | 0503 |  | 501 | 04 | 59 | 04 | 57 | 04 | 56 | 04 | 54 |
| 35 | 36 | 32 | 28 | 24 |  | 19 | 16 |  | 12 |  | 08 | 05 | 05 |  |  |  | 458 |  | 0455 | 04 | 453 |  | 50 |  | 48 |  | 46 |  | 44 |
| 40 | 34 | 29 | 24 | 19 |  | 15 | 10 | 050 | 06 | 050 | 02 | 045 | 57 |  |  |  | 50 |  | 46 |  | 43 |  | 40 |  | 37 |  | 34 |  | 32 |
| 45 | 31 | 26 | 20 | 15 |  | 09 | 0504 | 045 | 59 | 045 | 54 |  | 49 |  | 44 |  | 40 |  | 35 |  | 31 |  | 27 |  | 24 |  | 21 |  | 18 |
| 50 | 28 | 22 | 15 | 09 | 05 | 02 | 0456 |  | 50 |  | 44 |  | 38 |  | 33 |  | 27 |  | 22 |  | 17 |  | 13 |  | 08 | 04 | 04 | 04 | 00 |
| 52 | 0527 | 0520 | 0513 | 0506 | 04 | 59 | 0453 | 044 | 46 | 04 | 40 | 04 | 34 | 04 | 28 |  | 422 |  | 0416 | 04 | 411 | 04 | 05 | 04 | 01 | 03 | 56 | 03 | 52 |
| 54 | 25 | 18 | 10 | 03 |  | 56 | 49 |  | 42 |  | 35 |  | 28 |  | 22 |  | 15 |  | 09 | 04 | 403 | 03 | 58 | 03 | 52 |  | 48 |  | 43 |
| 56 | 23 | 15 | 08 |  |  | 52 | 44 |  | 37 |  | 30 |  | 22 |  | 15 |  | 08 |  | 0402 |  |  |  | 49 |  | 43 |  | 38 |  | 32 |
| 58 | 21 | 13 | 05 | 0456 |  | 48 | 40 |  | 32 |  | 24 |  | 16 |  | 08 |  | 400 |  | 0353 |  | 46 |  | 39 |  | 33 |  | 26 |  | 21 |
| 60 | 0519 | 0510 | 0501 | 0452 | 04 | 43 | 0434 | 042 | 25 | 041 | 17 | 04 | 08 | 04 | 00 |  | 351 |  | 0343 | 03 | 335 | 03 | 28 | 03 | 20 | 03 | 13 | 03 | 06 |

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| Lat | October |  |  |  |  |  |  |  |  |  | November |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 5 | 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 |  | 1 |  | 4 | 7 | 10 | 13 |  | 6 |  | 19 |
|  | \|h m | \|h m | h m | h m | h m | h m | h m | h m | h m | h m | h |  | h |  | h m | h m | h m | h | m | h | m |
| N72 | 1716 | 1701 | 1645 | 1630 | 1614 | 1558 | 1541 | 1525 | 1508 | 1450 | 14 | 32 | 141 | 13 | 1352 | 1328 | 1300 | 12 | 14 |  |  |
| 70 | 20 | 06 | 2 | 38 | 24 | 1610 | 1556 | 42 | 27 | 1513 | 14 | 58 | 14 | 42 | 1427 | 1411 | 1354 | 13 | 35 | 13 | 15 |
| 68 | 23 | 10 | 1658 | 45 | 33 | 20 | 1608 | 1555 | 43 | 30 | 15 | 17 | 15 | 05 | 1452 | 1439 | 1426 | 14 | 13 | 4 | 00 |
| 66 | 26 | 14 | 1703 | 51 | 40 | 29 | 17 | 1606 | 1555 | 44 |  | 33 |  | 22 | 1511 | 1500 | 1450 | 14 | 39 |  | 29 |
| 64 | 28 | 17 | 07 | 1656 | 46 | 36 | 25 | 15 | 1605 | 1555 |  | 45 |  | 36 | 26 | 17 | 1508 | 15 | 00 | 14 | 51 |
| 62 | 30 | 20 | 10 | 1701 | 51 | 42 | 32 | 23 | 14 | 1605 |  | 56 |  | 48 | 39 | 31 | 23 |  | 16 | 15 | 09 |
| N60 | 1731 | 1722 | 1713 | 1704 | 1656 | 1647 | 1638 | 1630 | 1622 | 1613 | 16 | 06 | 15 | 58 | 1551 | 1543 | 1536 | 15 | 30 | 15 | 23 |
| 58 | 33 | 25 | 16 | 08 | 1700 | 52 | 44 | 36 | 28 | 21 |  | 13 | 16 | 06 | 1600 | 1553 | 47 |  | 41 |  | 36 |
| 56 | 34 | 26 | 18 | 11 | 03 | 56 | 48 | 41 | 34 | 27 |  | 21 |  | 14 | 08 | 1602 | 1556 | 15 | 51 |  | 46 |
| 54 | 35 | 28 | 1 | 14 | 06 | 1659 | 53 | 46 | 39 | 33 |  | 27 |  | 21 | 15 | 10 | 1605 | 16 | 00 | 15 | 56 |
| 52 | 36 | 29 | 23 | 16 | 09 | 1703 | 1656 | 50 | 44 | 38 |  | 33 |  | 27 | 22 | 17 | 12 |  | 08 | 16 | 04 |
| N50 | 1738 | 1731 | 1725 | 1718 | 1712 | 1706 | 1700 | 1654 | 1649 | 1643 | 16 | 38 | 16 | 33 | 1628 | 1623 | 1619 | 16 | 15 | 16 | 12 |
| 45 | 40 | 34 | 29 | 23 | 18 | 13 | 08 | 1703 | 1658 | 1653 |  | 49 |  | 45 | 41 | 37 | 34 |  | 31 |  | 28 |
| 40 | 42 | 37 | 2 | 27 | 23 | 18 | 14 | 10 | 1706 | 1702 | 16 | 58 | 165 | 55 |  | 49 | 46 |  | 44 |  | 41 |
| 35 | 43 | 39 | 35 | 31 | 27 | 23 | 20 | 16 | 13 | 09 | 17 | 06 | 17 | 04 | 1701 | 1659 | 1656 | 16 | 54 | 16 | 53 |
| 30 | 45 | 41 | 38 | 34 | 31 | 28 | 25 | 22 | 19 | 16 |  | 14 |  | 11 | 09 | 1707 | 1705 | 17 | 04 | 17 | 03 |
| N20 |  | 1745 |  |  | 1738 | 1736 | 1733 | 1731 |  |  | 17 |  | 17 | 24 | 1723 | 1722 | 1721 | 17 | 20 | 17 | 20 |
| N10 | 50 | 49 | 47 | 45 | 44 | 42 | 41 | 40 | 39 | 38 |  | 37 |  | 36 | 35 | 35 | 35 |  | 35 |  | 35 |
| 0 | 53 | 52 | 51 | 50 | 49 | 49 | 48 | 48 | 47 | 47 |  | 47 |  | 47 | 47 | 1747 | 1748 | 17 | 48 | 17 | 49 |
| S10 | 55 | 55 | 1755 | 1755 | 1755 | 1755 | 1756 | 1756 | 1757 | 1757 | 17 | 58 | 175 | 58 | 1759 | 1800 | 1801 | 18 | 02 | 18 | 03 |
| 20 |  |  |  |  | 1802 |  | 1804 | 1805 |  |  |  |  |  | 10 |  | 14 | 15 |  | 17 |  | 19 |
| S30 | 1802 | 1803 | 1805 | 1807 | 1809 | 1811 | 1813 | 1815 | 1817 | 1820 | 18 | 22 | 18 | 24 | 1827 | 1829 | 1832 | 18 | 34 | 18 | 37 |
| 35 | 04 | 06 | 09 | 11 | 13 | 16 | 19 | 21 | 24 | 27 |  | 30 |  | 32 | 35 | 38 | 41 |  | 44 |  | 47 |
| 40 | 06 | 09 | 2 | 15 | 18 | 21 | 25 | 28 | 31 | 35 |  | 38 |  | 42 | 45 | 1849 | 1852 | 18 | 56 | 18 | 59 |
| 45 | 09 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 |  | 48 | 185 | 53 | 1857 | 1901 | 1905 | 19 | 10 | 19 | 14 |
| 50 | 12 | 17 | 21 | 26 | 31 | 36 | 41 | 46 | 51 | 1856 | 19 | 01 | 190 | 06 | 1911 | 16 | 21 |  | 26 |  | 31 |
| 52 | 1813 | 1818 | 1824 | 1829 | 1834 | 1839 | 1845 | 1850 | 1856 | 1901 | 19 |  | 191 | 12 | 1918 | 1924 | 1929 | 19 | 34 | 19 | 40 |
| 54 | 15 | 21 | 26 | 32 | 37 | 43 | 49 | 1855 | 1901 | 07 |  | 13 |  | 19 | 25 | 31 | 37 |  | 43 | 19 | 49 |
| 56 | 17 | 23 | 29 | 35 | 41 | 48 |  | 1901 | 07 | 14 |  | 20 |  | 27 | 34 | 40 | 47 | 19 | 53 | 20 | 00 |
| 58 | 19 | 25 | 32 | 39 | 46 | 53 | 1900 | 07 | 14 | 21 |  | 28 |  | 36 | 43 | 1950 | 1958 | 20 | 05 |  | 12 |
| 60 | 1821 | 1828 | 1836 | 1843 | 1851 | 1858 | 1906 | 1914 | 1922 | 1930 | 19 | 38 | 19 | 46 | 1954 | 2002 | 2010 | 20 | 18 | 20 | 26 |


| Lat | 2 |  | 5 |  | 8 |  | October |  |  |  |  |  |  |  | 23 |  | 26 |  | 29 |  | 1 |  | November |  |  |  |  |  |  |  |  |  | 19 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 11 | 14 |  |  | 7 | 20 |  | 4 |  | 7 |  | 10 |  | 13 |  |  |  |  |  |  |  |  |  |
| 。 | h | m |  |  | h | m | h | m | h | m | h | m | h | m |  |  | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h m |  | h m |  |
| N72 | 05 | 14 | 05 | 27 |  |  | 05 | 41 | 05 | 54 | 06 | 07 | 06 | 20 | 06 | 34 |  |  | 06 | 47 | 07 | 00 | 07 | 14 | 07 | 27 | 07 | 41 | 07 | 54 | 08 | 08 | 08 | 23 | 08 | 37 | 05 | 51 |
| 70 |  | 17 |  | 29 |  | 41 |  | 52 |  | 04 |  | 16 |  | 27 |  | 39 | 06 | 51 | 07 | 03 |  | 14 |  | 26 |  | 38 | 07 | 50 | 08 | 02 | 08 | 14 |  | 25 |
| 68 |  | 19 |  | 30 |  | 41 |  | 51 | 06 | 01 |  | 12 |  | 22 |  | 33 |  | 43 | 06 | 54 | 07 | 04 |  | 14 |  | 25 |  | 35 | 07 | 46 | 07 | 56 | 08 | 06 |
| 66 |  | 21 |  | 31 |  | 40 |  | 50 | 05 | 59 |  | 09 |  | 18 |  | 27 |  | 37 |  | 46 | 06 | 55 | 07 | 05 |  | 14 |  | 23 |  | 32 |  | 41 | 07 | 50 |
| 64 |  | 23 |  | 32 |  | 40 |  | 49 |  | 57 |  | 06 |  | 14 |  | 23 |  | 31 |  | 40 |  | 48 | 06 | 56 | 07 | 05 |  | 13 |  | 21 |  | 29 |  | 37 |
| 62 |  | 24 |  | 32 |  | 40 |  | 48 |  | 55 |  | 03 |  | 11 |  | 18 |  | 26 |  | 34 |  | 41 |  | 49 | 06 | 57 | 07 | 04 |  | 12 |  | 19 |  | 26 |
| N60 | 05 | 25 | 05 | 33 | 05 | 40 | 05 | 47 | 05 | 54 | 06 | 01 | 06 | 08 | 06 | 15 | 06 | 22 | 06 | 29 | 06 | 36 | 06 | 43 | 06 | 50 | 06 | 57 | 07 | 03 | 07 | 10 | 07 | 16 |
| 58 |  | 26 |  | 33 |  | 39 |  | 46 |  | 52 | 05 | 59 |  | 05 |  | 12 |  | 18 |  | 24 |  | 31 |  | 37 |  | 44 |  | 50 | 06 | 56 | 07 | 02 |  | 08 |
| 56 |  | 27 |  | 33 |  | 39 |  | 45 |  | 51 |  | 57 |  | 03 |  | 09 |  | 15 |  | 20 |  | 26 |  | 32 |  | 38 |  | 44 |  | 49 | 06 | 55 | 07 | 01 |
| 54 |  | 27 |  | 33 |  | 38 |  | 44 |  | 49 |  | 55 | 06 | 00 |  | 06 |  | 11 |  | 17 |  | 22 |  | 28 |  | 33 |  | 38 |  | 44 |  | 49 | 06 | 54 |
| 52 |  | 28 |  | 33 |  | 38 |  | 43 |  | 48 |  | 53 | 05 | 58 |  | 03 |  | 08 |  | 13 |  | 18 |  | 23 |  | 28 |  | 33 |  | 38 |  | 43 |  | 48 |
| N50 | 05 | 29 | 05 | 33 | 05 | 38 | 05 | 42 | 05 | 47 | 05 | 51 | 05 | 56 | 06 | 01 | 06 | 06 | 06 | 10 | 06 | 15 | 06 | 20 | 06 | 24 | 06 | 29 | 06 | 33 | 06 | 38 | 06 | 42 |
| 45 |  | 29 |  | 33 |  | 37 |  | 40 |  | 44 |  | 48 |  | 52 | 05 | 56 | 05 | 59 | 06 | 03 |  | 07 |  | 11 |  | 15 |  | 19 |  | 23 |  | 26 |  | 30 |
| 40 |  | 30 |  | 33 |  | 35 |  | 38 |  | 41 |  | 44 |  | 48 |  | 51 |  | 54 | 05 | 57 | 06 | 00 | 06 | 04 |  | 07 |  | 10 |  | 13 |  | 17 |  | 20 |
| 35 |  | 30 |  | 32 |  | 34 |  | 37 |  | 39 |  | 42 |  | 44 |  | 47 |  | 49 |  | 52 | 05 | 54 | 05 | 57 | 06 | 00 | 06 | 02 | 06 | 05 |  | 08 |  | 11 |
| 30 |  | 30 |  | 31 |  | 33 |  | 35 |  | 37 |  | 39 |  | 41 |  | 43 |  | 45 |  | 47 |  | 49 |  | 51 | 05 | 53 | 05 | 55 | 05 | 58 | 06 | 00 | 06 | 02 |
| N20 | 05 | 29 | 05 | 30 | 05 | 30 | 05 | 31 | 05 | 32 | 05 | 33 | 05 | 34 | 05 | 35 | 05 | 36 | 05 | 37 | 05 | 39 | 05 | 40 | 05 | 41 | 05 | 43 | 05 | 44 | 05 | 46 | 05 | 48 |
| N10 |  | 28 |  | 27 |  | 27 |  | 27 |  | 27 |  | 27 |  | 28 |  | 28 |  | 28 |  | 28 |  | 29 |  | 29 |  | 30 |  | 31 |  | 32 |  | 33 |  | 34 |
| 0 |  | 25 |  | 25 |  | 24 |  | 23 |  | 22 |  | 21 |  | 21 |  | 20 |  | 19 |  | 19 |  | 19 |  | 19 |  | 18 |  | 19 |  | 19 |  | 19 |  | 20 |
| S10 |  | 23 |  | 21 |  | 19 |  | 18 |  | 16 |  | 14 |  | 13 |  | 11 |  | 10 | 05 | 09 | 05 | 08 | 05 | 07 | 05 | 07 | 05 | 06 | 05 | 06 | 05 | 05 | 05 | 05 |
| 20 |  | 19 |  | 16 |  | 14 |  | 11 | 05 | 08 | 05 | 06 | 05 | 04 | 05 | 02 | 05 | 00 | 04 | 58 | 04 | 56 | 04 | 54 | 04 | 53 | 04 | 51 | 04 | 50 | 04 | 49 | 04 | 48 |
| S30 | 05 | 14 | 05 | 10 | 05 | 07 | 05 | 03 | 04 | 59 | 04 | 56 | 04 | 53 | 04 | 49 | 04 | 46 | 04 | 43 | 04 | 41 | 04 | 38 | 04 | 36 | 04 | 33 | 04 | 31 | 04 | 30 | 04 | 28 |
| 35 |  | 11 |  | 06 | 05 | 02 |  | 58 |  | 54 |  | 50 |  | 46 |  | 42 |  | 38 |  | 35 |  | 31 |  | 28 |  | 25 |  | 22 |  | 20 |  | 18 |  | 16 |
| 40 |  | 07 | 05 | 02 | 04 | 57 |  | 52 |  | 47 |  | 42 |  | 38 |  | 33 |  | 29 |  | 25 |  | 21 |  | 17 | 04 | 13 | 04 | 10 | 04 | 07 | 04 | 04 | 04 | 01 |
| 45 | 05 | 02 | 04 | 56 |  | 50 |  | 44 |  | 39 |  | 33 |  | 28 |  | 23 |  | 18 | 04 | 12 | 04 | 08 | 04 | 03 | 03 | 59 | 03 | 54 | 03 | 50 | 03 | 47 | 03 | 43 |
| 50 | 04 | 56 |  | 49 |  | 42 |  | 36 |  | 29 |  | 22 |  | 16 |  | 10 | 04 | 03 | 03 | 57 | 03 | 51 | 03 | 46 |  | 40 |  | 35 |  | 30 |  | 25 |  | 21 |
| 52 | 04 | 53 | 04 | 45 | 04 | 38 | 04 | 31 | 04 | 24 | 04 | 17 | 04 | 10 | 04 | 03 | 03 | 57 | 03 | 50 | 03 | 44 | 03 | 37 | 03 | 31 | 03 | 25 | 03 | 20 | 03 | 14 | 03 | 09 |
| 54 |  | 49 |  | 42 |  | 34 |  | 26 |  | 19 |  | 11 | 04 | 04 | 03 | 56 |  | 49 |  | 42 |  | 35 |  | 28 |  | 21 |  | 14 | 03 | 08 | 03 | 02 | 02 | 57 |
| 56 |  | 46 |  | 38 |  | 29 |  | 21 |  | 13 | 04 | 05 | 03 | 57 |  | 48 |  | 41 |  | 33 |  | 25 |  | 17 | 03 | 10 | 03 | 02 | 02 | 55 | 02 | 48 |  | 42 |
| 58 |  | 42 |  | 33 |  | 24 |  | 15 | 04 | 06 | 03 | 57 |  | 48 |  | 40 |  | 31 |  | 22 |  | 13 | 03 | 05 | 02 | 56 | 02 | 48 |  | 39 |  | 31 |  | 24 |
| 60 | 04 | 37 | 04 | 27 | 04 | 18 | 04 | 08 | 03 | 58 | 03 | 49 | 03 | 39 | 03 | 29 | 03 | 19 | 03 | 09 | 03 | 00 | 02 | 50 | 02 | 40 | 02 | 30 | 02 | 21 | 02 | 11 | 02 | 01 |

## EVENING CIVIL TWILIGHT

| Lat | 2 |  | 5 |  | 8 |  | October |  |  |  |  |  | 20 |  | 23 |  | 26 |  | 29 |  | November |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 11 |  |  |  | 4 |  | 17 | 1 |  |  |  | 4 | 7 |  | 10 |  | 13 |  | 16 |  | 19 |  |
| 。 | h |  |  |  | h | m | h | m | h | m | h | m | h | m |  |  | h | m |  |  | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N72 | 18 | 23 | 18 | 08 | 17 | 53 | 17 | 38 | 17 | 23 | 17 | 09 | 16 | 55 | 16 | 40 |  |  | 16 | 26 | 16 | 12 | 15 | 59 | 15 | 45 | 15 | 31 | 15 | 18 | 15 | 05 | 14 | 51 | 14 | 38 |
| 70 |  | 20 |  | 07 |  | 53 |  | 40 |  | 27 |  | 14 | 17 |  |  | 48 |  | 36 |  | 24 | 16 |  | 16 | 00 | 15 |  |  | 37 |  | 26 | 15 | 15 | 15 | 04 |
| 68 |  | 18 |  | 06 |  | 53 |  | 41 |  | 29 |  | 18 |  | 06 | 16 | 55 |  | 44 |  | 33 |  | 22 |  | 11 | 16 |  | 15 | 51 |  | 42 |  | 33 |  | 24 |
| 66 |  | 16 |  | 05 |  | 54 |  | 43 |  | 32 |  | 21 |  | 11 | 17 | 00 |  | 50 |  | 40 |  | 31 |  | 21 |  | 12 | 16 | 04 | 15 | 55 |  | 47 |  | 40 |
| 64 |  | 15 |  | 04 |  | 54 |  | 44 |  | 34 |  | 24 |  | 15 |  | 05 | 16 | 56 |  | 47 |  | 38 |  | 30 |  | 22 |  | 14 | 16 | 06 | 15 | 59 | 15 | 53 |
| 62 |  | 14 |  | 04 |  | 54 |  | 45 |  | 36 |  | 27 |  | 18 |  | 09 | 17 | 01 |  | 53 |  | 45 |  | 37 |  | 30 |  | 23 |  | 16 | 16 | 10 | 16 | 04 |
| N60 | 18 | 13 | 18 | 04 | 17 | 55 | 17 | 46 | 17 | 38 | 17 | 29 | 17 | 21 | 17 | 13 | 17 | 05 | 16 | 58 | 16 | 50 | 16 | 43 | 16 | 37 | 16 | 30 | 16 | 24 | 16 | 19 | 16 | 13 |
| 58 |  | 12 |  | 03 |  | 55 |  | 47 |  | 39 |  | 32 |  | 24 |  | 16 |  | 09 | 17 | 02 | 16 | 56 |  | 49 |  | 43 |  | 37 |  | 32 |  | 27 |  | 22 |
| 56 |  | 11 |  | 03 |  | 56 |  | 48 |  | 41 |  | 33 |  | 26 |  | 19 |  | 13 |  | 06 | 17 | 00 |  | 54 |  | 49 |  | 43 |  | 38 |  | 34 |  | 29 |
| 54 |  | 10 |  | 03 |  | 56 |  | 49 |  | 42 |  | 35 |  | 29 |  | 22 |  | 16 |  | 10 |  | 04 | 16 | 59 |  | 54 |  | 49 |  | 44 |  | 40 |  | 36 |
| 52 |  | 10 |  | 03 |  | 56 |  | 50 |  | 43 |  | 37 |  | 31 |  | 25 |  | 19 |  | 14 |  | 08 | 17 | 03 | 16 | 58 |  | 54 |  | 50 |  | 46 |  | 42 |
| N50 | 18 | 10 | 18 | 03 | 17 | 57 | 17 | 51 | 17 | 45 | 17 | 39 | 17 | 33 | 17 | 27 | 17 | 22 | 17 | 17 | 17 | 12 | 17 | 07 | 17 | 03 | 16 | 58 | 16 | 55 | 16 | 51 | 16 | 48 |
| 45 |  | 09 |  | 03 |  | 58 |  | 53 |  | 47 |  | 42 |  | 37 |  | 33 |  | 28 |  | 24 |  | 20 |  | 16 |  | 12 | 17 | 09 | 17 | 05 | 17 | 03 | 17 | 00 |
| 40 |  | 09 |  | 04 | 17 | 59 |  | 55 |  | 50 |  | 46 |  | 42 |  | 38 |  | 34 |  | 30 |  | 27 |  | 23 |  | 20 |  | 17 |  | 15 |  | 12 |  | 10 |
| 35 |  | 09 |  | 04 | 18 | 00 |  | 56 |  | 53 |  | 49 |  | 45 |  | 42 |  | 39 |  | 35 |  | 33 |  | 30 |  | 27 |  | 25 |  | 23 |  | 21 |  | 20 |
| 30 |  | 09 |  | 05 |  | 02 | 17 | 58 | 17 | 55 |  | 52 |  | 49 |  | 46 |  | 43 |  | 41 |  | 38 |  | 36 |  | 34 |  | 32 |  | 31 |  | 29 |  | 28 |
| N20 | 18 | 10 | 18 | 07 | 18 | 05 | 18 | 02 | 18 | 00 | 17 | 58 | 17 | 55 | 17 | 53 | 17 | 52 | 17 | 50 | 17 | 49 | 17 | 47 | 17 | 46 | 17 | 45 | 17 | 44 | 17 | 44 | 17 | 43 |
| N10 |  | 11 |  | 10 |  | 08 |  | 06 |  | 05 | 18 | 03 | 18 | 02 | 18 | 01 | 18 | 00 | 17 | 59 | 17 | 58 | 17 | 58 | 17 | 57 | 17 | 57 | 17 | 57 | 17 | 57 | 17 | 57 |
| 0 |  | 13 |  | 13 |  | 12 |  | 11 |  | 10 |  | 10 |  | 09 |  | 09 |  | 09 | 18 | 09 | 18 | 09 | 18 | 09 | 18 | 09 | 18 | 09 | 18 | 10 | 18 | 10 | 18 | 11 |
| S10 |  | 16 |  | 16 |  | 16 |  | 16 |  | 17 |  | 17 |  | 17 |  | 18 |  | 18 |  | 19 |  | 20 |  | 20 |  | 21 |  | 22 |  | 23 |  | 24 |  | 26 |
| 20 |  | 20 |  | 21 |  | 22 |  | 23 |  | 24 |  | 25 |  | 26 |  | 28 |  | 29 |  | 30 |  | 32 |  | 33 |  | 35 |  | 37 |  | 39 | 18 | 41 | 18 | 43 |
| S30 | 18 | 26 | 18 | 27 | 18 | 29 | 18 | 31 | 18 | 33 | 18 | 36 | 18 | 38 | 18 | 40 | 18 | 42 | 18 | 45 | 18 | 47 | 18 | 50 | 18 | 52 | 18 | 55 | 18 | 58 | 19 | 00 | 19 | 03 |
| 35 |  | 29 |  | 32 |  | 34 |  | 37 |  | 39 |  | 42 |  | 45 |  | 48 | 18 | 50 | 18 | 53 | 18 | 57 | 19 | 00 | 19 | 03 | 19 | 06 | 19 | 09 |  | 12 |  | 16 |
| 40 |  | 33 |  | 36 |  | 39 |  | 43 |  | 46 |  | 49 | 18 | 53 | 18 | 56 | 19 | 00 | 19 | 04 | 19 | 07 |  | 11 |  | 15 |  | 19 |  | 23 |  | 27 |  | 30 |
| 45 |  | 38 |  | 42 |  | 46 |  | 50 | 18 | 54 | 18 | 58 | 09 | 03 | 19 | 07 |  | 12 |  | 16 |  | 21 |  | 25 |  | 30 |  | 35 | 19 | 39 | 19 | 44 | 19 | 48 |
| 50 |  | 45 |  | 49 |  | 54 | 18 | 59 | 19 | 05 | 19 | 10 |  | 15 |  | 21 |  | 26 |  | 32 |  | 37 |  | 43 |  | 49 | 19 | 54 | 20 | 00 | 20 | 06 | 20 | 11 |
| 52 | 18 | 47 | 18 | 53 | 18 | 58 | 19 | 04 | 19 | 09 | 19 | 15 | 19 | 21 | 19 | 27 | 19 | 33 | 19 | 39 | 19 | 45 | 19 | 51 | 19 | 58 | 20 | 04 | 20 | 10 | 20 | 17 | 20 | 23 |
| 54 |  | 51 | 18 | 57 | 19 | 03 |  | 09 |  | 15 |  | 21 |  | 27 |  | 34 |  | 41 |  | 47 | 19 | 54 | 20 | 01 | 20 | 08 |  | 15 |  | 22 |  | 29 |  | 36 |
| 56 |  | 55 | 19 | 01 |  | 08 |  | 14 |  | 21 |  | 28 |  | 35 |  | 42 |  | 49 | 19 | 57 | 20 | 04 |  | 12 |  | 20 |  | 28 |  | 35 | 20 | 43 | 20 | 51 |
| 58 | 18 | 59 |  | 06 |  | 13 |  | 20 |  | 18 |  | 36 |  | 43 | 19 | 51 | 19 | 59 | 20 | 08 |  | 16 |  | 25 |  | 33 | 20 | 42 | 20 | 51 | 21 | 00 | 21 | 09 |
| 60 | 19 | 04 | 19 | 12 | 19 | 19 | 19 | 28 | 19 | 36 | 19 | 44 | 19 | 53 | 20 | 02 | 20 | 11 | 20 | 20 | 20 | 30 | 20 | 40 | 20 | 50 | 21 | 00 | 21 | 11 | 21 | 21 | 21 | 32 |

## SUNRISE

| Lat | November |  |  |  |  |  |  |  | December |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \hline \text { Jan } \\ 3 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 19 |  | 22 |  | $25 \quad 28$ |  |  |  | 1 |  | 4 |  | 7 |  | 10 |  | 13 |  | 16 |  | 19 |  | 22 |  | 25 |  | 28 |  | 31 |  |  |  |
|  | h |  | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N72 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | - |  |  |  |
| 70 | 10 | 14 | 10 | 40 | 11 | 19 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 68 | 9 | 30 | 09 | 45 | 10 | 00 | 10 | 16 | 10 | 33 | 10 | 52 | 11 | 14 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 66 | 09 | 01 | 09 | 12 | 09 | 23 | 09 | 34 | 09 | 45 | 09 | 55 | 10 | 05 | 10 | 14 | 10 | 22 | 10 | 28 | 10 | 32 | 10 | 35 | 10 | 35 | 10 | 33 | 10 | 30 | 10 | 25 |
| 64 | 08 | 39 | 08 | 48 | 08 | 57 | 09 | 06 | 09 | 15 | 09 | 23 | 09 | 30 | 09 | 36 | 09 | 42 | 09 | 47 | 09 | 50 | 09 | 52 | 09 | 53 | 09 | 52 | 09 | 51 | 09 | 48 |
| 62 |  | 21 |  | 29 |  | 37 | 08 | 45 | 08 | 52 | 08 | 59 | 09 | 05 | 09 | 10 | 09 | 15 | 09 | 19 |  | 22 |  | 24 |  | 24 |  | 24 |  | 23 |  | 22 |
| N60 | 08 | 07 | 08 | 14 | 08 | 21 | 08 | 28 | 08 | 34 | 08 | 840 | 08 | 45 | 08 | 50 | 08 | 54 | 08 | 58 | 09 | 00 | 09 | 02 | 09 | 03 | 09 | 03 | 09 | 03 | 9 | 01 |
| 58 | 07 | 54 | 08 | 01 | 08 | 07 |  | 13 |  | 19 |  | 24 |  | 29 |  | 33 |  | 37 |  | 40 | 08 | 43 | 08 | 45 | 08 | 46 | 08 | 46 | 08 | 46 | 08 | 45 |
| 56 |  | 43 | 07 | 49 | 07 | 55 | 08 | 01 | 08 | 06 | 08 | 11 |  | 15 |  | 19 |  | 23 |  | 26 |  | 28 |  | 30 |  | 31 |  | 32 |  | 32 |  | 31 |
| 54 |  | 34 |  | 40 |  | 45 | 07 | 50 | 07 | 55 | 07 | 59 | 08 | 03 | 08 | 07 | 08 | 10 |  | 13 |  | 15 |  | 17 |  | 19 |  | 19 |  | 19 |  | 19 |
| 52 |  | 26 |  | 31 |  | 36 |  | 40 |  | 45 |  | 49 | 07 | 53 | 07 | 56 | 07 | 59 | 08 | 02 | 08 | 04 | 08 | 06 | 08 | 07 | 08 | 08 | 08 | 08 | 08 | 08 |
| N50 | 07 | 18 | 07 | 23 | 07 | 28 | 07 | 32 | 07 | 36 | 07 | 70 | 07 | 43 | 07 | 47 | 07 | 50 | 07 | 52 | 07 | 54 | 07 | 56 | 07 | 57 | 07 | 58 | 07 | 59 | 07 | 58 |
| 45 | 07 | 02 | 07 | 06 | 07 | 10 | 07 | 14 |  | 17 |  | 21 |  | 24 |  | 27 |  | 29 |  | 32 |  | 34 |  | 35 |  | 37 |  | 38 |  | 38 |  | 38 |
| 40 | 06 | 49 | 06 | 53 | 06 | 56 | 06 | 59 | 07 | 02 | 07 | 705 | 07 | 08 | 07 | 10 | 07 | 13 |  | 15 |  | 17 |  | 18 |  | 20 |  | 21 |  | 22 |  | 22 |
| 35 |  | 38 |  | 41 |  | 44 |  | 46 | 06 | 49 | 06 | 52 | 06 | 54 | 06 | 57 | 06 | 59 | 07 | 01 | 07 | 03 | 07 | 04 | 07 | 06 | 07 | 07 | 07 | 08 | 07 | 08 |
| 30 |  | 28 |  | 30 |  | 33 |  | 35 |  | 38 |  | 40 |  | 43 |  | 45 |  | 47 | 06 | 49 | 06 | 50 | 06 | 52 | 06 | 53 | 06 | 54 | 06 | 55 | 06 | 56 |
| N20 | 6 | 11 | 06 | 13 | 06 | 15 | 06 | 16 | 06 | 18 | 06 | , 20 | 06 | 22 | 06 | 24 | 06 | 26 | 06 | 28 | 06 | 29 | 06 | 31 | 06 | 32 | 06 | 33 | 06 | 35 | 06 | 36 |
| N10 | 05 | 56 | 05 | 57 | 05 | 58 | 06 | 00 | 06 | 01 | 06 | 03 | 06 | 04 | 06 | 06 | 06 | 07 | 06 | 09 | 06 | 10 | 06 | 12 | 06 | 13 | 06 | 15 | 06 | 16 |  | 17 |
| 0 |  | 42 |  | 42 |  | 43 | 05 | 44 | 05 | 45 | 05 | 546 | 05 | 48 | 05 | 49 | 05 | 50 | 05 | 52 | 05 | 53 | 05 | 55 | 05 | 56 | 05 | 58 | 05 | 59 | 06 | 01 |
| S10 |  | 28 |  | 28 |  | 28 |  | 28 |  | 29 |  | 30 |  | 31 |  | 32 |  | 33 |  | 34 |  | 35 |  | 37 |  | 39 |  | 40 |  | 42 | 05 | 43 |
| 20 | 05 | 12 | 05 | 12 | 05 | 11 | 05 | 11 | 05 | 12 | 05 | 12 | 05 | 13 | 05 | 13 | 05 | 14 | 05 | 15 | 05 | 17 | 05 | 18 | 05 | 20 | 05 | 21 |  | 23 |  | 25 |
| S30 | 04 | 54 | 04 | 53 | 04 | 52 | 04 | 52 | 04 | 51 | 04 | 51 | 04 | 52 | 04 | 52 | 04 | 52 | 04 | 53 | 04 | 55 | 04 | 56 | 04 | 58 | 04 | 59 | 05 | 01 | 5 | 03 |
| 35 |  | 44 |  | 42 |  | 41 |  | 40 |  | 39 |  | 39 |  | 39 |  | 39 |  | 40 |  | 41 |  | 42 |  | 43 |  | 45 |  | 46 | 04 | 48 | 04 | 51 |
| 40 |  | 32 |  | 30 |  | 28 |  | 27 |  | 26 |  | 25 |  | 25 |  | 25 |  | 25 |  | 26 |  | 27 |  | 28 |  | 29 |  | 31 |  | 34 |  | 36 |
| 45 |  | 18 | 04 | 15 | 04 | 13 | 04 | 11 | 04 | 09 | 04 | 408 | 04 | 07 | 04 | 07 | 04 | 07 | 04 | 08 | 04 | 08 | 04 | 10 | 04 | 11 | 04 | 13 | 04 | 16 | 04 | 19 |
| 50 | 04 | 00 | 03 | 57 | 03 | 54 | 03 | 51 | 03 | 49 | 03 | 347 | 03 | 45 | 03 | 45 | 03 | 45 | 03 | 45 | 03 | 46 | 03 | 47 | 03 | 49 | 03 | 51 | 03 | 54 | 03 | 57 |
| 52 | 03 | 52 | 03 | 48 | 03 | 45 | 03 | 42 | 03 | 39 | 03 | 37 | 03 | 35 | 03 | 35 | 03 | 34 | 03 | 34 | 03 | 35 | 03 | 36 | 03 | 38 | 03 | 40 | 03 | 43 | 03 | 46 |
| 54 |  | 43 |  | 39 |  | 35 |  | 31 |  | 29 |  | 26 |  | 23 |  | 23 |  | 22 |  | 22 |  | 23 |  | 24 |  | 26 |  | 28 |  | 31 |  | 34 |
| 56 |  | 32 |  | 28 |  | 23 |  | 19 |  | 16 | 03 | 13 | 03 | 09 | 03 | 30 | 03 | 09 | 03 | 08 | 03 | 09 | 03 | 10 | 03 | 12 | 03 | 14 |  | 17 |  | 21 |
| 58 |  | 21 |  | 15 | 03 | 10 | 03 | 06 | 03 | 02 | 02 | 58 | 02 | 53 | 02 | 53 | 02 | 52 | 02 | 52 | 02 | 52 | 02 | 53 | 02 | 55 | 02 | 57 | 03 | 01 | 03 | 05 |
| 60 | 03 | 06 | 03 | 00 | 02 | 54 | 02 | 49 | 02 | 44 | 02 | 40 | 02 | 34 | 02 | 34 | 02 | 32 | 02 | 31 | 02 | 31 | 02 | 32 | 02 | 34 | 02 | 37 | 02 | 41 | 02 | 45 |

SUNSET

| Lat | November |  |  |  |  |  |  |  | December |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} \mathrm{Jan} \\ 3 \\ \hline \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 19 |  | 22 |  | 25 |  | 28 |  | 1 |  | 4 |  | 7 |  | 10 |  | 13 |  | 16 |  | 19 |  | 22 |  | 25 |  | 28 |  | 31 |  |  |  |
|  | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m | h | m |
| N72 |  |  | - |  | - |  | - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | - |  |  |  |  |  |  |  | - |  |
| 70 | 13 | 15 | 12 | 51 | 12 | 13 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 68 | 14 | 00 | 13 | 47 | 13 | 33 | 13 | 19 | 13 | 04 | 12 | 247 | 12 | 227 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 66 |  | 29 | 14 | 19 | 14 | 10 | 14 | 01 | 13 | 52 | 13 | 344 | 13 | 37 | 13 | 30 | 13 | 26 | 13 | 22 | 13 | 21 | 13 | 21 | 13 | 24 | 13 | 29 | 13 | 35 | 13 | 43 |
| 64 | 14 | 51 | 14 | 43 |  | 36 |  | 29 | 14 | 22 | 14 | 417 | 14 | 412 | 14 | 08 | 14 | 05 | 14 | 04 | 14 | 03 | 14 | 04 | 14 | 06 | 14 | 10 | 14 | 15 | 14 | 21 |
| 62 | 15 | 09 | 15 | 02 | 14 | 56 | 14 | 50 | 14 | 45 | 14 | 41 |  | 37 |  | 34 |  | 32 |  | 31 |  | 32 |  | 33 |  | 35 |  | 38 | 14 | 42 | 14 | 47 |
| N60 | 15 | 23 | 15 | 18 | 15 | 12 | 15 | 08 | 15 | 03 | 15 | 500 | 14 | 57 | 14 | 55 | 14 | 54 | 14 | 53 | 14 | 53 | 14 | 55 | 14 | 57 | 14 | 59 | 15 | 03 | 15 | 07 |
| 58 |  | 36 |  | 31 |  | 26 |  | 22 |  | 19 |  | 16 | 15 | 513 | 15 | 12 | 15 | 11 | 15 | 10 | 15 | 11 | 15 | 12 | 15 | 14 | 15 | 17 |  | 20 |  | 24 |
| 56 |  | 46 |  | 42 |  | 38 |  | 35 |  | 32 |  | 29 |  | 27 |  | 26 |  | 25 |  | 25 |  | 26 |  | 27 |  | 29 |  | 31 |  | 34 |  | 38 |
| 54 | 15 | 56 | 15 | 52 |  | 48 |  | 45 |  | 43 |  | 41 |  | 39 |  | 38 |  | 38 |  | 38 |  | 39 |  | 40 |  | 42 |  | 44 |  | 47 | 15 | 50 |
| 52 | 16 | 04 | 16 | 01 | 15 | 58 | 15 | 55 | 15 | 53 | 15 | 51 |  | 50 |  | 49 |  | 48 |  | 49 |  | 49 | 15 | 51 | 15 | 52 | 15 | 55 | 15 | 58 | 16 | 01 |
| N50 | 16 | 12 | 16 | 09 | 16 | 06 | 16 | 03 | 16 | 01 | 16 | 600 | 15 | 559 | 15 | 58 | 15 | 58 | 15 | 58 | 15 | 59 | 16 | 01 | 16 | 02 | 16 | 05 | 16 | 07 | 16 | 10 |
| 45 |  | 28 |  | 26 |  | 24 |  | 22 |  | 21 |  | 20 | 16 | 16 | 16 | 19 | 16 | 19 | 16 | 19 | 16 | 20 |  | 21 |  | 23 |  | 25 |  | 28 |  | 30 |
| 40 |  | 41 |  | 39 |  | 38 |  | 37 |  | 36 |  | 35 |  | 35 |  | 35 |  | 35 |  | 36 |  | 37 |  | 38 |  | 40 |  | 42 |  | 44 |  | 47 |
| 35 | 16 | 53 | 16 | 51 | 16 | 50 | 16 | 49 | 16 | 49 | 16 | 648 | 16 | 648 | 16 | 48 | 16 | 49 | 16 | 50 | 16 | 51 | 16 | 52 | 16 | 54 | 16 | 56 | 16 | 58 | 17 | 00 |
| 30 | 17 | 03 | 17 | 01 | 17 | 01 | 17 | 00 | 17 | 00 | 17 | 700 | 17 | 700 | 17 | 17 | 17 | 01 | 17 | 02 | 17 | 04 | 17 | 05 | 17 | 07 | 17 | 09 | 17 | 10 |  | 13 |
| N20 | 17 | 20 | 17 | 19 | 17 | 19 | 17 | 19 | 17 | 19 | 17 | 720 | 17 | 1720 | 17 | 21 | 17 | 22 | 17 | 23 | 17 | 25 | 17 | 26 | 17 | 28 | 17 | 29 | 17 | 31 | 17 | 33 |
| N10 |  | 35 |  | 35 |  | 35 |  | 36 |  | 37 |  | 37 |  | 38 |  | 39 |  | 41 |  | 42 | 17 | 43 | 17 | 45 | 17 | 46 | 17 | 48 | 17 | 50 | 17 | 51 |
| 0 | 17 | 49 | 17 | 50 | 17 | 51 | 17 | 52 | 17 | 53 | 17 | 754 | 17 | 755 | 17 | 56 | 17 | 58 | 17 | 59 | 18 | 01 | 18 | 02 | 18 | 04 | 18 | 05 | 18 | 07 | 18 | 08 |
| S10 | 18 | 03 | 18 | 05 | 18 | 06 | 18 | 07 | 18 | 09 | 18 | 810 | 18 | 1812 | 18 | 14 | 18 | 15 | 18 | 17 |  | 18 |  | 20 |  | 21 |  | 23 |  | 24 |  | 25 |
| 20 |  | 19 |  | 21 |  | 23 |  | 25 |  | 26 |  | 28 |  | 30 |  | 32 |  | 34 |  | 36 |  | 37 | 18 | 39 | 18 | 40 | 18 | 41 | 18 | 43 | 18 | 44 |
| S30 | 18 | 37 | 18 | 39 | 18 | 42 | 18 | 44 | 18 | 47 | 18 | 849 | 18 | 851 | 18 | 54 | 18 | 56 | 18 | 58 | 18 | 59 | 19 | 01 | 19 | 02 | 19 | 03 | 19 | 04 | 19 | 05 |
| 35 |  | 47 | 18 | 50 | 18 | 53 | 18 | 56 | 18 | 59 | 19 | 02 | 19 | 9 04 | 19 | 06 | 19 | 09 | 19 | 11 | 19 | 12 |  | 14 |  | 15 |  | 16 |  | 17 |  | 18 |
| 40 | 18 | 59 | 19 | 03 | 19 | 06 | 19 | 09 | 19 | 13 |  | 16 |  | 18 |  | 21 |  | 23 |  | 26 |  | 27 |  | 29 |  | 30 |  | 31 |  | 32 |  | 32 |
| 45 | 19 | 14 |  | 18 |  | 22 |  | 25 |  | 29 |  | 32 |  | 36 | 19 | 39 | 19 | 41 | 19 | 44 | 19 | 45 | 19 | 47 | 19 | 48 | 19 | 49 | 19 | 50 | 19 | 50 |
| 50 |  | 31 |  | 36 |  | 41 |  | 45 |  | 50 | 19 | 954 | 19 | 97 | 20 | 01 | 20 | 03 | 20 | 06 | 20 | 08 | 20 | 10 | 20 | 11 | 20 | 12 | 20 | 12 | 20 | 11 |
| 52 | 19 | 40 | 19 | 45 | 19 | 50 | 19 | 55 | 19 | 59 | 20 | 04 | 20 | 07 | 20 | 11 | 20 | 14 | 20 | 17 | 20 | 19 | 20 | 21 | 20 | 22 | 20 | 22 | 20 | 22 | 20 | 22 |
| 54 | 19 | 49 | 19 | 55 | 20 | 00 | 20 | 05 | 20 | 10 |  | 15 |  | 19 |  | 23 |  | 26 |  | 29 |  | 31 |  | 33 |  | 34 |  | 34 |  | 34 |  | 34 |
| 56 | 20 | 00 | 20 | 06 |  | 12 |  | 18 |  | 23 |  | 28 |  | 33 |  | 37 |  | 40 | 20 | 43 | 20 | 45 | 20 | 47 | 20 | 48 | 20 | 49 | 20 | 48 | 20 | 47 |
| 58 |  | 12 |  | 19 |  | 25 |  | 32 |  | 38 | 20 | 043 | 20 | 048 | 20 | 53 | 20 | 57 | 21 | 00 | 21 | 02 | 21 | 04 | 21 | 05 | 21 | 05 | 21 | 05 | 21 | 03 |
| 60 | 20 | 26 | 20 | 34 | 20 | 41 | 20 | 48 | 20 | 55 | 21 | 101 | 21 | 107 | 21 | 112 | 21 | 17 | 21 | 20 | 21 | 23 | 21 | 25 | 21 | 25 | 21 | 25 | 21 | 24 | $21$ | 22 |

MORNING CIVIL TWILIGHT

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| N72 | 08 | 51 | 09 | 05 | 09 | 20 | 09 | 34 | 09 | 48 | 10 | 02 | 10 | 15 | 10 | 1027 | 10 | 38 | 10 | 48 | 10 | 54 | 10 | 57 | 10 | 57 | 10 | 53 | 10 | 46 | 10 | 38 |
| 70 |  | 25 | 08 | 37 | 08 | 48 | 08 | 59 | 09 | 09 | 09 | 19 | 09 | 28 | 09 | 36 | 09 | 43 | 09 | 48 | 09 | 52 | 09 | 54 | 09 | 55 | 09 | 54 | 09 | 51 | 09 | 47 |
| 68 | 08 | 06 | 08 | 15 |  | 25 |  | 34 | 08 | 42 | 08 | 50 | 08 | 57 | 09 | 04 | 09 | 09 | 09 | 14 | 09 | 17 | 09 | 19 | 09 | 20 | 09 | 20 | 09 | 18 | 09 | 15 |
| 66 | 07 | 50 | 07 | 58 | 08 | 06 | 08 | 14 |  | 22 |  | 29 |  | 35 | 08 | 40 | 08 | 45 | 08 | 49 | 08 | 52 | 08 | 54 | 08 | 55 | 08 | 55 | 08 | 54 | 08 | 52 |
| 64 |  | 37 |  | 44 | 07 | 52 | 07 | 59 | 08 | 05 | 08 | 11 |  | 17 |  | 22 |  | 26 |  | 29 |  | 32 |  | 34 |  | 35 |  | 35 |  | 34 |  | 33 |
| 62 |  | 26 |  | 33 |  | 39 |  | 45 | 07 | 51 | 07 | 57 | 08 | 02 | 08 | 06 | 08 | 10 |  | 13 |  | 16 |  | 18 |  | 19 |  | 19 |  | 19 |  | 18 |
| N60 | 07 | 16 | 07 | 23 | 07 | 29 | 07 | 34 | 07 | 40 | 07 | 45 | 17 | 49 | 07 | 53 | 07 | 57 | 08 | 00 | 08 | 02 | 08 | 04 | 08 | 05 | 08 | 06 | 08 | 06 | 08 | 05 |
| 58 |  | 08 |  | 14 |  | 19 |  | 24 |  | 29 |  | 34 |  | 38 |  | 42 |  | 45 | 07 | 48 | 07 | 51 | 07 | 52 | 07 | 54 | 07 | 54 | 07 | 54 | 07 | 54 |
| 56 | 07 | 01 | 07 | 06 |  | 11 |  | 16 |  | 21 |  | 25 |  | 29 |  | 32 |  | 36 |  | 38 |  | 40 |  | 42 |  | 43 |  | 44 |  | 44 |  | 44 |
| 54 | 06 | 54 | 06 | 59 | 07 | 04 |  | 08 |  | 13 |  | 17 |  | 20 |  | 24 |  | 27 |  | 29 |  | 32 |  | 33 |  | 35 |  | 35 |  | 36 |  | 35 |
| 52 |  | 48 |  | 53 | 06 | 57 | 07 | 01 | 07 | 705 |  | 09 |  | 13 |  | 16 |  | 19 |  | 21 |  | 23 |  | 25 |  | 26 |  | 27 |  | 28 |  | 27 |
| N50 | 06 | 42 | 06 | 47 | 06 | 51 | 06 | 55 | 06 | 59 | 07 | 02 | 07 | 06 | 07 | 709 | 07 | 11 | 07 | 14 | 07 | 16 | 07 | 18 | 07 | 19 | 07 | 20 | 07 | 20 | 07 | 20 |
| 45 |  | 30 |  | 34 |  | 38 |  | 41 |  | 44 | 06 | 47 | 06 | 50 | 06 | 53 | 06 | 56 | 06 | 58 | 07 | 00 | 07 | 02 | 07 | 03 | 07 | 04 | 07 | 05 | 07 | 05 |
| 40 |  | 20 |  | 23 |  | 26 |  | 29 |  | 32 |  | 35 |  | 38 |  | 40 |  | 42 |  | 44 | 06 | 46 | 06 | 48 | 06 | 49 | 06 | 50 | 06 | 51 | 06 | 52 |
| 35 |  | 11 |  | 13 |  | 16 |  | 19 |  | 21 |  | 24 |  | 26 |  | 29 |  | 31 |  | 33 |  | 34 |  | 36 |  | 38 |  | 39 |  | 40 |  | 40 |
| 30 | 06 | 02 | 06 | 05 | 06 | 07 | 06 | 10 | 06 | 12 | 06 | 14 | 06 | 16 |  | 18 |  | 20 |  | 22 |  | 24 |  | 26 |  | 27 |  | 28 |  | 29 |  | 30 |
| N20 | 05 | 48 | 05 | 49 | 05 | 51 | 05 | 53 | 05 | 55 | 05 | 56 | 05 | 58 | 06 | 00 | 06 | 02 | 06 | 04 | 06 | 05 | 06 | 07 | 06 | 08 | 06 | 09 | 06 | 11 | 06 | 12 |
| N10 |  | 34 |  | 35 |  | 36 |  | 37 |  | 39 |  | 40 |  | 42 | 05 | 543 | 05 | 45 | 05 | 46 | 05 | 48 | 05 | 49 | 05 | 51 | 05 | 52 | 05 | 54 | 05 | 55 |
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| S10 | 05 | 05 | 05 | 05 | 05 | 05 | 05 | 06 | 05 | 06 | 05 | 07 | 05 | 08 | 05 | 09 | 05 | 10 | 05 | 11 | 05 | 13 | 05 | 14 | 05 | 16 | 05 | 17 | 05 | 19 |  | 20 |
| 20 | 04 | 48 | 04 | 48 | 04 | 48 | 04 | 47 | 04 | 47 | 04 | 48 | 04 | 48 | 04 | 449 | 04 | 50 | 04 | 51 | 04 | 52 | 04 | 53 | 04 | 45 | 04 | 57 | 04 | 59 | 05 | 00 |
| S30 | 04 | 28 | 04 | 27 | 04 | 26 | 04 | 25 | 04 | 424 | 04 | 24 | 04 | 24 | 04 | 425 | 04 | 25 | 04 | 26 | 04 | 27 | 04 | 28 | 04 | 30 | 04 | 32 | 04 | 34 | 04 | 36 |
| 35 |  | 16 | 04 | 14 | 04 | 12 | 04 | 11 | 04 | 410 | 04 | 10 | 04 | 10 | 04 | 10 | 04 | 10 | 04 | 11 | 04 | 12 | 04 | 13 | 04 | 15 | 04 | 17 |  | 19 |  | 21 |
| 40 | 04 | 01 | 03 | 59 | 03 | 57 | 03 | 55 | 03 | 54 | 03 | 53 | 03 | 52 | 03 | 52 | 03 | 52 | 03 | 53 | 03 | 54 | 03 | 55 | 03 | 57 | 03 | 59 | 04 | 01 | 04 | 04 |
| 45 | 03 | 43 |  | 40 |  | 38 |  | 35 |  | 33 |  | 32 |  | 31 |  | 30 |  | 30 |  | 30 |  | 31 |  | 32 |  | 34 |  | 36 | 03 | 39 | 03 | 42 |
| 50 |  | 21 |  | 17 |  | 13 | 03 | 10 | 03 | 37 | 03 | 04 | 03 | 03 | 03 | 01 | 03 | 01 | 03 | 01 | 03 | 01 | 03 | 03 | 03 | 04 | 03 | 07 | 03 | 10 | 03 | 13 |
| 52 | 03 | 09 | 03 | 05 | 03 | 00 | 02 | 57 | 02 | 253 | 02 | 51 | 02 | 49 | 02 | 247 | 02 | 46 | 02 | 46 | 02 | 46 | 02 | 47 | 02 | 249 | 02 | 52 | 02 | 55 | 02 | 58 |
| 54 | 02 | 57 | 02 | 51 | 02 | 46 |  | 42 |  | 38 |  | 35 |  | 32 |  | 30 |  | 29 |  | 28 |  | 28 |  | 29 |  | 31 |  | 34 |  | 37 |  | 41 |
| 56 |  | 42 |  | 35 |  | 30 |  | 24 | 02 | 20 | 02 | 15 | 02 | 12 | 02 | 209 | 02 | 07 | 02 | 06 | 02 | 06 | 02 | 07 | 02 | 29 | 02 | 12 | 02 | 16 | 02 | 20 |
| 58 |  | 24 | 02 | 16 | 02 | 09 | 02 | 02 | 01 | 56 | 01 | 50 | 01 | 46 | 01 | 142 | 01 | 39 | 01 | 37 | 01 | 36 | 01 | 37 | 01 | 139 | 01 | 42 | 01 | 47 | 01 | 53 |
| 60 | 02 | 01 | 01 | 52 | 01 | 42 | 01 | 33 | 01 | 124 | 01 | 16 | 01 | - 07 | 00 | 59 | 00 | 53 | 00 | 49 | 00 | 47 | 00 | 47 | 00 | 49 | 00 | 54 | 01 | 01 | 01 | 10 |

EVENING CIVIL TWILIGHT

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| N72 | 14 | 38 | 14 | 26 | 14 | 13 | 14 | 01 |  | 349 | 13 | 37 | 13 | 27 | 13 | 17 | 13 | 09 | 13 | 02 | 12 | 59 | 12 | 59 | 13 | 02 | 13 | 09 | 13 | 19 | 13 | 31 |
| 70 | 15 | 04 | 14 | 54 | 14 | 45 | 14 | 36 | 14 | 428 | 14 | 20 | 14 | 14 | 14 | 08 | 14 | 04 | 14 | 02 | 14 | 01 | 14 | 02 | 14 | 04 | 14 | 09 | 14 | 14 | 14 | 21 |
| 68 |  | 24 | 15 | 16 | 15 | 08 | 15 | 01 | 14 | 455 | 14 | 49 | 14 | 44 | 14 | 41 | 14 | 38 | 14 | 36 | 14 | 36 | 14 | 37 | 14 | 39 | 14 | 43 | 14 | 47 | 14 | 53 |
| 66 |  | 40 |  | 33 |  | 26 |  | 20 |  | 515 | 15 | 11 | 15 | 07 | 15 | 04 | 15 | 02 | 15 | 01 | 15 | 01 | 15 | 03 | 15 | 05 | 15 | 08 | 15 | 12 | 15 | 17 |
| 64 | 15 | 53 |  | 47 |  | 41 |  | 36 |  | 32 |  | 28 |  | 25 |  | 23 |  | 21 |  | 21 |  | 21 |  | 22 |  | 24 |  | 27 |  | 31 |  | 35 |
| 62 | 16 | 04 | 15 | 58 | 15 | 54 | 15 | 49 |  | 46 |  | 42 |  | 40 |  | 38 |  | 37 |  | 37 |  | 37 |  | 39 |  | 41 |  | 43 | 15 | 46 | 15 | 51 |
| N60 | 16 | 13 | 16 | 09 | 16 | 04 | 16 | 01 | 15 | 557 | 15 | 55 | 15 | 53 | 15 | 51 | 15 | 50 | 15 | 50 | 15 | 51 | 15 | 52 | 15 | 54 | 15 | 57 | 16 | 00 | 16 | 03 |
| 58 |  | 22 |  | 17 |  | 14 |  | 10 |  | 608 | 16 | 05 | 16 | 04 | 16 | 02 | 16 | 02 | 16 | 02 | 16 | 03 | 16 | 04 | 16 | 06 | 16 | 08 |  | 11 |  | 15 |
| 56 |  | 29 |  | 26 |  | 22 |  | 19 |  | 17 |  | 15 |  | 13 |  | 13 |  | 12 |  | 12 |  | 13 |  | 14 |  | 16 |  | 19 |  | 21 |  | 25 |
| 54 |  | 36 |  | 33 |  | 30 |  | 27 |  | 25 |  | 23 |  | 22 |  | 21 |  | 21 |  | 21 |  | 22 |  | 24 |  | 25 |  | 28 |  | 30 |  | 33 |
| 52 |  | 42 |  | 39 |  | 36 |  | 34 |  | 32 |  | 31 |  | 30 |  | 29 |  | 29 |  | 29 |  | 30 |  | 32 |  | 34 |  | 36 |  | 39 |  | 41 |
| N50 | 16 | 48 | 16 | 45 | 16 | 43 | 16 | 41 |  | 639 | 16 | 38 | 16 | 37 | 16 | 36 | 16 | 37 | 16 | 37 | 16 | 38 | 16 | 39 | 16 | 41 | 16 | 43 | 16 | 46 | 16 | 48 |
| 45 | 17 | 00 | 16 | 58 | 16 | 56 | 16 | 55 | 16 | 653 | 16 | 53 | 16 | 52 | 16 | 52 | 16 | 52 | 16 | 53 | 16 | 54 | 16 | 55 | 16 | 57 | 16 | 59 | 17 | 01 | 17 | 04 |
| 40 |  | 10 | 17 | 09 | 17 | 07 | 17 | 06 |  | 706 | 17 | 05 | 17 | 05 | 17 | 05 | 17 | 06 | 17 | 06 | 17 | 08 | 17 | 09 | 17 | 11 | 17 | 13 |  | 15 |  | 17 |
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| N20 | 17 | 43 | 17 | 43 | 17 | 43 | 17 | 43 |  | 1743 | 17 | 44 | 17 | 45 | 17 | 45 | 17 | 46 | 17 | 48 | 17 | 49 | 17 | 50 | 17 | 52 | 17 | 53 | 17 | 55 | 17 | 57 |
| N10 | 17 | 57 | 17 | 57 | 17 | 58 | 17 | 58 | 17 | 759 | 18 | 00 | 18 | 01 | 18 | 02 | 18 | 03 | 18 | 05 | 18 | 06 | 18 | 08 | 18 | 09 | 18 | 11 | 18 | 12 | 18 | 14 |
| 0 | 18 | 11 | 18 | 12 | 18 | 13 | 18 | 14 |  | 1815 |  | 16 |  | 17 |  | 19 |  | 20 |  | 22 |  | 23 |  | 25 |  | 26 |  | 28 |  | 29 |  | 30 |
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| 20 | 18 | 43 | 18 | 45 | 18 | 47 | 18 | 49 |  | 851 | 18 | 53 | 18 | 55 | 18 | 57 | 18 | 58 | 19 | 00 | 19 | 02 | 19 | 03 | 19 | 05 | 19 | 06 | 19 | 07 | 19 | 08 |
| S30 | 19 | 03 | 19 | 06 | 19 | 09 | 19 | 11 |  | 914 | 19 | 16 | 19 | 19 | 19 | 21 | 19 | 23 | 19 | 25 | 19 | 27 | 19 | 28 | 19 | 30 | 19 | 31 | 19 | 32 | 19 | 32 |
| 35 |  | 16 |  | 19 |  | 22 |  | 25 |  | 28 |  | 31 |  | 33 |  | 36 |  | 38 |  | 40 | 19 | 42 | 19 | 44 | 19 | 45 | 19 | 46 | 19 | 47 | 19 | 47 |
| 40 |  | 30 |  | 34 |  | 38 | 19 | 41 |  | 945 | 19 | 48 | 19 | 51 | 19 | 54 | 19 | 56 | 19 | 58 | 20 | 00 | 20 | 02 | 20 | 03 | 20 | 04 | 20 | 05 | 20 | 05 |
| 45 | 19 | 48 | 19 | 53 | 19 | 57 | 20 | 01 |  | 2005 | 20 | 09 | 20 | 12 | 20 | 16 | 20 | 18 | 20 | 21 |  | 23 |  | 25 |  | 26 |  | 26 |  | 27 |  | 27 |
| 50 | 20 | 11 | 20 | 17 | 20 | 22 |  | 27 |  | 32 |  | 37 |  | 41 |  | 44 | 20 | 48 | 20 | 50 | 20 | 53 | 20 | 54 | 20 | 55 | 20 | 56 | 20 | 56 | 20 | 55 |
| 52 | 20 | 23 | 20 | 29 | 20 | 35 | 20 | 40 | 20 | 2045 | 20 | 50 | 20 | 55 | 20 | 59 | 21 | 03 | 21 | 06 | 21 | 08 | 21 | 10 | 21 | 11 | 21 | 11 | 21 | 11 | 21 | 10 |
| 54 |  | 36 |  | 42 | 20 | 49 | 20 | 55 |  | 2101 | 21 | 07 | 21 | 12 | 21 | 16 |  | 20 |  | 23 |  | 26 |  | 27 |  | 28 |  | 29 |  | 28 |  | 27 |
| 56 | 20 | 51 | 20 | 58 | 21 | 06 | 21 | 13 |  | 20 |  | 26 |  | 32 | 21 | 37 | 21 | 42 | 21 | 45 | 21 | 48 | 21 | 50 | 21 | 51 | 21 | 50 | 21 | 49 | 21 | 47 |
| 58 | 21 | 09 | 21 | 18 |  | 27 | 21 | 36 |  | 2144 | 21 | 51 | 21 | 59 | 22 | 05 | 22 | 10 | 22 | 15 | 22 | 18 | 22 | 20 | 22 | 20 | 22 | 20 | 22 | 18 | 22 | 15 |
| 60 | 21 | 32 | 21 | 43 | 21 | 55 | 22 | 06 |  | 2217 | 22 | 27 |  | 39 | 22 | 48 | 22 | 57 | 23 | 04 | 23 | 08 | 23 | 10 | 23 | 09 | 23 | 07 | 23 | 03 | 22 | 56 |

## Chapter

## 27

## Gridded Charts

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## Introduction

Why is grid navigation used? The main reason is that, at high latitudes, the amount of meridian convergence becomes significant. Consider the case at Figure 27.1.

A is at longitude 70W. The line defining True North is the line joining A to the North Pole - the meridian of 70 W . The track $A B$ makes an initial track angle of $020^{\circ}(\mathrm{T})$.

So suppose we set off using a compass based on True heading and keep applying the appropriate drift to maintain a track of $020^{\circ}(\mathrm{T})$.

But as we cross each meridian going eastwards, the direction of True North changes. Relative to the 'top of the map', i.e. the 12 o'clock position on the page, the direction of North is turning anticlockwise. This means that our direction of $020^{\circ}(\mathrm{T})$ will also turn anticlockwise on this map.


Figure 27.1 A High latitude track
If we maintain a track of $020^{\circ}(\mathrm{T})$, our path over the ground will look something like this:


Figure 27.2 Changing true track

Exactly the same problem occurs at lower latitudes (unless a direct Mercator chart is being used), but it is usually not noticed.


Figure 27.3 A Lambert chart
We wish to fly from C to D, which are both at 53 N . The mean track is $090^{\circ}(\mathrm{T})$. The initial track is $082^{\circ}(\mathrm{T})$. We are using a Lambert chart. We now have a choice:

1. We can maintain a track of $090^{\circ}(\mathrm{T})$. If we do, we will still finish up at D , but we will fly the rhumb line track. If we were to take a succession of fixes whilst flying, but not alter track, they would appear like this:


Figure 27.4 Flying the rhumb line track
However, this is not how most navigation actually takes place. The idea of having a straight line on your chart is that it is supposed to be the track you are trying to fly. If you wanted to fly the rhumb line track, you would use a Mercator chart. So most nonautomatic navigation is done using the other option.
2. We can set off on the initial track of $082^{\circ}(T)$. We then settle into our normal regular cycle of fixing and correcting back to track. After a certain time we take our first fix. There will probably be some cross-track error, mainly caused by a combination of the following components.

- Changes in the wind from the forecast value.
- Inaccurate heading hold.

The cross-track error will also include a small component caused by holding a track of $082^{\circ}(T)$ whilst the straight line track has actually altered to, say, $083^{\circ}(T)$, but this will be completely swamped by other random navigation errors, of which the above will be far the greatest components. We do not even notice the cross-track position error caused by convergency because it is usually so small at low to medium latitudes.

We plot our fix, then use our protractor to make a normal correction to get back to track, locally measuring the track near the fix, thereby automatically correcting for the convergency. During the course of the leg, the datum track will change from $082^{\circ}(\mathrm{T})$ to $098^{\circ}(\mathrm{T})$, but the process happens simply by correcting back to the straight-line track on the chart and measuring using the local meridian.


Figure 27.5 Flying the great circle track

## Gridded Charts

This technique of sweeping the change of track into the normal navigation corrections breaks down at high latitudes, however, because the amount of convergency becomes too great to ignore. We therefore have to abandon True North as our direction datum and use a grid and its associated Grid North on our chart instead. We also have to align our compass to Grid North, not True North.


Figure 27.6 Standard North Polar Grid
There are various different methods of selecting the datum direction for Grid North but, for the EASA ATPL syllabus, the datum chosen will always be aligned with a nominated meridian of longitude (the Greenwich meridian, for instance, in the examples in Figure 27.6 and Figure 27.7).

If the Greenwich meridian is the chosen datum, the grid is known as a Standard Grid. Standard grids are widely used in printed grid maps for the European area. Another favourite datum often used in the USA and Canada is 060W.

At the datum meridian, there is no difference between Grid North and True North. As the longitude changes further and further east or west from the datum, the angle between True North and Grid North increases (see Figure 27.7) because Grid North remains parallel to the datum meridian but True North changes with the increasing convergency of the meridians.


Figure 27.7 Creation of grid

## Convergence

In order to obtain a constant straight line track direction, a datum meridian is selected and lines parallel to this meridian are drawn at suitable intervals on the chart to form a grid.

The direction of the datum meridian is known as Grid North. The direction of the track measured from Grid North is constant and is known as Grid direction.

This enables an aircraft to fly along approximate great circle tracks.
The difference between True direction and Grid direction at any point is known as convergence. You should be able to see from Figure 27.7 that it is equal to the chart convergence between the meridian at that point and the datum meridian, because grid north will be parallel to the datum meridian whilst true north will have changed by the amount of the chart convergence between the datum meridian and the local meridian at the point being considered.

Convergence is annotated Easterly when True North lies to the East of Grid North and Westerly when True North lies to the West of Grid North.

This enables the conversion of Grid direction to True direction by applying the convergence in the sense:

Convergence EAST True LEAST
GRID
$105^{\circ}(\mathrm{G})$
$105^{\circ}(\mathrm{G})$

Figure 27.8 Convergence west


Convergence WEST True BEST
TRUE
$125^{\circ}(\mathrm{T})$
$085^{\circ}(\mathrm{T})$


Figure 27.9 Convergence east

Convergence follows the same pattern of conversion of direction as our previous conversions of:-

True $\longrightarrow$ Variation $\longrightarrow$ Magnetic $\longrightarrow$ Deviation $\longrightarrow$ Compass

| True | Variation <br> West $\longrightarrow$ | Magnetic |
| :---: | :---: | :---: |
| Best |  |  | | Deviation |
| :---: |
| West $\longrightarrow$ | | Compass |
| :---: |

Except that it goes on the left-hand side of the columns (if going from True to Compass)
Grid Convergence
West $\longrightarrow$ True
Best $\begin{gathered}\text { Variation } \\ \text { West } \longrightarrow\end{gathered} \begin{gathered}\text { Magnetic }\end{gathered} \begin{gathered}\text { Deviation } \\ \text { West } \longrightarrow \text { Best }\end{gathered}$

So, to give an example:

| Grid | Convergence | True | Variation | Magnetic | Deviation | Compass |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $090^{\circ}(\mathrm{G})$ | $10^{\circ} \mathrm{W}$ | $100^{\circ}(\mathrm{T})$ | $8^{\circ} \mathrm{W}$ | $108^{\circ}(\mathrm{M})$ | $2^{\circ} \mathrm{E}$ | $106^{\circ}(\mathrm{C})$ |

## East or West Convergence?

There is a logical way of working out whether convergence will be East or West. It depends on whether we are in the Northern or Southern hemisphere and whether the aircraft position is East or West of the datum meridian. See the diagram at Figure 27.10.


Figure 27.10 Putting a grid on the Earth

In this diagram, the vertical centre line represents the datum meridian. In this particular figure, it is the Greenwich meridian, but it does not have to be. The red lines represent other meridians on the Earth and the green lines represent Grid North.

Now consider the top left quarter of the diagram. This represents the Northern hemisphere, with the aircraft at some position out to the left, i.e. on a meridian somewhere to the West of the datum.

In this quarter of the diagram, except at the Equator itself, True North leans to the right (the East) of Grid North, like this:

Therefore, by definition, convergence must be East.


Figure 27.11 True north east of grid north

Now work through the other 3 quadrants of the diagram and we finish up with the following:


Figure 27.12 Direction of convergence
Now that we can convert from True to Grid on the chart, we need to find a method of steering Grid direction. This can either be done using a compass or using a gyro.

## Steering by Compass

If grid technique is being used because of the problem of meridian convergence at high latitudes, then the aircraft may also be fairly near one of the Magnetic Poles. In this case, it may not be practical to use a magnetic compass because variation may be changing too rapidly or the magnetic field strength may be too weak. However, even at high latitudes this may not always be the case. Flying in parts of Northern Canada would probably give a problem, but flying in Northern Europe or Northern Russia, for instance, probably would not, because these areas are a greater distance from the Magnetic North Pole. So it may be possible, under some circumstances, to steer by magnetic compass even when using grid technique.

The problem is not too complicated and is very comparable to the problem of steering True heading when using a magnetic compass. When using True steering, we apply Deviation to convert the Compass heading to Magnetic heading, then apply the difference between Magnetic direction and True direction (Variation). For Grid steering, all we have to do is apply Deviation to convert the Compass heading to Magnetic heading, then apply the difference between Magnetic direction and Grid direction. We call this Grivation and it is the algebraic sum of Variation and Convergence.

If a Magnetic direction is required from a Grid direction, the Convergence and Variation must be applied.

| GRID | CONV | TRUE | VAR |  | MAG |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $090^{\circ}$ | $10^{\circ} \mathrm{E}$ | $080^{\circ}$ | $10^{\circ} \mathrm{E}$ | $=$ | $070^{\circ}$ |

In the example above Grivation is $20^{\circ} \mathrm{E}$. Grivation can be applied directly to Grid direction to give Magnetic direction and vice-versa.

| GRID | GRIV | MAG |
| :--- | :--- | :--- |
| $090^{\circ}$ | $20^{\circ} \mathrm{E}$ | $070^{\circ}$ |

## Examples

| CONVERGENCE VARIATION |  | GRIVATION |
| :---: | :---: | :---: |
| $17^{\circ} \mathrm{W}+4{ }^{\circ} \mathrm{E}$ | $=13^{\circ} \mathrm{W}$ |  |
| $11^{\circ} \mathrm{E}+4^{\circ} \mathrm{E}$ | $=15^{\circ} \mathrm{E}$ |  |
| $14^{\circ} \mathrm{E}+4^{\circ} \mathrm{W}$ | $=10^{\circ} \mathrm{E}$ |  |

Lines on a chart joining places of equal Grivation are named isogrivs.
For some practice, consider the following situation:
A straight line track is drawn between two points on a Lambert or Polar Stereographic chart as shown in Figure 27.13.


Figure 27.13 A track drawn on a chart with converging meridians

Now complete the table below.
The trick is to remember that the Grid track does not change direction. If you have found the Grid track at one longitude, it must be the same Grid track at all longitudes.

| POS | ${ }^{\circ}$ GRID | CONV | ${ }^{\circ}$ TRUE | VAR | GRIV | ${ }^{\circ}$ MAG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  |  | 066 | 8 W |  |  |
| B |  |  |  | 6 W |  | 090 |
| C | 099 |  |  | 4 W |  |  |
| D |  |  |  | 0 |  | 117 |
| E |  |  | 133 | 2 E |  |  |

CHECK: GRID $\pm$ GRIV $=$ MAG
TRUE $\pm$ VAR $=$ MAG

| POS | ${ }^{\circ}$ GRID | CONV | ${ }^{\circ}$ TRUE | VAR | GRIV | ${ }^{\circ}$ MAG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 099 | 33 E | 066 | 8 W | 25 E | 074 |
| B | 099 | 15 E | 084 | 6 W | 9 E | 090 |
| C | 099 | 0 | 099 | 4 W | 4 W | 103 |
| D | 099 | 18 W | 117 | 0 | 18 W | 117 |
| E | 099 | 34 W | 133 | 2 E | 32 W | 131 |

Now let us see how we would use a magnetic compass to steer a True heading as the variation changes, and then we will see that much the same principles can be applied to steering a Grid heading.


Figure 27.14 Updating variation on crossing isogonals

In Figure 27.14 we have a rhumb line track. After correction for drift, we need to maintain a True heading of, say, $080^{\circ}(\mathrm{T})$. To the west of the $3.5^{\circ} \mathrm{W}$ isogonal, the variation is, to the nearest degree, $3^{\circ} \mathrm{W}$. To the east of it, the variation is $4^{\circ} \mathrm{W}$. Similarly, at the $4.5^{\circ} \mathrm{W}$ isogonal, tracking eastwards, the variation changes from $4^{\circ} \mathrm{W}$ to $5^{\circ} \mathrm{W}$.

The pilot starts by flying a Magnetic heading of $083^{\circ}(\mathrm{M})$ in order to maintain a True heading of $080^{\circ}(\mathrm{T})$. As he crosses the $3.5^{\circ} \mathrm{W}$ isogonal, he alters Magnetic heading to $084^{\circ}(\mathrm{M})$, and then as he crosses the $4.5^{\circ} \mathrm{W}$ isogonal, he alters Magnetic heading to $085^{\circ}(\mathrm{M})$, maintaining a True heading of $080^{\circ}(\mathrm{T})$.

If a chart is constructed with isogrivs, then the same principles can be used to maintain a Grid heading when using a magnetic compass.

Figure 27.15 shows a printed grid chart. The blue graticule is latitude and longitude. The grey graticule is the grid. The Blue dotted lines are isogonals. The grey ones are isogrivs.


Figure 27.15 A gridded chart
You can see from the above chart that the rate at which the isogrivs are changing is much the same as the rate at which the isogonals are changing, so it is no more difficult, in many parts of the world, to fly a Grid heading using a magnetic compass than it is to fly a True heading when using a magnetic compass.

## Steering by Gyro

However, one of the reasons that we may be using Grid is because the aircraft is at high latitudes. In this case, a magnetic compass may be unusable in some parts of the world because the route
is within the 6 microtesla zone, and the horizontal component of the Earth's magnetic field is too weak. In other parts, the rate of variation may be changing too rapidly to be usable.

In these cases, we would steer by gyro. The errors of a gyro are Real Drift (also known as Random Wander), Earth Rate, and Transport Wander.

## Real Drift

Real Drift in INS-quality gyros is of the order of one-hundredth of a degree per hour. However, if an INS were fitted, it would not normally be necessary to use Grid, because most INS are polar-programmed and can continue to operate in True up to very high latitudes. Nevertheless, if no INS is fitted, on a typical modern airline-quality Slaved Gyro Compass, the gyro drift-rate will still be only about one-tenth of a degree per hour when operating in DG mode. In this case, Real Drift is low enough to be insignificant.

## Earth Rate

Earth Rate occurs because the gyro is aligned with True North at some particular meridian. The aircraft then rotates with the Earth, along with the meridian, but the gyro continues to remain in alignment with the point in space where it was originally aligned. The correction factor is $15 \times$ sine latitude degrees / hour. Many modern Slaved Gyro Compasses have a Latitude Correction control, which is a knob turned by the crew to be set to the local latitude, and the correction is then fed to the gyro. The crew then keep resetting the latitude as it changes.

## Transport Wander

Transport Wander is another name for meridian convergence. The gyro is aligned with True North at some particular meridian. Then, as the aircraft travels either east or west, the gyro remains aligned to the original direction in space, but the direction of the meridians (and therefore the local direction of True North) changes. (See Figure 27.10, which also covers this case). To operate in True, a correction has to be calculated for this meridian convergence. The correction factor is change of longitude $\times$ sine mean latitude.

However, to operate in Grid, no correction is required. What is required is a direction that does not change with change of longitude - a grid. Therefore we use the gyro, but without any correction for Transport Wander (or meridian convergence).

Note: There is, in fact, a small error caused by the difference between Earth Convergence, which is the gyro correction, and Chart Convergence, which will depend on the projection and the Standard Parallels being used. A correction has to be applied, known as Residual Transport Wander. This is fed in either by slewing the gyro or as a fiddle-factor to the Latitude Control, but its use is beyond the scope of this course, and is not discussed further.

Therefore, to operate with a gyro in Grid, all that is needed is to align it to Grid North initially, then feed in a correction for Earth Rate. Real Drift will be small with a good gyro, and Transport Wander will be non-existent (or there may be a small amount of Residual Transport Wander to be corrected).

## Polar Grids

A gridded chart is invariably used in polar regions. In both the north and south polar areas the plotting charts are:

A Polar Stereographic or a *Transverse/*Oblique Mercator. (*Not covered in syllabus)
On each of these charts the $360^{\circ}$ of longitude on the Earth are represented by $360^{\circ}$ on the chart.

This property makes these charts ideally suited for the use of a standard polar grid which can be printed on the charts for all users.

The use of a standard grid eliminates any confusion over the alignment of those few VOR/TACAN stations which are not aligned with Magnetic or True North but with Grid North. In the Northern regions the datum meridian used is the Greenwich meridian. At any True meridian the chart convergency will be equal to the longitude with sign reversed.


Figure 27.16 Standard North Polar Grid

For example, at $45^{\circ} \mathrm{W}$ chart convergency $=45^{\circ} \mathrm{E}$.
At $45^{\circ} \mathrm{E}$ chart convergency $=45^{\circ} \mathrm{W}$.
Consider a route from A to B in Figure 27.16. The Grid Track is $090^{\circ}(\mathrm{T})$

```
At 45'` Track = 090
At 45'E Track = 090
```

In the Southern hemisphere the Greenwich meridian is again used as the datum meridian.
In the Southern hemisphere the chart convergency is always equal to the longitude when a standard grid is used.

To maintain the orientation with Grid North at the top, diagrams illustrating the standard south polar grid are normally drawn with the $180^{\circ} \mathrm{E} / \mathrm{W}$ meridian at the bottom of the diagram.

From Figure 27.17, Grid Track $A$ to $B=$ constant $070^{\circ}(\mathrm{G})$.

At A long. $=45^{\circ} \mathrm{W}$ so conv $=45^{\circ} \mathrm{W}$.
$070^{\circ}(\mathrm{G})+45^{\circ} \mathrm{W}$ conv $=115^{\circ}(\mathrm{T})$.
At point $B$ long. $=45^{\circ} \mathrm{E}$ so conv $=45^{\circ} \mathrm{E}$
$070^{\circ}(\mathrm{G})-45^{\circ} \mathrm{E}$ conv $=025^{\circ}(\mathrm{T})$.
At point $X$ track $=090^{\circ}(\mathrm{G})$
Conv $=180^{\circ} \mathrm{E} / \mathrm{W}$
$090^{\circ}(\mathrm{G})+/-180^{\circ} \mathrm{E} / \mathrm{W}=270^{\circ}(\mathrm{T})$.


Figure 27.17 Standard South Polar Grid

## Grid / True Conversion Problems

## Northern Hemisphere



Figure 27.18 North Polar Stereographic Projection
Figure 27.18 represents a North Polar Stereographic Projection. Therefore the ' $n$ ' factor (the relationship between the inclination of the meridians and the change of longitude) is 1.00 .

Aircraft No 1. Consider Aircraft No 1. Just by inspection of the diagram, you can see its Grid heading is $360^{\circ}(\mathrm{G})$ whilst the True heading is $270^{\circ}(\mathrm{T})$. Therefore the True heading is 90 degrees less than the Grid by simple visual inspection.

Now work it out using convergence. If Grid North is at the 12 o'clock position, then the datum meridian must be the Greenwich meridian - we now have a Standard Grid. Aircraft 1 is at longitude $090^{\circ} \mathrm{W}$, which means that the aircraft is west of the datum. From the following diagram:


Northern hemisphere, aircraft west of datum, therefore convergence east.

Convergence East, True Least
Therefore True heading should be 90 degree less than Grid - which it is. Both the convergence formula and the diagram give the same answer.

Now try the same exercise with Aircraft Nos. 2 to 5.

Here are the answers:

| Northern Hemisphere |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Aircraft | Longitude | Convergence | Grid Hdg | True <br> Hdg |
| 1 | 90 W | 90 E | 360 | 270 |
| 2 | 45 W | 45 E | 225 | 180 |
| 3 | 45 E | 45 W | 315 | 360 |
| 4 | 90 E | 90 W | 000 | 090 |
| 5 | 135 E | 135 W | 090 | 225 |

## Southern Hemisphere

This is more of a challenge.


Figure 27.19 South Polar Stereographic Projection
Aircraft No 1. Visual Inspection $-000^{\circ}(\mathrm{G}), 090^{\circ}(\mathrm{T})$ (remember, True North is away from the South Pole). Therefore True is 90 degrees greater than Grid.

Using convergence - If Grid North is at the 12 o'clock position, then the datum meridian must be the Greenwich meridian, i.e. we have a Standard Grid. Aircraft 1 is at longitude $090^{\circ} \mathrm{W}$, which means that the aircraft is west of the datum. From the following diagram:


Southern Hemisphere, aircraft west of datum, therefore convergence west.

Convergence West, True Best
Therefore True heading should be 90 degrees greater than Grid - which it is. Both the convergence formula and the diagram give the same answer.

Now try the same exercise with Aircraft Nos 2 to 5.
Here are the answers:

| Southern Hemisphere |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Aircraft | Longitude | Convergence | Grid Hdg | True <br> Hdg |
| 1 | 90 W | 90 W | 000 | 090 |
| 2 | 135 W | 135 W | 225 | 360 |
| 3 | 135 E | 135 E | 315 | 180 |
| 4 | 90 E | 90 E | 360 | 270 |
| 5 | 45 E | 45 E | 090 | 045 |

More Grid / True Conversion Problems

## Polar Stereographic Charts

## Example 1

Aircraft Position 45N 110W
Grid Track $=132^{\circ}(\mathrm{G})$
Datum $=060^{\circ} \mathrm{W}$
What is True Track?
Is the aircraft east or west of the datum meridian?
Is that east or west convergence?
Ans: From diagram below, Northern hemisphere,therefore east convergence


With a Polar Stereo Chart, ' n ' = 1
so 50 degree change of longitude $=50$ degrees convergence
Given: Convergence East, True Least, Grid Track is $132^{\circ}(\mathrm{G})$.
Therefore True $\operatorname{Track}=132^{\circ}(\mathrm{G})-50$ degrees $=082^{\circ}(\mathrm{T})$

Or you could use a diagram:


Figure 27.20 Example 1
Plot the aircraft position (45N 110W) - the blue dot on Figure 27.20.
Draw in the datum (from 060W to the North Pole, and continuing) - the red solid line. This gives you the direction of Grid North.

Draw in Grid North through the aircraft position (the dotted red line).
Draw in the direction of True North (the blue line).
Now compare the aircraft True track with the Grid track. You can see that the
True track is about $082^{\circ}(\mathrm{T})$ - which is the same answer as the arithmetical calculation.

## Example 2

Aircraft position 28 S 118 E .
True Track $=042^{\circ}(\mathrm{T})$
Grid Track $=133^{\circ}(\mathrm{G})$
What is the datum meridian?

What is the convergence?

Is the aircraft east or west of the datum?


Answer: Difference between Grid and True, Grid is greater. $133^{\circ}(\mathrm{G})-042^{\circ}(\mathrm{T})=91^{\circ}$ East
Answer: Southern hemisphere, look at the diagram - east of datum

If the aircraft is 91 degrees east of the datum and the aircraft is at $118^{\circ} \mathrm{E}$, where is the datum?
Answer: $118^{\circ} \mathrm{E}-91$ degrees $=027^{\circ} \mathrm{E}$ longitude.
Or you could use a diagram:


Figure 27.21 Example 2
Plot the aircraft position (28S 118E) - the blue dot on Figure 27.21.
Draw in the direction of True North (the blue line).
Draw in the True track of $042^{\circ}(\mathrm{T})$ (the green line).
The Grid Track is $133^{\circ}(\mathrm{G})$. This gives you the direction of the grid (the red dotted line).
Parallel the grid through the South Pole. This gives you the datum meridian.
(Note: These diagrams are only supposed to be sketches, to allow you to visualize the situation. They are not intended to be scale drawing solutions. But they allow you to see which way to apply the convergence, and then do a numerical calculation).

## Questions

1. A North Polar Stereographic chart is overprinted with a false grid aligned with the prime meridian. At position 80 N 135 E the grid track is $235^{\circ}$, will the true track be:
a. $\quad 010^{\circ}$
b. $\quad 100^{\circ}$
c. $\quad 190^{\circ}$
d. $280^{\circ}$
2. A South Polar Stereographic chart is overprinted with a false grid aligned with the prime meridian. At position 87S 123W the true track of an aircraft is $179^{\circ}$. What will the grid track be?
a. $056^{\circ}$
b. $\quad 123^{\circ}$
c. $\quad 237^{\circ}$
d. $302^{\circ}$
3. A South Polar Stereographic chart is overprinted with a false grid aligned with the anti-meridian of the prime meridian. At 78S 101E with a grid track of $180^{\circ}$ the true track will be:
a. $079^{\circ}$
b. $\quad 101^{\circ}$
c. $\quad 259^{\circ}$
d. $\quad 281^{\circ}$
4. A North Polar Stereographic chart is overprinted with a false grid. At 77N 37W the grid track is $175^{\circ}$ and the true track is $093^{\circ}$. At which longitude is the false grid aligned?
a. 045 W
b. 045 E
c. 082E
d. 119W
5. Using the data in the diagram, the false grid datum is $\qquad$ and we are in the. $\qquad$ hemisphere
a. $\quad 180^{\circ} \mathrm{E} / \mathrm{W}$, Northern
b. $180^{\circ} \mathrm{E} / \mathrm{W}$, Southern
c. $\quad 000^{\circ} \mathrm{E} / \mathrm{W}$, Northern
d. $\quad 000^{\circ} \mathrm{E} / \mathrm{W}$, Southern

6. A Southern Lambert's conformal conic chart is overprinted with a false grid aligned with the $180^{\circ} \mathrm{W}$ meridian. The constant of the cone is 0.843 . If the true track at $145^{\circ} \mathrm{E}$ is $333^{\circ}$, what will the grid track be?
a. $\quad 002.5^{\circ}$
b. $\quad 029.5^{\circ}$
c. $\quad 303.5^{\circ}$
d. $\quad 360^{\circ}$

## Answers

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a | a | c | b | d | c |

## Chapter <br> 28 <br> Plotting

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## Introduction

So far in your studies you have become aware of two methods of navigation. One is solo-pilot VFR navigation based on the 1 in 60 rule. The other is the use of automatic computing systems such as INS, IRS, FMS, GPS, and RNAV. However, there is a whole group of aircraft which are more sophisticated than light training aircraft but less sophisticated than the full jet airliner glass cockpit standard. These aircraft are routinely fitted with VOR, DME, ADF and Slaved Gyro Compasses.

For this sort of intermediate complexity aircraft, it becomes necessary to plot on a Radio Navigation chart or a specialist plotting chart, usually a Mercator's chart or a Lambert's Conical Orthomorphic chart. The EASA ATPL requirement is for the use of the Lambert's Conical Orthomorphic and occasionally the Polar Stereographic. Techniques for these charts, but not Mercator's, are given in these notes.

The charts used for most of the questions are the Jeppesen $E(L O) 1 \& 2$ and the $E(L O) 1 A$. Students sitting the EASA exams are to bring their own copy of each chart because the CAA will not provide one at the exam venues.

## Equipment

The following items of equipment are required for this section of the course:-

- A navigational computer
- A navigational straight-edge
- Plotting protractor (square type)
- Compass/Dividers
- 2H and HB pencils for plotting and writing, and a soft eraser


## Definitions, Abbreviations, Symbols

## True or Magnetic?

Plotting is normally carried out using true north as the datum. It is possible to plot some types of radio bearing directly in magnetic without converting to true, but usually true tracks and bearings are used.

## Heading

The direction in which the nose of the aircraft is pointing measured in degrees (000-360) clockwise from the true, magnetic, or compass north, abbreviations being $\operatorname{Hdg}(\mathrm{T}), \operatorname{Hdg}(\mathrm{M})$, Hdg(C).

- If $\mathrm{Hdg}(\mathrm{M})$ is given, it must be converted to true by taking into account the variation on the chart.
- If $\mathrm{Hdg}(\mathrm{C})$ is given, it must be converted to true by taking into account variation and deviation.

Fix. The ground position of the aircraft obtained from two or three visual or radio position lines, or by radar.

Position Line. A line somewhere along which the aircraft is known to be at a particular time.

QTE. The true bearing of the aircraft from a DF station.
QDM. The track (magnetic) from the aircraft to the DF station.
QDR. The bearing (magnetic) of the aircraft from the DF station.
VOR Radial. Sometimes referred to as VOR QDR. The magnetic bearing of the aircraft from a VOR beacon.

VOR QDM. The magnetic track from the aircraft to a VOR beacon.
Isogonal. A line on the chart joining places of equal magnetic variation.
Convergency. The angle between two selected meridians at a given latitude or latitudes.

## Symbols

The standard symbols used in plotting are shown below, with their meanings. You will find it a great help to your plotting if you always use these symbols and always record the appropriate time of each occurrence.
$\stackrel{1115}{\longleftrightarrow}$ Position line at 1115 UTC. Plotted as a true bearing


- 1510 Radar fix at 1510 UTC


DF station, VOR, NDB or VDF facility
The simplest form of plot is obtained by plotting the positions of two fixes. Provided that the aircraft has been flying a single heading during the interval between the fixes, a straight line joining them represents the track.


Figure 28.1 Track Made Good
A track of this sort determined between two fixes between which one heading only has been flown is called a Track Made Good (TMG).

This track made good is easily measured on the chart in the usual way. The distance measured along the TMG gives a method of calculating the actual ground speed. In Figure 28.1, since the distance flown in 30 minutes is 100 NM the actual ground speed must be 200 kt .

## Fixing Position

If an aircraft is to be navigated safely from one position to another, it is necessary to make frequent checks on its ground position, and to make any required correction.

A 'fix' is a position defined from radio aids whilst a 'pinpoint' is a position found by mapreading.

Some fixes give a position directly (for instance a GPS fix or a fix from an ATC ground radar) whilst others are made from a combination of position lines. A position line is a line, somewhere along which an aircraft is known to be at a particular time.

Examples of position lines which are obtainable visually are long straight stretches of railways, roads, rivers or coastlines. The position and direction of line can then be transferred to the plotting chart.

Position lines can be visual, radio or radar. These can be plotted on charts and represent lines, somewhere along which the aircraft is known to be at the time the position line was obtained. Any two position lines obtained at the same time will give a fix, for, as the aircraft is known to be somewhere along each, it must be at the point where they cut each other.

The plotting of visual position lines is quite straightforward. They are simply drawn in on the plotting chart in the same position and direction as on the topographical map. The plotting of radio position lines is not quite so straightforward and needs further explanation.

## Bearings Measured by a Ground Station

It must be remembered that radio waves follow great circle paths and that all directions measured are great circle bearings.

Bearings measured by ground stations are referred to by groups of code letters beginning with the letter Q. These are taken from the old ' Q code', most of which is now obsolete.

QTE The true bearing of an aircraft from a ground station.
QDM The magnetic track from an aircraft to a ground station.
QDR The magnetic bearing from a ground station to an aircraft. (QDR is the reciprocal of QDM).

QTEs are obtainable by an aircraft when a ground station measures the direction in ${ }^{\circ}(\mathrm{T})$ from which a radio signal sent out by the aircraft, approaches it. The direction so measured is passed to the aircraft by RT.

QDMs and QDRs are obtained in two ways:-
VDF An aircraft sends out a radio signal and a ground station measures a QTE. The ground station converts the QTE into a QDR by applying variation at the ground station where the measuring is done. Then by adding or subtracting $180^{\circ}$ to change it into the form of a direction from the aircraft to the ground station, a QDM is obtained.

| True bearing of aircraft measured by ground station (QTE) | $140^{\circ}(\mathrm{T})$ |
| :--- | :--- |
| Variation of ground station | $\underline{10^{\circ} \mathrm{W}}$ |
| Magnetic bearing of aircraft from ground station (QDR) | $150^{\circ}(\mathrm{M})$ |
| Add | $\underline{180^{\circ}}$ |
| Magnetic track of ground station from aircraft (QDM) | $330^{\circ}(\mathrm{M})$ |

VOR A QDM or QDR, depending on which is selected by the operator in the aircraft, is displayed on an indicator in the aircraft.

With VDF, it is obvious that the measuring is done at the ground station.
However, with VOR, the bearing received by the aircraft is also determined at the ground station. Since the bearings are measured at the ground station, the variation of the ground station is used to convert them to magnetic direction.

## Plotting of VOR Bearings

QDMs and QDRs are magnetic bearings. If the chart has a magnetic north arrow, they can be plotted from the magnetic datum. Otherwise, variation must be applied and they must be plotted as QTEs (true bearing of aircraft from ground station).

## Plotting QDMs and QDRs

With QDMs, apply the reciprocal to convert the QDM to a radial. There now 3 ways to plot it.
You could align the protractor with the magnetic north arrow, if there is one. The problem is that these tend to be very short (usually about $11 / 2$ centimetres) and it is difficult to align the protractor accurately.

A better method is to find the variation from the isogonals, then use your protractor to plot the variation at the VOR using the nearest meridian as true north. You now have a magnetic north arrow, but a much more accurate one. You can now plot the magnetic radial directly without needing to arithmetically apply variation.

However, if the VOR is on an airway, or better still, is used to define several airways, then the airway centre lines will be defined by printed radials on the chart. Take, for instance, Shannon (SHA 5243.3N 00853.1W) on the $\mathrm{E}(\mathrm{LO}) 1$ chart. There are radials of $052(\mathrm{M}), 071(\mathrm{M}), 080(\mathrm{M})$, $115(\mathrm{M})$, and $173(\mathrm{M})$. Put the centre of your protractor over the VOR symbol and then turn it until the 052 on its scale goes through the 052 radial, the 071 goes through the 071 radial, and so on. You have now aligned the protractor with the same magnetic north that Jeppesen have used and you can now plot a magnetic radial directly.

Alternatively, you can convert the magnetic radial to a true bearing by applying variation arithmetically and then plot by reference to the nearest meridian.

## Bearings Measured by Aircraft - ADF or Airborne Weather Radar (AWR)

There are 2 sources of bearings measured at the aircraft. One is ADF and the other is Airborne Weather Radar (AWR). ADF is the more common and familiar method, but it is possible to use AWR in the MAP mode. In this case the normal pencil beam becomes a cosecant ${ }^{2}$ beam which gives a better performance as a map-painting radar and ranges and bearings can be obtained from suitable ground returns, particularly headlands and islands.

Since the bearings are measured relative to the fore-and-aft axis of the aircraft, it is necessary to convert them into true directions by use of the aircraft's compass. The compass is affected by the local variation at the aircraft. Therefore AIRCRAFT variation is used.

ADF. The measurement of the bearing of the ground transmitter from the aircraft is determined by the aircraft. The bearing measured by the aircraft equipment is normally shown as either a relative bearing or a magnetic bearing.


Figure 28.2 True and relative bearings

## Relative Bearing

This is a bearing measured not from the direction of true or magnetic north, but from the direction in which the aircraft is pointing. For example, if an aircraft is heading in the direction of $030^{\circ}(\mathrm{T})$ and there is a NDB due east of the aircraft, the bearing of the NDB from the aircraft is $060^{\circ}$ (relative). This bearing would appear on a relative bearing indication (RBI) in the aircraft.

It can be seen from Figure 28.2 that adding together the relative bearing and the true heading will give the true bearing.

## Magnetic Bearing

This is a bearing measured from the direction of magnetic north, and such a bearing would appear on a radio magnetic indicator (RMI) in the aircraft. To change this into a true bearing, all that is necessary is the application of variation at the aircraft's position, where the measuring is being done.

## AWR

An AWR bearing is a relative bearing from the aircraft. This is combined with true heading of the aircraft to give a true bearing TO the response, as above. This is then reciprocated to give the true bearing to plot FROM the response.

## Converting a Relative Bearing into a True Bearing

AWR bearings are always in the form of relative bearings. ADF bearings will also be relative if the presentation is on an RBI. It is necessary to convert these into true bearings which can then be plotted from the radar response or the NDB, as appropriate.

Apply AIRCRAFT variation to the magnetic heading to get true heading.
Add relative bearing to true heading in order to get true bearing TO the NDB / radar response. If the answer comes to more than $360^{\circ}(\mathrm{T})$, subtract 360 .

Take the reciprocal in order to get the True Bearing to plot FROM the NDB / radar response.

## Example

RBI reading 207
Aircraft heading $315^{\circ}(\mathrm{M})$
NDB variation 7W
Aircraft variation 9W
What is the true bearing to plot from the NDB?

| Mag Heading | $315^{\circ}(\mathrm{M})$ |  |
| :--- | :--- | :---: |
| Aircraft Variation | $\frac{9 \mathrm{~W}}{}$ |  |
| True Heading | $306^{\circ}(\mathrm{T})$ |  |
| Relative Bearing | $\frac{207^{\circ}(\mathrm{R})}{513^{\circ}(\mathrm{T})}$ (TO the NDB) |  |
| True Bearing | $\frac{-360}{153^{\circ}(\mathrm{T}) \text { (TO the NDB) }}$ |  |
| True Bearing |  |  |
| Take the Reciprocal | $333^{\circ}(\mathrm{T})$ (FROM the NDB) |  |

## Distance Measuring Equipment (DME)

DME indicates slant range from the ground DME facility to the aircraft. Strictly, this should be converted to plan range before plotting but, unless the height is given in the question, it is not possible to do so. If the range is more than about 10 NM the difference is insignificant unless the aircraft is at high altitude.

Often the question will simply require you to combine a VOR bearing with a DME range to arrive at a fix, and this is straightforward. However, sometimes the question is to derive position from 2 DME ranges. This creates a problem of ambiguity, because there will be 2 positions where the ranges intersect.


In a real situation you would probably be able to work out which was the correct intersection from a knowledge of your approximate position.

However, the exam question will normally tell you your heading and whether the ranges are increasing or decreasing.

If you are told that the aircraft is heading $270^{\circ}(\mathrm{T})$ and the ranges are decreasing then you must be in position $B$, because if you were at $A$, the ranges would be
 increasing.

## The 'Cocked Hat'

The intersection of three position lines usually forms a triangle - called a 'cocked hat' - and the fix is then taken to be at the point where the bisectors of the angles of the triangle would meet. This is shown in Figure 28.3.

The size of the cocked hat is an indication of the probable accuracy of your fix.


Figure 28.3 The Cocked Hat

## Climb and Descent

Climb and descent are not really plotting problems, but it is convenient to group them together. They can come as 2 types of question, but they are essentially the same problem - to make the time taken in a vertical direction in the climb or descent the same as the time taken in the horizontal direction. The vertical distance divided by the rate of climb / descent must be the same as the horizontal distance divided by the ground speed.

## Example 1

At 65 NM from a VOR you commence a descent from FL330 in order to arrive over the VOR at FL100. Your mean ground speed in the descent is 240 knots. What rate of descent is required?

| a | 1420 feet/min |
| :--- | :--- |
| b | 1630 feet/ $/ \mathrm{min}$ |
| c | 1270 feet/min |
| d | 1830 feet/ min |

Answer: 65 NM @ 240 knots will take 16.25 minutes. You need to lose 23000 feet, also in 16.25 minutes. The required ROD is $23000 / 16.25=1415$ feet per minute.

Answer (a)

## Example 2

You are homing to overhead a VORTAC and will descend from 7500 QNH to be 1000 AMSL by 6 NM DME. Your ground speed is 156 knots and the ROD will be 800 feet/min. At what range from the VORTAC do you commence the descent?

| a | 27.1 NM |
| :--- | :--- |
| b | 15.8 NM |
| c | 11.7 NM |
| d | 30.2 NM |

Answer: You need to lose 6500 feet @ 800 fpm ROD. 6500/800 $=8.125$ minutes in the descent.
156 knots for 8.125 minutes will cover 21.1 NM. You need to be down at 1000 feet 6 NM before the DME, so range to start the descent is 27.1 NM .

## Answer (a)

## Lambert and Polar Stereo Charts - Effects of Convergence

For all practical purposes, a straight line on a Lambert Conical Orthomorphic (conformal) chart or a Polar Stereographic chart at high latitudes can be considered to be a great circle. However, the type of plotting with which the student is concerned deals with aircraft which fly rhumb line tracks.

## Measuring Tracks

If an aircraft is to fly from A to B, the two positions are joined by a straight line (great circle) and its direction is measured as the mid-meridian to give the rhumb line track on the 1:1000000 scale chart.

## Plotting Headings and Tracks

If a known heading or track has to be plotted, this is done using the nearest meridian to the position from which it has to be drawn. Any subsequent error is ignored.

## Measuring Distances

If, on a particular chart, scale cannot be considered constant, distances must be measured using the latitude scale locally.

## Plotting Positions

This takes a little longer than on a Mercator chart, because the Lambert graticule is not rectangular. The procedure is as follows:-

Mark the longitude of the position to be plotted in the longitude scale to the north and also on the one to the south of the position.

Lay a straight edge between the two points.
Using a pair of dividers, plot the latitude along the length of the straight edge, upwards or downwards from the nearest parallel of latitude.

## Plotting of ADF Bearings

If the ch.long is $2^{\circ}$ or more, draw a line parallel to the aircraft's meridian through the NDB and plot as shown in Figure 28.4. If ch. long is less than $2^{\circ}$, plot from the meridian of NDB. This is explained in more detail in the next paragraph.

## Plotting on a Lambert Chart

Plotting bearings on a Lambert chart is less complicated than on a Mercator because great circles (radio waves) are straight or near-straight lines, and so there is no conversion angle to apply. However, there may be a correction for chart convergence to apply. It depends on whether the bearing is measured at the ground station or at the aircraft.

The bearing measurement process for VDF (QDMs and QTEs) and for VOR takes place at the ground station. It may appear with VOR that the bearing is displayed in the aircraft and has been decoded in the aircraft, but it has not been measured in the aircraft. The different between the VOR reference phase and the bearing phase is encoded at the moment of transmission on that particular bearing at the ground station. All that the aircraft VOR receiver does is de-modulate the encoded signal and display it. It does not actually measure a bearing, unlike ADF.

For bearings measured at the ground station (VDF or VOR), the plotting process is simple. You simply correct for variation at the point where it has been added (the ground station), and plot the true bearing from the ground station. This will be the correct great circle track at the ground station. The straight-line bearing will change direction with respect to north as it crosses meridians and encounters convergence and therefore will not be the same great circle track direction at the aircraft, but it is not being plotted from the aircraft, so this does not matter. It is the correct radio wave path.

For bearings measured at the aircraft (ADF / NDB and AWR (airborne weather radar)), the situation is complicated because there is convergence between the meridian where the bearing is measured (the aircraft) and from where it will be plotted (the NDB). Consider the situation at Figure 28.4. An aircraft measures a bearing of an NDB of $245^{\circ}(\mathrm{R})$ while on a heading of $025^{\circ}(\mathrm{T})$. It is required to plot the position line on a Lambert chart.

Adding together the relative bearing of $245^{\circ}$ and the true heading of $025^{\circ}$ gives the true great circle bearing of the NDB from the aircraft, $270^{\circ}$.

But if the reciprocal, $090^{\circ}$, is plotted from the meridian of the NDB, the position line will not pass through the aircraft position, as the meridian through the aircraft is not parallel to that through the NDB. See Figure 28.4(a).


Figure 28.4 Plotting an ADF bearing

If, however, the angle of $090^{\circ}$ is measured from a line parallel to the aircraft's meridian drawn through the NDB, the position line will pass through the aircraft as shown in Figure 28.4(b).

Chart convergence has been automatically allowed for by drawing a line parallel to the aircraft's meridian through the NDB. This method is always used in plotting.

If it is necessary to obtain the bearing to plot from the meridian of the NDB, reference to Figure $28.4(b)$ will show that in this case it is equal to $090^{\circ}$ minus chart convergence.

## Example

An aircraft flying on a heading of $330^{\circ}(T)$ measures a bearing of $090^{\circ}(R)$ of an NDB.
What is the bearing to plot on a Lambert chart: -
a. from a line parallel to the aircraft's meridian drawn through the NDB?
b. from the meridian of the NDB. Chart convergence is $3^{\circ}$ ?

## Solution

a. $\quad 330^{\circ}+090^{\circ}(R)=060^{\circ}(T)$ (True great circle bearing of NDB from aircraft) Plot $240^{\circ}$ from a line parallel to the aircraft's meridian drawn through the NDB. See Figure 28.5(a).
b. Bearing to plot from meridian of NDB $=240^{\circ}+3^{\circ}=243^{\circ}$


Figure 28.5 Plotting an ADF bearing
In practice, we would not normally use method (b), plotting from the true meridian. For practical plotting, we draw in a parallel false meridian, as in method (a).

Note: In the case of bearings measured at the aircraft, it is the aircraft's compass which is used to add to relative bearing in order to obtain true bearing. Therefore, if the heading is magnetic and it is necessary to correct it to true, we always use variation at the aircraft.

Bearings measured at ground station (VOR, VDF) - variation at ground station
Bearings measured at aircraft (NDB/ADF, AWR) - variation at aircraft

## Questions

1. What is the average magnetic course and distance between 6000N 02000W and Sumburgh VOR? (in the exam they gave an attached chart as an Annex - for revision practice use your Jeppesen Manual chart AT(H/L)1 or 5 AT(HI))

|  | Course | Distance |
| :--- | :--- | :--- |
| a. | 095 | 562 |
| b. | 095 | 468 |
| c. | 105 | 562 |
| d. | 105 | 468 |

2. What is the average true track and distance between WTD NDB (5211.3N 00705.0W) and FOY NDB (5234.0N 00911.7W)? - use your Jeppesen E(LO)1

|  | Track | Distance |
| :--- | :--- | :--- |
| a. | 294 | 76 |
| b. | 286 | 76 |
| c. | 294 | 81 |
| d. | 286 | 81 |

3. You are on a heading of $105^{\circ}(\mathrm{C})$, deviation $3^{\circ} \mathrm{E}$. WTD NDB (5211.3N 00705.0W) bears $013^{\circ}(\mathrm{R})$, CRK VOR ( 5150.4 N 00829.7 W ) QDM is $211^{\circ}$. What is your position? (Use Jeppesen E(LO)1)
a. $\quad 5245 \mathrm{~N} 00757 \mathrm{~W}$
b. $\quad 5228 \mathrm{~N} 00802 \mathrm{~W}$
c. $\quad 5412 \mathrm{~N} 00639 \mathrm{~W}$
d. $\quad 5217 \mathrm{~N} 00745 \mathrm{~W}$
4. The airport at 5211N 00932W is: (use Jeppesen $E(L O) 1)$
a. Kerry
b. Cork
c. Shannon
d. Waterford
5. Reference Jeppesen $\mathrm{E}(\mathrm{LO}) 1$, position 5211 N 00931 W , which of the following denotes all the symbols?
a. military airport, ILS, NDB
b. civil airport, VOR, ILS
c. military airport, VOR, ILS
d. civil airport, ILS, NDB
6. What is the aircraft position in lat and long given the following (use Jeppesen chart E(LO)1):
CRN (5318N 00857W) 18 DME, SHA (5243N 00853W) 20 DME, Heading $270^{\circ}(\mathrm{M})$
Both ranges DME decreasing
a. $\quad 5201 \mathrm{~N} 00908 \mathrm{~W}$
b. $\quad 5301 \mathrm{~N} 00908 \mathrm{~W}$
c. $\quad 5302 \mathrm{~N} 00843 \mathrm{~W}$
d. $\quad 5203 \mathrm{~N} 00843 \mathrm{~W}$

Chart For Questions 11-15.

7. What is at 5211 N 00932 W - - (use $\mathrm{E}(\mathrm{LO}) 1$ )
a Kerry VOR
b Cork Airport
c Waterford NDB
d Kerry Airport
8. What is the mean true track and distance from the BAL VOR ( 5318 N 00627 W ) to CFN NDB (5502N 00820W)? - (use E(LO)1)

|  | $328^{\circ}$ | 125 NM |
| :--- | :--- | :--- |
| a |  | $148^{\circ}$ |
| b | 125 NM |  |
| c | $328^{\circ}$ | 134 NM |
| d | $148^{\circ}$ | 134 NM |

9. You are at position 5340N 00800W. What is the QDR from the SHA VOR (5243N 00853W)? - (use E(LO)1)

| a | 217 |
| :--- | :--- |
| b | 037 |
| c | 209 |
| d | 029 |

10. Your radial from the SHA VOR ( 5243 N 00853 W ) is $120^{\circ}(\mathrm{M})$. From the CRK VOR ( 5151 N 00830 W ), the radial is $033^{\circ}(\mathrm{M})$. What is your position? - (use $\mathrm{E}(\mathrm{LO}) 1$ )
```
a 5320N 00800W
b 5240N 00821W
c 5220N 00821W
d 5230N 00800W
```

For the following questions use the chart opposite. This is for practice only because the chart is not used in the EASA exams.
11. QTE ' H ' $326^{\circ}(\mathrm{T})$

QTE 'G' $\quad 101^{\circ}(\mathrm{T})$
Give the latitude and longitude of where the $\mathbf{2}$ position lines cross.
12. QTE ' $G$ ' $140^{\circ}(\mathrm{T})$

QTE 'H' $\quad 270^{\circ}$ (T)
Give the latitude and longitude of where the $\mathbf{2}$ position lines cross.
13. DME 'L' 150 NM

DME 'M' 150 NM
Give the latitude and longitude of where the $\mathbf{2}$ position lines cross.
14. VOR 'I'
$\begin{array}{ll}\text { QDR } & 339^{\circ}(\mathrm{M}) \\ \text { DME } & 120 \text { NM }\end{array}$
DME 'I' 120 NM
Give the latitude and longitude of where the 2 position lines cross.
Do not forget to apply variation to the VOR.
15. Your DR position is 64 N 004 W .

RMI QDM NDB 'J' (in Norway) $\mathbf{1 2 0}^{\circ}$ (M) (Meridian Transfer)
QTE 'H' $042^{\circ}(\mathrm{T})$
Give the latitude and longitude of where the 2 position lines cross.

## Answers

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| c | d | b | a | d | c | d | a | b | d |



## Chapter

## 29

## The Direct Indicating Compass

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## The Magnetic Compass

A compass is an instrument designed to indicate direction on the surface of the Earth, relative to some known datum. The magnetic compass uses the horizontal component of the Earth's field as its directional datum. Unfortunately, the Earth's field is normally not aligned with the true meridian - the most desirable datum from which to measure direction. The angular difference between true and magnetic meridians is called the magnetic variation discussed in chapter 3.

The purpose of a magnetic 'steering' compass in an aircraft is to indicate heading, the direction in which the aircraft is pointing.

Magnetic influences - iron/steel components, electric currents - distort the Earth's field so that the compass magnet assembly deviates from the magnetic meridian. This is called compass deviation.

The rules for applying variation and deviation to the compass heading indication in order to determine true heading are detailed in the Navigation notes.

## Direct Indicating Magnetic Compass

This chapter deals with the direct indicating or direct reading magnetic compass, where the pilot directly reads his heading in relation to the pivoted magnet assembly.

There are two basic types of direct reading magnetic compasses used in aircraft, the vertical card and, less commonly, the grid ring compass.


Figure 29.1 A vertical card compass

## The Vertical Card Compass

The vertical card compass - which is also known as the B-type or E-type - is the direct reading compass in general use. It is usually the main magnetic heading reference in light aircraft and the standby compass in larger aircraft.

It consists of a circular compass card attached directly to the magnet assembly. This combined unit is suspended in liquid within the compass bowl. A vertical lubber line on the glass window of the bowl enables the heading to be read off the compass card.

## The Grid Ring Compass

The P-type compass or grid ring compass is found on older aircraft. It is more accurate than the vertical card compass and is more stable.

It is, however, heavier, bulkier and more expensive. In addition it can only be read in straight and level flight, as the grid ring has to be unclamped and aligned with the north reference before a reading can be taken against the lubber line.

The grid ring compass also differs from the vertical card compass in that it achieves a greater periodicity by the addition of damping wires which also rotate through the compass liquid.


Figure 29.2 A grid ring compass

## Compass Requirements

The direct reading magnetic compass contains a pivoted magnet which must be able to align itself, and remain aligned, with the horizontal component of the Earth's magnetic field. For the compass to succeed, certain requirements must be satisfied. The most important of these are that the magnet system must be:

- Horizontal
- Sensitive
- Aperiodic


## Horizontality

In order to measure direction in the horizontal, the magnets must lie as nearly as possible in the horizontal plane during normal straight and level flight. A freely suspended magnet assembly would align itself with the Earth's total field so the magnets would only be horizontal at the magnetic equator.

To achieve horizontality, the magnet assembly is 'pendulously suspended', the centre of gravity of this assembly being lower than its supporting pivot, as shown in Figure 29.3.


Figure 29.3 Equilibrium (Northern hemisphere) viewed from west
In this way, the tilting effect caused by the vertical component $Z$ of the Earth's field is opposed by the weight of the magnet assembly, this equilibrium being achieved at the cost of only a very slight residual tilt of the magnets (North-seeking ends down) - by about $2^{\circ}$ in mid-latitudes - in the Northern hemisphere. (South-seeking ends down in the Southern hemisphere). Figure 29.3. shows the two turning couples involved.

One is produced by Z which exerts a downward force on the red (North-seeking) end of the compass magnet and an upward force on the blue end.

The other couple is produced by the weight W acting downwards through the centre of gravity (displaced because of the tilt) and the reaction $R$ acting upwards through the pivot.

For equilibrium, the magnet takes up the amount of tilt necessary to make the couples balance. (A third - very weak - couple produced by the horizontal component, H, of the Earth's field, opposing the tilt has been omitted for simplicity).

## Sensitivity

The magnet system is required to seek the horizontal component H of the Earth's field in all areas except near the magnetic poles - where the horizontal component is inadequate.

The notes on magnetism show that the ability of a pivoted magnet to align itself with an external field - its sensitivity - depends on the strength of the external field and on the magnetic moment of the magnet. The weak external field $(\mathrm{H})$ at a place cannot be changed, but the
magnetic moment of the magnet can be increased - by increasing the magnet's length and/or pole strength.

It is, however, undesirable to increase the magnet length so the pole strength is increased by using two, four or six short magnets or a circular magnet, made of an alloy which will accept and retain the high degree of magnetism required.

Sensitivity is further increased by reducing friction. This is achieved in three ways:

- By using an iridium-tipped pivot in a jewelled cup
- By lubricating the pivot with the liquid which fills the compass bowl.
- By reducing the effective weight of the magnet assembly acting down through the pivot, because the liquid that the magnet assembly is displacing is denser than air .


## Aperiodicity

The magnetic assembly is required to be aperiodic or 'dead beat', which means that it should settle down quickly on a steady indication after being displaced by turbulence or manoeuvres.

Any tendency to oscillate must be quickly 'damped out'. The desired aperiodicity is achieved as follows:

- Several short magnets are used instead of one longer one. This keeps the mass of the assembly near the centre, so reducing the moment of inertia and consequently making any oscillations easier to damp out. Light alloy is utilized wherever possible in order to minimize the weight of the assembly framework.
- The primary purpose of the liquid in the compass bowl is to act as a damping liquid on the compass assembly. The grid ring compass dampens oscillations more rapidly than the vertical card compass, due to the addition of damping wires. These wires are attached to the magnet assembly and also pass through the damping liquid. (See Figure 29.2.)


## Serviceability Checks

Compass body. Check that there is no obvious damage such as dents or cracks. Any lighting system should be checked, as should the efficacy of the luminous paint.

Compass Liquid. The compass liquid should be checked and be free from:

- Sediment and discolouration - either of which would indicate corrosion which would result in increased pivot friction.
- Bubbles - which would probably indicate a leaking seal. Turbulence and manoeuvres would cause any bubbles to move about, creating eddies which could disturb the magnet system.

Accuracy Limit - CS-OPS1: $\pm 10^{\circ}$

## Deviation

Deviation is produced by the iron/steel components in the aircraft. It is the angle between the local magnetic meridian and the direction in which the compass magnets are lying.

Deviation is named Easterly (or plus) if the North-seeking (red) ends of the magnets point to the East of magnetic North. If the North-seeking ends points to the West of magnetic North, deviation is said to be Westerly (or minus).

Deviation varies with heading so it has to be measured on a series of different headings. This is usually done by conducting a compass swing (which is fully covered in the chapter on aircraft magnetism). Once deviation has been reduced as far as possible, the residual deviation is recorded on a compass deviation card, which is located in the aircraft.

During the swing, normal flying conditions should be simulated as far as possible, with engines running, electrical / radio services switched on, and the aircraft in a level flight attitude.

It is obviously most important that no ferromagnetic objects such as tools, or watches should be placed near the compass as this would introduce unknown amounts of deviation. Furthermore, ferromagnetic payloads should be stowed as far away from the compass as permissible within the loading limits. With exceptionally large ferromagnetic loads, a compass swing may have to be carried out before flight with the load aboard.

## Chapter <br> 30

## Aircraft Magnetism

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## Deviation

The compass needle would accurately define the magnetic meridian were it not for the aircraft's own internal magnetism deflecting it. Deviation is the angular difference measured between the direction taken up by a compass needle and the magnetic meridian. Deviation is named easterly or westerly depending on whether the North seeking end of the compass needle lies to the East or West of the magnetic meridian.

|  | Compass <br> Heading | Deviation | Magnetic <br> Heading |
| :---: | :---: | :---: | :---: |
| Deviation West <br> Compass Best | 095 | -5 | 090 |
| Deviation East <br> Compass Least | 090 | +5 | 095 |



Figure 30.1

## Compass Swing

The basic method of determining deviation is to compare the aircraft's heading compass reading with magnetic heading as defined by a high quality 'land or datum' compass. This comparison of aircraft compass and magnetic datum readings is carried out in an area selected specifically for this purpose.

Therefore the aims of a compass swing are as follows:

- To observe / determine the deviations / differences between Magnetic North (observed on a landing compass) and Compass North (observed in the aircraft) on a series of headings
- To correct / remove as much deviation as possible
- To record the residual deviation which is left after the compass has been adjusted

The magnetic deviation observed during a compass swing can be said to be derived from Hard Iron and Soft Iron magnetism and this total field can in turn, for our purposes, be later resolved into two further combined components (coefficients B and C).

## Hard Iron Magnetism

The total force at the compass position produced by permanent hard iron magnetism can be resolved into three components. These components will be fixed for a given aircraft and will not change with change of heading.


Figure 30.2

## Soft Iron Magnetism

Soft iron magnetism is induced in parts of the aircraft structure by surrounding fields - the most important of these being the Earth. The Earth's field has a vertical as well as horizontal component. However, again for our purposes we will within the constraints of the syllabus only consider vertical soft iron (VSI) magnetism ( $Z$ is the vertical component of the Earth's field and H is the horizontal component). The component Z has an increasing effect with latitude as the compass magnets try to follow the Earth's flux lines; therefore, VSI magnetism must also vary with latitude. However, Z is zero at the Equator, where the horizontal component H is greatest, so no VSI magnetism is induced there.

When we examine the effective positioning of the imaginary magnets found when completing a compass swing we must remember that we use a real system (the compass) to give us aircraft heading and that this readout is affected by these magnetic forces which we have gone to some trouble to discover.

We can see from Figure 30.3 that the positioning can


Figure 30.3 the compass swing which can cater for any positioning as long as we follow the basic rules.

For example we may examine the case where the effect of the blue pole is said to be in the nose or forward of the aircraft compass.

Heading North the isolated Blue pole is in the same horizontal direction as the Earth's blue pole and so the needle is not deviated. The directive force or alignment of the Earth's field is being augmented by the blue pole, effectively they are pulling together.

As the aircraft now turns right on to $045^{\circ}$ deviation begins to take place and as we can see from the table by $090^{\circ}$ this has become maximum and then starts to become less as we approach $180^{\circ}$.

$\mathrm{Hdg} 000^{\circ} \mathrm{C}$


Hdg $135^{\circ} \mathrm{C}$

Remember that the blue pole represents a magnetic force which on this heading acts along the same line but in opposition to the stronger Earth's field.

On the remaining headings $180^{\circ}$ to $360^{\circ}$ the effects of the blue pole in the nose are as expected i.e. the red end of the compass needle is being attracted to the West of Magnetic North giving the maximum westerly deviation on $270^{\circ}$.



If the deviations caused by the blue pole in the nose are plotted against compass heading, a positive sine curve is obtained. Had the blue pole been aft of the compass a negative sine curve would have been obtained. This would mean that on a heading of $090^{\circ}$ the deviation would reach a maximum westerly value instead of a maximum easterly value. The changes in directive force would also be revised, the maximum occurring on $180^{\circ}$ and the minimum on $360^{\circ}$.

Figure 30.4

| Heading ${ }^{\circ}$ (C) | Deviation | Directive Force |
| :---: | :---: | :---: |
| 000 | Zero | Maximum |
| 045 | East + some | More than Earth's |
| 090 | East + max | Earth's approximately |
| 135 | East + some | Less than Earth's |
| 180 | Zero | Minimum |
| 225 | West - some | Less than Earth's |
| 270 | West - max | Earth's approximately |
| 315 | West - some | More than Earth's |
| 000 | Zero | Maximum |

What we have examined here is known for compass swinging as Coefficient B which we could view as that component which is resolved along the body of the aircraft. The forces resolved follow a simple sine curve which in our case here would be 'positive' although negative curves occur just as frequently.


Figure 30.5


Figure 30.6

Equally we should be able to see that if a further magnetic source is resolved to the right wing, we would achieve a positive cosine curve along the same lines and this is more usually described as Coefficient C .

The combination of Coefficients A (a mechanical function yet to be discussed) + B + C are resolved during the compass swing and to some extent can be removed by adjustment but other factors are at work here and will probably leave us with some errors at the end.

## Correction of Coefficients

The principle for correcting coefficients is the same for any system and can be summed up as follows:

Coefficient A - a mechanical problem of a displaced lubber line corrected by loosening the bolts holding the compass body or in the case of the RIMC the detector unit and carefully turning it until the correct heading is in place.

Coefficient B - correction required because of magnetic deviating forces acting upon the DRMC or the detector unit and giving errors known as deviation. Firstly calculate the error to be removed or more correctly the heading you wish to make the compass read and this will be done on an Easterly or Westerly heading.

Coefficient C - correction required because of magnetic deviating forces acting upon the DRMC or the detector unit and giving errors known as deviation. Firstly calculate the error to be removed or more correctly the heading you wish to make the compass read and this will be done on a Northerly or Southerly heading.

We can see that the correction for B and C are very similar but that we must remember to apply the sign of the correction properly to ensure an accurate correction to our compass system. When the compass swing is completed we of course have to check our work and this 'check swing' is carried out using eight or perhaps twelve points of the compass to allow us to derive a compass card that will be placed in the aircraft. This compass card indicates to us the residual deviations that we have been unable to resolve within the essentially horizontal procedure. Alternatively, the residual deviations affecting the compass after the completion of a compass swing may be shown by the use of a graphical table or a curve constructed from the information obtained. Either set of calculations will allow for the placing of a Compass Deviation Card near to the compass in the aircraft.

## Accuracy Limits

In accordance with CS Ops-1 (European Regulations) the aircraft's compasses must, after correction, be within the following limits:

## Change of Magnetic Latitude

$$
\tan \operatorname{dip}=\frac{\mathrm{Z}}{\mathrm{H}}
$$

The changes in deviation due to change of magnetic latitude have to be considered firstly with regard to hard iron and secondly to vertical soft iron.

Hard Iron. The hard iron deviating force, remains constant regardless of change of latitude. However, H varies with latitude being maximum at the Equator and zero at the poles. Thus the smaller the directive force $H$ the greater the maximum deviation, caused by hard iron deviating force. Hard iron deviating effect increases towards the poles and is minimum at the magnetic equator. The sign of the deviation will be the same in both hemispheres.

To summarize, maximum deviation due to hard iron magnetism is inversely proportional to the value of H , which changes with change of magnetic latitude.

Vertical Soft Iron. The directive force at the compass position is H , whilst the magnetizing agent of any VSI component is Z. The maximum deviation due to VSI magnetism will vary inversely as H .

The VSI maximum deviation will vary directly as Z . Z has no effect on hard iron deviation. The maximum deviation produced by VSI magnetism increases towards the magnetic poles.

$$
\text { Max deviation }=\frac{Z}{H}=\tan \operatorname{dip}
$$

so maximum deviation varies directly with tan dip.

## Occasions for Swinging the Compass

- When compass components are installed or replaced.
- Whenever the accuracy of the compass is in doubt.
- After a maintenance inspection if required by the schedule.
- After a significant aircraft modification, repair or replacement involving magnetic material.
- When carrying unusual ferromagnetic payloads!
- When the compass has been subjected to significant shock.
- If the aircraft has been struck by lightning.
- After significant modification to aircraft radio/electrical systems.
- After the aircraft has been given a new theatre of operations if the move involves a large change of magnetic latitude.
- If the aircraft has been in long term storage standing on one heading.


## Questions

1. European regulations (CS Ops-1) state that the maximum permissible deviations after compensation for the DRC are:
a. Ten degrees
b. Three degrees
c. One degree
d. Two degrees
2. Compass swings should be carried out:
a. on the apron.
b. only on the compass swinging base or site.
c. at the holding point.
d. on the active runway.
3. Aircraft magnetism caused by vertical soft iron:
a. varies with magnetic heading but not with magnetic latitude.
b. varies with magnetic latitude but not with heading.
c. it is not affected magnetic latitude or heading.
d. varies as the cosine of the compass heading.
4. Aircraft magnetism caused by hard iron:
a. is not usually influenced by the Earth's magnetic field.
b. varies directly with magnetic latitude.
c. varies indirectly with magnetic latitude.
d. is maximum on east and west.
5. The aim of a compass swing is:
6. to find deviation on the cardinal headings and to calculate coefficients $A, B$ and C.
7. to eliminate or reduce the coefficients found.
8. to record any residual deviation and to prepare a compass correction card.
a. only answer 1 is correct.
b. answers 1 and 3 are correct.
c. answers 1, 2 and 3 are all correct.
d. none of the above answers are correct.
9. Deviation due to coefficient A is mainly caused by:
a. hard iron force acting along the longitudinal axis.
b. hard and soft iron forces acting along the lateral axis.
c. vertical soft iron forces.
d. a misaligned lubber line.

Answers

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a | b | b | b | c | d |

## Chapter

## 31

## General Navigation Problems

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31

## Introduction

The General Navigation exam includes many questions on general principles. They fall into the following broad categories:

Triangular problems
Implications of geometry on the Triangle of Velocities
Calculation of Rhumb Line track angles
Cross-track displacement between Rhumb Line and Great Circle tracks
Timing to a beacon

## Triangular Problems

Question. The relative bearing to a beacon is $270^{\circ}(\mathrm{R})$. Three minutes later, at a ground speed of 180 knots, it has changed to $225^{\circ}(\mathrm{R})$. What was the distance of the closest point of approach of the aircraft to the beacon?
a. $\quad 45 \mathrm{NM}$
b. $\quad 18 \mathrm{NM}$
c. $\quad 9 \mathrm{NM}$
d. $\quad 3 \mathrm{NM}$

Answer: The question does not specify any heading or track, so draw the situation assuming that the aircraft is travelling due north. That way, the relative bearing and the true bearing are the same.


The angle A within the triangle when the $270^{\circ}(\mathrm{R})$ bearing is taken is $90^{\circ}$. Three minutes later, the relative bearing is $225^{\circ}(\mathrm{R})$, so the angle B within the triangle at that corner is $45^{\circ}$. The two of them add up to $135^{\circ}$, leaving $45^{\circ}$ remaining (angle C) of the $180^{\circ}$ which must make up the internal angles of a triangle.

Therefore angles $B$ and $C$ are equal and we have another isosceles triangle. So length $A B=$ length AC.

We know that from $A$ to $B$ is 3 minutes at 180 knots, which is 9 nautical miles. Therefore $A C$, which is the closest point of approach to the beacon, is also 9 nautical miles. Answer $\mathbf{c}$ is the right one here.

Question. At 1000 hours an aircraft is on the $310^{\circ}$ radial from a VOR/DME, at 10 nautical miles range. At 1010 the radial and range are 040/10 NM. What is the aircraft's track and ground speed?
a. $\quad 080^{\circ}(\mathrm{M}) / 85$ knots
b. $\quad 085^{\circ}(\mathrm{M}) / 85$ knots
c. $\quad 080^{\circ}(\mathrm{M}) / 80$ knots
d. $\quad 085^{\circ}(\mathrm{M}) / 90$ knots

Answer. Start by drawing a diagram. This is just a sketch - not a scale drawing. It would be possible to solve this question by drawing it out accurately but this is not the ideal method.

Draw a Magnetic North reference arrow, and draw in the two fixes.


If radial $B$ is $310^{\circ}(M)$, then the angle between Magnetic North and $A B$ is $50^{\circ}$. Similarly the angle up to $A C$ is $40^{\circ}$ - right-angled triangle. $A B$ and $A C$ are both 10 nautical miles so, again, we have an isosceles triangle $A B C$. The internal angles must add up to $180^{\circ}$ and 90 of them are at corner A, so the remaining $90^{\circ}$ are equally split between corners $B$ and $C$. Therefore both $B$ and $C$ are $45^{\circ}$.

There are now several ways of establishing the track angle. The easiest is to consider the lefthand of the two smaller triangles above. The bottom angle is $50^{\circ}$ and angle $B$ is $45^{\circ}$. These add up to $95^{\circ}$, so angle $x$ must be $85^{\circ}$. If angle $x$ is $85^{\circ}$, then so is angle $y$. Angle $y$ is the track angle.

Another way is to say that if direction $A B$ is $310^{\circ}(M)$, then direction $B A$ must be $130^{\circ}(M)$. We know that angle $B$ is $45^{\circ}$, so the track direction is $(130-45)=085^{\circ}(\mathrm{M})$.

Now for the ground speed.
You can do it by sin or cosine of $45^{\circ}$, using your 10 nautical mile sides for $A B$ or $A C$, but it is easier to find the distance $B C$ using Pythagoras.
$\mathrm{BC}^{2}=\mathrm{AB}^{2}+\mathrm{AC}^{2}$
so $\mathrm{BC}=\sqrt{200}=14.14$ nautical miles.

Use your navigation computer to calculate a ground speed if 14.14 NM are covered in 10
minutes. It works out at 85 knots.
Thus answer $\mathbf{b}$ is correct.

## Implications of Geometry on the Triangle of Velocities

It is often assumed, wrongly, that if the wind is at right angles to heading or track, there is no head or tail wind component. For many problems, especially in Flight Planning, you are expected to assume zero wind component.

However, strictly speaking, this is not true. Consider the following question:
Question. An aircraft is tracking $090^{\circ}(\mathrm{T})$ at a TAS of 180 knots. The wind is from due North and the wind component is 5 knots head. What will the wind component be if the aircraft flies a track of $270^{\circ}(\mathrm{T})$ ?
a. $\quad 5$ knots tail
b. zero
c. 5 knots head
d. not possible to tell

Answer. You could solve this one on your navigation computer, though not easily, and not by one of the conventional methods. Put the wind direction ( $360^{\circ}$ ) at the $12 o^{\prime}$ clock position and draw a straight line vertically downwards from the blue circle (about $3-4 \mathrm{~cm}$ long). Put 180 knots under the blue circle. Bring the track ( $090^{\circ}$ ) up to the heading index to start. Your initial drift will be to starboard. Rotate the wind face to the right, select a heading off to the left until the amount of difference between heading and track is the same as the drift-line which crosses the wind direction at 175 knots ground speed. It is a little difficult to explain without a diagram, but try it practically and you will see that it occurs at a heading of $077^{\circ}$, using $13^{\circ} \mathrm{S}$ drift. This gives a wind of $360^{\circ} / 42$. Now apply $360^{\circ} / 42$ to a track of $270^{\circ}(\mathrm{T})$ and you will find you need to fly a heading of $283^{\circ}(\mathrm{T})$. This gives you a ground speed of 175 knots.

We get a headwind of 5 knots whether we track $090^{\circ}(\mathrm{T})$ or $270^{\circ}(\mathrm{T})$. Why is this? Draw a diagram.


This makes it clear that on a track of $090^{\circ}(\mathrm{T})$ with a TAS of 180 knots, the ground speed is less than the TAS. The hypotenuse will be longer than the opposite or adjacent, to use trigonometrical terms.

If we now reverse the track, we get a mirror image of the situation.


180 knots TAS still gives us 175 knots ground speed - ( 5 knots headwind).
Answer c is correct.
This shows that a zero head / tail wind component does not occur, strictly speaking, with the wind at right angles to track. You can also show, by a similar drawing exercise, that it does not occur at right angles to heading. It is rather an academic point for practical purposes, but in fact, zero wind component is present when the wind is at right angles to the bisector between heading and track.

## Calculation of Rhumb Line Track Angles

You know how to calculate the distance in nautical miles of change of latitude. Also how to calculate distance involved in change of longitude using the departure formula. You will also need to calculate Rhumb Line track angle using the tangent relationship. This is a typical example:-

Question. What is the Rhumb Line track from A (4500N 01000W) to B (4830N 01500W)?
a. $\quad 315^{\circ}(\mathrm{T})$
b. $\quad 330^{\circ}(\mathrm{T})$
c. $\quad 215^{\circ}(\mathrm{T})$
d. $\quad 150^{\circ}(\mathrm{T})$

Answer. You have to draw these diagrams as though they were a Mercator chart-a rectangular graticule. A straight line will be a Rhumb Line.

The change of latitude from 4500 N to 4830 N is $3^{\circ} 30^{\prime}$, or 210 nautical miles. To solve the E-W distance we use the departure. Strictly speaking, it should be the departure at mid-latitude, at 4645 N in this example. However, in the examples noted so far the changes of latitude are small and it does not make a great deal of difference to the departure. In fact, on this example, you get an answer slightly nearer to the right answer if you use the departure at 45N.

```
Departure = change of longitude (in minutes) x cos latitude
    = 5 < 60 }\times0.707
    = 212.13 nautical miles
```

The tangent of angle $x=\frac{\text { opposite }}{\text { adjacent }}=\frac{210}{212}$

Therefore angle $x \quad=\quad 44.7^{\circ}$

The Rhumb Line track from A westwards along the parallel of latitude is $270^{\circ}(\mathrm{T})$, add $x$ to that to get $314.7^{\circ}(\mathrm{T})$ - answer a.


## Timing to a Beacon

Most timing problems require the wind component to be taken into consideration. Ground speed is a combination of TAS and head / tail wind component. To get the timing right requires the right ground speed. However, what the pilot actually adjusts is the Mach No. or CAS, which is directly proportional to TAS, not G/S.

Question. You are flying at a True Mach No. of .77 in a SAT of $-55^{\circ} \mathrm{C}$. At 1000 hours you are 150 NM from a DME with an ETA of 1017. ATC ask you to slow down to arrive at 1021. What is your new TMN if you reduce speed at 150 NM distance to go?
a. M. 72
b. M. 62
c. M. 67
d. M. 59

Answer. Start by calculating the current ground speed. 150 NM to go in 17 minutes gives 530 knots G/S. We need to alter the G/S to cover the 150 NM in 21 minutes, which is a new required ground speed of 429 knots.

Avoid the temptation to say that the required new Mach No. must be:
New Mach No. $=\quad$ M. $77 \times \frac{429}{530}=\quad$ M. 62

You have to take the wind into consideration:-

| Old ground speed | $=$ | 150 NM in 17 min |  | 530 knots |
| :---: | :---: | :---: | :---: | :---: |
| TAS: |  | M. 77 @ -55 ${ }^{\circ} \mathrm{C}$ | $=$ | 442 knots |
| Wind component |  |  | = | +88 knots |
| New required G/S | = | 150 NM in 21 min | $=$ | 429 knots |
| Wind component |  |  | = | -88 knots |
| New required TAS |  |  | = | 341 knots |
| New Mach No. | = | M. $77 \times \frac{341}{442}$ |  | M. 59 |

Answer d is correct.

## Chapter

## 32

## Revision Questions

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## Questions

1 Pressure Altitude is 28000 feet, $\mathrm{OAT}=-45^{\circ} \mathrm{C}$, Mach No = $0.46, \mathrm{~W} / \mathrm{V}=270 / 85$,
Track $=200^{\circ} \mathrm{T}$. What is the drift and groundspeed?
a. 18L / 252 knots
b. $\quad 15 \mathrm{R} / 310$ knots
c. 17L / 228 knots
d. 17R / 287 knots
$2 \mathrm{G} / \mathrm{S}=240$ knots, Distance go = 530 NM. What is time to go?
a. $\quad 27$ minutes
b. $\quad 29$ minutes
c. $\quad 2 \mathrm{~h} 05 \mathrm{~m}$
d. $\quad 2 \mathrm{~h} 12 \mathrm{~m}$

3 'ISA Dev $=+30^{\circ} \mathrm{C}$, Pressure alt $=5000$ feet. What is true alt?
a. 4550 feet
b. $\quad 5600$ feet
c. 4290 feet
d. 5320 feet

4 Course $040^{\circ}(\mathrm{T})$, TAS 120 kt , Wind speed = 30 knots. From which direction will the wind give the greatest drift?
a. $\quad 220^{\circ}(\mathrm{T})$
b. $\quad 230^{\circ}(\mathrm{T})$
c. $\quad 235^{\circ}(\mathrm{T})$
d. $\quad 245^{\circ}(\mathrm{T})$

5 Required course $045^{\circ}(\mathrm{T}), \mathrm{W} / \mathrm{V}=190 / 30, \mathrm{FL}=55$ @ ISA, Variation = $15^{\circ} \mathrm{W}$. CAS = 120 knots. What is mag heading and G/S?
a. $052^{\circ}(\mathrm{M}) \quad 154$
b. $067^{\circ}(\mathrm{M}) \quad 154$
c. $037^{\circ}(\mathrm{M}) \quad 154$
d. $\quad 037^{\circ}(\mathrm{M}) \quad 113$

6 An aircraft flies a great circle track from $56^{\circ} \mathrm{N} 070^{\circ} \mathrm{W}$ to $62^{\circ} \mathrm{N} 110^{\circ} \mathrm{E}$. The total distance travelled is:
a. $\quad 3720 \mathrm{NM}$
b. $\quad 5420 \mathrm{NM}$
c. $\quad 1788 \mathrm{NM}$
d. $\quad 2040$ NM
$7 \quad$ You are flying $090^{\circ}(\mathrm{C})$ heading. Deviation is 2 W and Variation is 12E. Your TAS is 160 knots. You are flying the 070 radial outbound from a VOR and you have gone 14 NM in 6 minutes. What is the W/V?
a. $\quad 158^{\circ}(\mathrm{T}) / 51$
b. $\quad 060^{\circ}(\mathrm{T}) / 50$
c. $\quad 340^{\circ}(\mathrm{T}) / 25$
d. $\quad 055^{\circ}(\mathrm{T}) / 25$

Please refer to Appendix B (attached at back).
Assume a North Polar Stereographic chart whose grid is aligned with the Greenwich meridian. An aircraft flies from the geographic North Pole for a distance of 480 NM along the $110^{\circ}$ E meridian, then follows a grid track of $154^{\circ}$ for a distance of 300 NM.

Its position is now approximately:
a. $\quad 78^{\circ} 45^{\prime} \mathrm{N} 087^{\circ} \mathrm{E}$
b. $\quad 70^{\circ} 15^{\prime} \mathrm{N} 080^{\circ} \mathrm{E}$
c. $\quad 79^{\circ} 15^{\prime} \mathrm{N} 074^{\circ} \mathrm{E}$
d. $\quad 80^{\circ} 00^{\prime} \mathrm{N} 080^{\circ} \mathrm{E}$

The sensitivity of a direct reading magnetic compass is:
a. Inversely proportional to the horizontal component of the Earth's magnetic field.
b. Inversely proportional to the vertical and horizontal components of the Earth's magnetic field.
c. Inversely proportional to the vertical component of the Earth's magnetic field.
d. Proportional to the horizontal component of the Earth's magnetic field.

An aircraft at position $60^{\circ} \mathrm{N} 005^{\circ} \mathrm{W}$ tracks $090^{\circ}(\mathrm{T})$ for 315 km . On completion of the flight the longitude will be:
a. $010^{\circ} 40^{\prime} \mathrm{W}$
b. $000^{\circ} 15^{\prime} \mathrm{E}$
c. $\quad 000^{\circ} 40^{\prime} \mathrm{E}$
d. $002^{\circ} 10^{\prime} \mathrm{W}$

What is the definition of magnetic variation?
a. The angle between the direction indicated by a compass and Magnetic North.
b. The angle between True North and Compass North.
c. The angle between Magnetic North and True North.
d. The angle between Magnetic Heading and Magnetic North.

At the magnetic equator:
a. Dip is zero
b. Variation is always maximum
c. Deviation is zero
d. The isogonal is an agonic line

Which of these is a correct statement about the Earth's magnetic field?
a. It has no effect on aircraft deviation.
b. The angle of dip is the angle between the vertical and the total magnetic force.
c. It may be temporary, transient, or permanent.
d. It acts as though there is a large blue magnetic pole in Northern Canada

Where is a compass most effective?
a. About midway between the equator and the Magnetic North Pole
b. In the region of the magnetic South Pole
c. In the region of the magnetic North Pole
d. On the geographic equator

15 The value of variation:
a. is zero at the magnetic equator
b. cannot exceed $180^{\circ}$
c. has a maximum value of $45^{\circ} \mathrm{E}$ or $45^{\circ} \mathrm{W}$
d. cannot exceed $180^{\circ}$

Grivation is the combination of
a. Variation and Deviation
b. Deviation and the Agonic value
c. Variation and Grid Convergence
d. Grid Convergence and Deviation

An aircraft leaves at 0900UTC on a 250 NM journey with a planned ground speed of 115 knots. After 74 NM the aircraft is 1.5 minutes behind the planned schedule. What is the revised ETA at the destination?
a. 1100
b. 1110
c. 1115
d. 1054

| A | 30 NM |
| :---: | :---: |
| ATA A is 1010. ETA B is 1030. | ETA C is 1043 |

ATA $A$ is 1010. ETA $B$ is 1030. ETA $C$ is 1043.
ATA $B$ is 1027. What is revised ETA $C$ ?
a. 1040
b. 1043
c. 1038
d. 1036

Isogrivs are lines that connect positions that have:
a. the same grivation
b. the same variation
c. $\quad 0^{\circ}(\mathrm{M})$ agnetic dip
d. the same horizontal magnetic field strength

What is the average magnetic track and distance between WTD NDB (5211.3N 00705.0W) and FOY NDB (5234.0N 00911.7W)? - use your Jeppesen E(LO)1

Track Dist
a. $294 \quad 76$
b. $286 \quad 76$
c. $294 \quad 81$
d. $286 \quad 81$

21 A useful method of a pilot resolving, on a visual flight, any uncertainty in the aircraft's position is to maintain visual contact with the ground and:
a. set heading towards a line feature such as a coastline, river or motorway
b. fly the reverse of the heading being flown prior to becoming uncertain until a pinpoint is obtained
c. fly expanding circles until a pinpoint is obtained
d. fly reverse headings and associated timings until the point of departure is reached

An aircraft is at FL140 with an IAS of 210 and a true OAT of $-5^{\circ}(\mathrm{C})$. The wind component is -35 knots. When the aircraft is at 150 NM from a reporting point, ATC request the crew to lose 5 minutes by the time they get to the beacon. How much do they need to reduce IAS?
a. $\quad 15$ knots
b. $\quad 25$ knots
c. $\quad 30$ knots
d. 20 knots


ATA $X$ is 1420. ETA $Y$ is 1447. ATA $Y$ is 1450. What is new ETA $Z$ ?
a. 1503
b. 1508
c. 1510
d. 1512

An aircraft starts at position 0410S 17822W and heads true north for 2950 NM, then turns 90 degrees right, and maintains a rhumb line track for 314 kilometres. What is its final position?
a. 5500 N 17422 W
b. 4500 N 17422W
c. 5500 N 17738 E
d. 4500 N 17738E

27 By what amount must you change your rate of descent given a 10 knot decrease in headwind on a $3^{\circ}$ glide slope?
a. $\quad 50$ feet per minute increase
b. $\quad 30$ feet per minute increase
c. $\quad 50$ feet per minute decrease
d. 30 feet per minute decrease

28 You are on a heading of $105^{\circ}(\mathrm{C})$, deviation $3^{\circ} \mathrm{E}$. WTD NDB (5211.3N 00705.0W) bears $013^{\circ}$, CRK VOR (5150.4N 00829.7W) QDM is $211^{\circ}$. What is your position? (Use Jeppesen E(LO)1)
a. $\quad 5245 \mathrm{~N} 00757 \mathrm{~W}$
b. $\quad 5217 \mathrm{~N} 00745 \mathrm{~W}$
c. $\quad 5412 \mathrm{~N} 00639 \mathrm{~W}$
d. $\quad 5228 \mathrm{~N} 00802 \mathrm{~W}$

The airport at 5211N 00706W is: (use Jeppesen E(LO)1)
a. Kerry
b. Cork
c. Shannon
d. Waterford

30 In which months is the difference between apparent noon and mean noon the greatest?
a November and February
b. January and July
c. March and September
d. June and December

315 hours 20 minutes and 20 seconds time difference is equivalent to which change of longitude?
a. $\quad 81^{\circ} 30^{\prime}$
b. $\quad 78^{\circ} 15^{\prime}$
c. $79^{\circ} 10^{\prime}$
d. $\quad 80^{\circ} 05^{\prime}$

32 The main reason that day and night, throughout the year, have different durations is due to the:
a. Earth's rotation
b. relative speed of the Sun along the ecliptic
c. inclination of the ecliptic to the Equator
d. gravitational effect of the Sun and the Moon on the speed of rotation of the Earth

A Lambert's Conical conformal chart has standard parallels at 63 N and 41 N . What is the convergence factor?
a. . 891
b. . 788
c. 656
d. . 707

34 On a chart, 90.74 km is represented by 7.0 centimetres. What is the scale?
a. $\quad 1 / 700000$
b. $\quad 1 / 2015396$
c. $\quad 1 / 1296400$
d. $\quad 1 / 1156600$

On a direct Mercator chart, great circles are shown as:
a. Curves convex to the nearer pole
b. Straight lines
c. Rhumb lines
d. Curves concave to the nearer pole

The scale on a Lambert's conformal conic chart
a. is constant along a meridian of longitude
b. is constant along a parallel of latitude
c. varies slightly as a function of latitude and longitude
d. is constant across the whole map

Please refer to Appendix A.
What is the symbol for a DME?
a. $\quad 9$
b. $\quad 10$
c. $\quad 12$
d. 4

38 Reference Jeppesen E(LO)1, position 5211N 00706W, which of the following denotes all the symbols?
a. military airport, ILS, NDB
b. civil airport, VOR, ILS
c. military airport, VOR, ILS
d. civil airport, ILS, NDB

39 Heading is $156^{\circ}(\mathrm{T})$, TAS is 320 knots, W/V is $130 / 45$ and the Variation is $10^{\circ} \mathrm{W}$. What is your magnetic track?
a. $\quad 170$
b. $\quad 150$
c. 160
d. 222

40 You are heading $345^{\circ}(\mathrm{M})$, the variation is $20^{\circ} \mathrm{E}$, and you take a radar bearing of $30^{\circ}$ left of the nose from an island. What bearing do you plot from the island?
a. $\quad 160^{\circ}(\mathrm{T})$
b. $\quad 155^{\circ}(\mathrm{T})$
c. $\quad 140^{\circ}(\mathrm{T})$
d. $\quad 180^{\circ}(\mathrm{T})$

41 Your pressure altitude is FL55, the QNH is 998, and the SAT is $+30^{\circ}(\mathrm{C})$. What is density altitude?
a. 6980 feet
b. $\quad 7750$ feet
c. 8620 feet
d. 10020 feet

42 On a particular take-off, you can accept up to 10 knots tailwind. The runway QDM is 047 , the variation is $17^{\circ} \mathrm{E}$ and the ATIS gives the wind direction as 210 . What is the maximum wind strength you can accept?
a. $\quad 18$ knots
b. 4 knots
c. 8 knots
d. 11 knots

43 The agonic line:
a. is midway between the magnetic North and South poles
b. follows the geographic equator
c. is the shorter distance between the respective True and Magnetic North and South poles
d. indicates zero variation

44 On a 5\% glide slope your groundspeed is 150 kt . What should be your rate of descent to maintain the glide slope?
a. 750 feet/min
b. 450 feet/min
c. 900 feet/min
d. 650 feet/min

45 At 65 NM from a VOR you commence a descent from FL330 in order to arrive over the VOR at FL80. Your mean ground speed in the descent is 240 knots. What rate of descent is required?
a. $\quad 1540$ feet/ min
b. $\quad 1630$ feet/min
c. $\quad 1270$ feet/min
d. 1830 feet/min

On the Jeppesen E(LO)1 chart, what are the symbols at Sligo (5354.8N 00849.1W)?
a. VOR, NDB, DME, compulsory reporting point
b. civil airport, NDB, DME, compulsory reporting point
c. civil airport, VOR, DME, non-compulsory reporting point
d. VOR, NDB, DME, non-compulsory reporting point

47 In which month does aphelion occur?
a. January
b. March
c. July
d. November

48 What is the highest latitude listed below at which the Sun will rise above the horizon and set every day?
a. $\quad 68^{\circ} \mathrm{N}$
b. $\quad 66^{\circ} \mathrm{N}$
c. $\quad 62^{\circ} \mathrm{N}$
d. $\quad 72^{\circ} \mathrm{N}$

49 The pressure alt is 29000 feet and the SAT is $-55^{\circ}(\mathrm{C})$. What is density altitude?
a. 27500 feet
b. 26000 feet
c. 30000 feet
d. 31000 feet

50 What is the UTC time of Sunrise in Vancouver, British Columbia, Canada (49N 123 30W) on the 6th December? (In the exam, tables were supplied. The answers given below are based on the tables in your Gen Nav notes).
a. 2324 UTC
b. 0724 UTC
c. 1552 UTC
d. 0738 UTC

How does scale change on a normal Mercator chart?
a. Expands as the cosine of the latitude
b. Expands directly with the secant of the latitude
c. Correct on the standard parallels, expands outside them, contracts within them
d. Expands as the secant of the E/W great circle distance

52 You are on an ILS 3-degree glide slope which passes over the runway threshold at 50 feet. Your DME range is $\mathbf{2 5}$ NM from the threshold. What is your height above the runway threshold elevation? (Use the 1 in 60 rule and 6000 feet = 1 nautical mile)
a. $\quad 7450$ feet
c. $\quad 6450$ feet
d. 7550 feet
d. 8010 feet

53 When it is 0600 Standard Time in Queensland, Australia, what is the Standard Time in Hawaii, USA? (Disregard Summer Time)
a. 0200 ST
b. 0600 ST
c. 1000 ST
d. $\quad 1200$ ST

Please refer to Appendix A.
Which of the following is the symbol for an exceptionally high (over 1000 feet AGL) unlit obstruction?
a. 6
b. $\quad 9$
c. $\quad 10$
d. 15

55 You are flying at a True Mach No. of 82 in a SAT of $-45^{\circ}(\mathrm{C})$. At 1000 hours you are 100 NM from the POL DME and your ETA at POL is 1012. ATC ask you to slow down to be at POL at 1016. What should your new TMN be if you reduce speed at 100 NM distance to go?
a. M. 76
b. M. 72
c. $\quad \mathrm{M} .68$
d. M . 61

56 The relative bearing to a beacon is $090^{\circ} \mathrm{R}$. Three minutes later, at a ground speed of 180 knots, it has changed to $135^{\circ}$ R. What was the distance of the closest point of approach of the aircraft to the beacon?
a. $\quad 45 \mathrm{NM}$
b. $\quad 18 \mathrm{NM}$
c. $\quad 9 \mathrm{NM}$
d. $\quad 3 \mathrm{NM}$

57 Ground speed is 540 knots. 72 NM to go. What is time to go?
a. $\quad 8 \mathrm{~min}$
b. $\quad 9 \mathrm{~min}$
c. $\quad 18 \mathrm{~min}$
d. $\quad 12 \mathrm{~min}$

An aircraft at position 2700 N 17000 W travels 3000 km on a track of $180^{\circ}(\mathrm{T})$, then 3000 km on a track of $090^{\circ}(\mathrm{T})$, then 3000 km on a track of $000^{\circ}(\mathrm{T})$, then 3000 km on a track of $270^{\circ}(\mathrm{T})$. What is its final position?
a. 2700 N 17318 W
b. $0000 \mathrm{~N} / \mathrm{S} 17000 \mathrm{~W}$
c. 2700 N 17000 W
d. 2700 N 14300 W

59 On the Jeppesen E(LO)1, Baldonnel (5318.0N 00626.9W) is 91 NM DME, Galway ( 5318.1 N 00856.5 W ) is 50 NM DME. What is your position?
a. $\quad 5242 \mathrm{~N} 00827 \mathrm{~W}$
b. $\quad 5230 \mathrm{~N} 00834 \mathrm{~W}$
c. $\quad 5255 \mathrm{~N} 00819 \mathrm{~W}$
d. $\quad 5219 \mathrm{~N} 00809 \mathrm{~W}$

60 An aircraft at FL370 is required to commence descent at 120 NM from a VOR and to cross the facility at FL130. If the mean GS for the descent is 288 kt , the minimum rate of descent required is:
a. $\quad 920 \mathrm{ft} / \mathrm{min}$
b. $\quad 890 \mathrm{ft} / \mathrm{min}$
c. $\quad 860 \mathrm{ft} / \mathrm{min}$
d. $\quad 960 \mathrm{ft} / \mathrm{min}$

61 You are homing to overhead a VORTAC and will descend from 7500 QNH to be 1000 AMSL by 6 NM DME. Your ground speed is 156 knots and the ROD will be $1000 \mathrm{feet} / \mathrm{min}$. At what range from the VORTAC do you commence the descent?
a. $\quad 22.9 \mathrm{NM}$
b. $\quad 15.8 \mathrm{NM}$
c. $\quad 16.9 \mathrm{NM}$
d. $\quad 30.2 \mathrm{NM}$

A rhumb line is:
a. the vertex of a conformal polyformic projection
b. a straight line on a Lambert's conformal chart
c. a line on the Earth which cuts all meridians at the same angle
d. the shortest distance between two points on the Earth's surface

63 You fly from 49N to 58 N along the $180 \mathrm{E} / \mathrm{W}$ meridian. What is the distance in kilometres?
a. $\quad 540 \mathrm{~km}$
b. $\quad 804 \mathrm{~km}$
c. $\quad 1222 \mathrm{~km}$
d. $\quad 1000 \mathrm{~km}$

64 On a particular direct Mercator wall chart, 'the full length of the parallel of latitude at 53 N is 133 cm long. What is the scale of the chart at 30S?
a. 1:30000000
b. 1: 18000000
c. 1: 21000000
d. 1: 26000000

65 What is the highest latitude on the Earth at which the Sun can be vertically overhead?
a. $231^{1 / 2}$
b. $\quad 6611_{2}{ }^{\circ}$
c. $45^{\circ}$
d. $\quad 90^{\circ}$

Track $=090^{\circ}(\mathrm{T})$,
Variation $=12^{\circ} \mathrm{E}$,
TAS $=460$ knots,
$W / V=360^{\circ}(T) / 100$,
What is compass heading and ground speed?

|  |  |  |
| :--- | :--- | :--- |
| a. | $079^{\circ}$ | 470 knots |
| b. | $067^{\circ}$ | 450 knots |
| c. | $068^{\circ}$ | 460 knots |
| d. | $070^{\circ}$ | 455 knots |

Please refer to Appendix A.
What symbol is used to show multi unlit obstacles on a map/chart?
a. 5
b. 7
c. $\quad 13$
d. $\quad 11$

The angle between True North and Magnetic North is known as:
a. deviation
b. dip
c. alignment error
d. variation

An aircraft is at $10^{\circ} \mathrm{N}$ and is flying North at $444 \mathrm{~km} /$ hour. After 3 hours the latitude is:
a. $\quad 10^{\circ} \mathrm{S}$
b. $\quad 02^{\circ} \mathrm{S}$
c. $\quad 22^{\circ} \mathrm{N}$
d. $\quad 00^{\circ} \mathrm{N} / \mathrm{S}$

Given that:
$A$ is $N 55^{\circ} \mathrm{E} / \mathrm{W} 000^{\circ}$
B is N54 $\mathrm{E} 010^{\circ}$,
If the average true course of the great circle is $100^{\circ}(\mathrm{T})$, the true course of the rhumb line at point $A$ is:
a. $096^{\circ}$
b. $\quad 107^{\circ}$
c. $\quad 104^{\circ}$
d. $100^{\circ}$

The circumference of the Earth is approximately:
a. $\quad 40000 \mathrm{NM}$
b. $\quad 10800 \mathrm{NM}$
c. $\quad 5400 \mathrm{NM}$
d. 21600 NM

72 The angle between the plane of the Equator and the plane of the Ecliptic is:
a. $\quad 66.5^{\circ}$
b. $\quad 23.5^{\circ}$
c. $\quad 25.3^{\circ}$
d. $\quad 65.6^{\circ}$

73 Position $A$ is at 70S 030W, position $B$ is 705060 E . What is the great circle track of $B$ from $A$, measured at $A$ ?
a. $\quad 132^{\circ}(\mathrm{T})$
b. $\quad 048^{\circ}(\mathrm{T})$
c. $\quad 090^{\circ}(\mathrm{T})$
d. $\quad 228^{\circ}(\mathrm{T})$

74 The value of magnetic variation on a chart changes with time. This is due to:
a. Movement of the magnetic poles, causing an increase
b. Increase in the magnetic field, causing an increase
c. Reduction in the compass deviation, causing a decrease
d. Movement of the magnetic poles, which can cause either an increase or a decrease

Isogonal lines converge as follows:
a. At the North Magnetic Pole
b. At the North and South Magnetic and both Geographical Poles
c. At the North and South Magnetic Poles
d. At the Magnetic equator.

76 Position $A$ is $55 N$ 30W. Position $B$ is $54 N$ 20W. The great circle track from $A$ to $B$, measured at $A$, is $100^{\circ}(T)$. What is the rhumb line bearing from $A$ to $B$ ?
a. $\quad 104^{\circ}(\mathrm{T})$
b. $\quad 090^{\circ}(\mathrm{T})$
c. $\quad 100^{\circ}(\mathrm{T})$
d. $\quad 284^{\circ}(\mathrm{T})$

77 An aircraft departs a point 0400N 17000W and flies 240 NM South, followed by 240 NM East, then 240 NM North, then 240 NM West. What is its final position?
a. 0400 N 17000 W
b. 0600 S 17000 W
c. $\quad 0400 \mathrm{~N} 170^{\circ} 35.9^{\prime} \mathrm{W}$
d. $\quad 0400 \mathrm{~N} 170^{\circ} 01.8^{\prime} \mathrm{W}$

At 1000 hours an aircraft is on the 310 radial from a VOR/DME, at 40 nautical miles range. At 1015 the radial and range are 040/40 NM. What is the aircraft's track and ground speed?
a. $080^{\circ} / 226$ knots
b. $085^{\circ} / 226$ knots
c. $080^{\circ} / 56$ knots
d. $085^{\circ} / 90$ knots

79 A straight line is drawn on a North Polar Stereographic chart joining Point $A$ (7000N 06000W) to Point B (7000N 06000E).
What is the initial track direction (going eastwards) of the line at A ?
a. $\quad 090^{\circ}(\mathrm{T})$
b. $\quad 030^{\circ}(\mathrm{T})$
c. $\quad 120^{\circ}(\mathrm{T})$
d. $\quad 330^{\circ}(\mathrm{T})$

80 What is the maximum possible value of Dip Angle at either Pole?
a. $\quad 66^{\circ}$
b. $\quad 180^{\circ}$
c. $\quad 90^{\circ}$
d. $\quad 45^{\circ}$

81 Given:
Magnetic heading $311^{\circ}$
Drift is $10^{\circ}$ left
Relative bearing of NDB 270
What is the magnetic bearing of the NDB measured from the aircraft?
a. $\quad 221^{\circ}$
b. $\quad 208^{\circ}$
c. $\quad 211^{\circ}$
d. $180^{\circ}$

82 The initial straight track from $A(75 N 60 E)$ to $B(75 N 60 W)$ on a Polar Stereographic chart is:
a. $030^{\circ}$
b. $360^{\circ}$
d. $060^{\circ}$
d. $330^{\circ}$

83 Given: Runway direction $083^{\circ}(\mathrm{M})$, Surface W/V 035/35 kt. Calculate the effective crosswind component.
a. $\quad 24 \mathrm{kt}$
b. $\quad 26 \mathrm{kt}$
c. $\quad 31 \mathrm{kt}$
d. $\quad 34 \mathrm{kt}$

Given: For take-off an aircraft requires a headwind component of at least 10 kt and has a cross-wind limit of 35 kt . The angle between the wind direction and the runway is $45^{\circ}$. Calculate the maximum and minimum allowable wind speeds.
a. $\quad 15 \mathrm{kt}$ and 50 kt
b. $\quad 15 \mathrm{kt}$ and 43 kt
c. $\quad 12 \mathrm{kt}$ and 38 kt
d. $\quad 18 \mathrm{kt}$ and 50 kt

85 What is the weight in kilograms of 380 US Gallons at a Specific Gravity of 0.78?
a. 1123
b. 2470
c. 5434
d. 543

86 You leave A to fly to B, 475 NM away, at 1000 hours. Your ETA at B is 1130. At 1040, you are 190 NM from A. What ground speed is required to arrive on time at B?
a. 342 knots
b. $\quad 330$ knots
c. $\quad 317$ knots
d. 360 knots

87 What is the aircraft position in lat and long given the following (use Jeppesen chart $\mathrm{E}(\mathrm{LO}) 1$ ):

CRN (5318N 00857W) 18 DME
SHA (5243N 00853W) 20 DME
Heading 270(M)
Both ranges DME increasing
a. 5201 N 00908 W
b. $\quad 5301 \mathrm{~N} 00908 \mathrm{~W}$
c. 5302 N 00843 W
d. 5203 N 00843 W

88 What is the NDB frequency and ident at 5211N 00932W? - Use E(LO)1)
a. Kerry NDB -113 MHz and KER
b. Cork NDB -343 KHz and OC
c. Waterford NDB -368 MHz and WTD
d. Kerry NDB - 334KHz and KER

89 What is the mean magnetic track and distance from the BAL VOR (5318N 00627W) to CFN NDB (5502N 00820W)? - (use E(LO)1)

| a. | $335^{\circ}$ | 125 |
| :--- | :--- | :--- |
| b. | $148^{\circ}$ | 125 |
| c. | $328^{\circ}$ | 134 |
| d. | $148^{\circ}$ | 134 |

90 You are at position 5340N 00800W. What is the QDR from the SHA VOR (5243N 00853W)? - (Use E(LO)1)
a. 217
b. 037
c. 209
d. 029

91 Your radial from the SHA VOR (5243N 00853W) is $120^{\circ}(\mathrm{M})$. From the CRK VOR ( 5151 N 00830 W ), the radial is $033^{\circ}(\mathrm{M})$. What is your position? - (use $\mathrm{E}(\mathrm{LO}) 1$ )
a. $\quad 5230 \mathrm{~N} 00820 \mathrm{~W}$
b. $\quad 5240 \mathrm{~N} 00821 \mathrm{~W}$
c. 5220 N 00821 W
d. 5230 N 00800 W

92 Which of the following differences in latitude will give the biggest difference in the initial great circle track and the mean great circle track between two points separated by $10^{\circ}$ change of longitude?
a. 60 N and 60 S
b. $\quad 60 \mathrm{~N}$ and 60 N
c. 30 S and 30 N
d. $\quad 30 \mathrm{~S}$ and 25 S

93 An aircraft is at 5530 N 03613 W , where the variation is 15 W . It is tuned to a VOR located at 5330 N 03613 W , where the variation is 12 W . What VOR radial is the aircraft on?
a. $\quad 348$
b. 012
c. $\quad 165$
d. 015

The wind velocity is 359/25. An aircraft is heading 180 at a TAS of 198 knots. (All directions are True). What is its track and ground speed?

|  |  | 179 |
| :--- | :--- | :--- |
| a. | 223 |  |
| b. | 179 | 220 |
| c. | 180 | 220 |
| d. | 180 | 223 |

95 An aircraft's compass must be swung:
a. If the aircraft has been in the hangar for a long time and has been moved several times.
b. If the aircraft has been subjected to hammering.
c. Every maintenance inspection
d. After a change of theatre of operations at the same magnetic latitude.

96 Civil Twilight occurs between:
a. Sunset and $6^{\circ}$ below the horizon
b $\quad 6^{\circ}$ and $12^{\circ}$ below the horizon
c $\quad 12^{\circ}$ and $18^{\circ}$ below the horizon
d. Sunrise and Sunset

97 What is the dip angle at the North Magnetic Pole?
a. $\quad 0^{\circ}$
b. $\quad 90^{\circ}$
c. $\quad 180^{\circ}$
d. $64^{\circ}$

98 What is a line of equal grivation?
a. An isocline
b. An isogonal
c. An isogriv
d. An isovar

99 What is the reason for seasonal changes in climate?
a. Because the Earth's spin axis is inclined to the plane of its orbit round the Sun
b. Because the distance between the Earth and the Sun varies over a year
c. Because the Earth's orbital speed round the Sun varies according to the time of the year
d. Because of the difference between the Tropical Year and the Calendar Year

100 The aircraft position is at 5330 N 00800W. The VORs are tuned to Shannon (SHA, 5243N 00853W) and Connaught (CON, 5355N 00849W). Which radials will be indicated?

|  | SHA. | CON |
| :--- | :--- | :--- |
| a. | 033 | 130 |
| b. | 221 | 318 |
| c. | 042 | 138 |
| d. | 213 | 310 |

101 You are on the 205 radial from the Shannon VOR (SHA, 5243N 00853W) and on the 317 radial from Cork VOR (CRK, 5150N 00830W). What is the aircraft position?
a. $\quad 5205 \mathrm{~N} 00915 \mathrm{~W}$
b. $\quad 5215 \mathrm{~N} 00917 \mathrm{~W}$
c. $\quad 5118 \mathrm{~N} 00913 \mathrm{~W}$
d. 5210 N 00909 W

102 What is the radial and DME distance from Connaught VOR/DME (CON, 5355N 00849W) to overhead Abbey Shrule aerodrome (5336N 00739W)?

| a. | 304 | 47 NM |
| :--- | :--- | :--- |
| b. | 124 | 47 NM |
| c. | 296 | 46 NM |
| d. | 116 | 46 NM |

103 What is the average magnetic track and distance between Kerry NDB (KER, 5211N 00932W) and CarNMore NDB (CRN, 5318N 00856W)?

| a. | 025 | 70 NM |
| :--- | :--- | :--- |
| b. | 197 | 71 NM |
| c. | 017 | 70 NM |
| d. | 205 | 71 NM |

104 What is the approximate course (T) and distance between Waterford NDB (WTD, 5212N 00705W) and Sligo NDB (SLG, 5417N 00836W)?

| a. | 344 | 139 NM |
| :--- | :--- | :--- |
| b. | 164 | 138 NM |
| c. | 156 | 136 NM |
| d. | 336 | 137 NM |

105 What is the rhumb line track from $A(4500 \mathrm{~N} 01000 \mathrm{~W})$ to $B(4830 \mathrm{~N} 00500 \mathrm{~W})$ ?
a. $\quad 045^{\circ} \mathrm{T}$
b. $\quad 030^{\circ} \mathrm{T}$
c. $\quad 225^{\circ} \mathrm{T}$
d. $\quad 150^{\circ} \mathrm{T}$

106 What is the effect on the Mach number and TAS in an aircraft that is climbing with constant CAS?
a. Mach number decreases; TAS decreases
b. Mach number increases; TAS remains constant
c. Mach number increases; TAS increases
d. Mach number remains constant; TAS increases

107 Please refer to Appendix A.
What is the chart symbol for a lightship?
a. 6
b. 8
c. $\quad 9$
d. $\quad 12$

108 Given:
Track $198^{\circ}$, Heading $184^{\circ}$, TAS 427 kt and GS 453kt, what are the W/V and Drift Angle?
a. $\quad 280^{\circ} / 110 \mathrm{kt}$ and $14^{\circ} \mathrm{P}$
b. $\quad 087^{\circ} / 109 \mathrm{kt}$ and $14^{\circ} \mathrm{S}$
c. $\quad 116^{\circ} / 110 \mathrm{kt}$ and $14^{\circ} \mathrm{S}$
d. $294^{\circ} / 110 \mathrm{kt}$ and $14^{\circ} \mathrm{P}$

109 An aircraft is on the 025 radial from Shannon VOR (SHA, 5243N 00853W) at 49 DME. What is its position?
a. 5329 N 00930 W
b. 5239 N 00830 W
c. $\quad 5229 \mathrm{~N} 00930 \mathrm{~W}$
d. 5329 N 00830 W

110 An island is observed to be $15^{\circ}(\mathrm{T})$ o the left.
The aircraft heading is $120^{\circ}(\mathrm{M})$, variation $17^{\circ}(\mathrm{W})$.
The bearing ( ${ }^{\circ} \mathrm{T}$ ) from the aircraft to the island is:
a. $\quad 268$
b. $\quad 302$
c. 088
d. 122

111 An aircraft is flying around the Earth eastwards along the 60 N parallel of latitude at a ground speed of 360 knots. At what ground speed would another aircraft have to fly eastwards along the Equator to fly once round the Earth in the same journey time?
a. 600 knots
b. $\quad 240$ knots
c. $\quad 720$ knots
d. 120 knots

112 If it is 0700 hours Standard Time in Kuwait, what is the Standard Time in Algeria?
a. 0500 hours
b. 0900 hours
c. 1200 hours
d. 0300 hours

If variation is East, then:
a. True North is West of Magnetic North
b. Compass North is West of Magnetic North
c. True North is East of Magnetic North
d. Magnetic North is West of Compass North

114 At what latitude does the maximum difference between geodetic and geocentric latitude occur?
a. $\quad 0^{\circ}$
b. $\quad 45^{\circ}$
c. $60^{\circ}$
d. $\quad 90^{\circ}$

115 At what times of the year does the length of the hours of daylight change most rapidly?
a. Spring Equinox and Autumn Equinox
b. Summer Solstice and Winter Solstice
c. Spring Equinox and Summer Solstice
d. Autumn Equinox and Winter Solstice

116 Given: Aircraft height $=2500$ feet, ILS GP angle $=3^{\circ}$, at what approximate distance from the threshold can you expect to intercept the glide-path?
a. $\quad 8.0 \mathrm{NM}$
b. $\quad 14.5 \mathrm{NM}$
c. $\quad 13.1 \mathrm{NM}$
d. $\quad 7.0 \mathrm{NM}$

Convert 80 metres/sec into knots.
a. $\quad 155$ knots
b. 55 knots
c. $\quad 160$ knots
d. $\quad 16$ knots

118 The chart that is generally used for navigation in polar areas is based on a:
a. Direct Mercator Projection
b. Gnomonic projection
c. Lambert conformal projection
d. Stereographic projection

Which of the following conversions from True to Compass is the correct one?

|  | T | V | M | D. | C |
| :--- | :--- | :--- | :--- | :--- | :--- |
| a. | 130 | $2 W$ | 132 | -1 | 131 |
| b. | 130 | 2 E | 132 | -1 | 133 |
| c. | 130 | $2 W$ | 132 | -1 | 133 |
| d. | 130 | $2 E$ | 132 | -1 | 133 |

120 Your position is 5833N 17400W. You fly exactly 6 NM westwards. What is your new position?
a. $\quad 5833 \mathrm{~N}$ 17411.5W
b. 5833 N 17355 W
c. 5833 N 17340W
d. $\quad 5833 \mathrm{~N}$ 17348.5W

121 TAS $=240$ knots. Track is $180^{\circ}(\mathrm{T})$. The relative bearing from an NDB is $315(\mathrm{R})$ at 1410. At 1420 the bearing has changed to 270(R). What is your distance from the NDB at 1420 ?
a. $\quad 40 \mathrm{NM}$
b. $\quad 50 \mathrm{NM}$
c. $\quad 60 \mathrm{NM}$
d. $\quad 70 \mathrm{NM}$

122 Given:
True Track $=352$
Variation $=11 \mathrm{~W}$
Deviation = -5
Drift $=8^{\circ} \mathbf{R}$
What is Heading (C)?
a. $\quad 078^{\circ}(\mathrm{C})$
b. $\quad 346^{\circ}(\mathrm{C})$
c. $\quad 000^{\circ}(\mathrm{C})$
d. $025^{\circ}(\mathrm{C})$

123 What is the radial and DME distance from CRK VOR (5151N 00830W) to position 5220N 00910W?
a. $\quad 322(\mathrm{M}) 39 \mathrm{NM}$
b. $330(\mathrm{M}) 41 \mathrm{NM}$
c. $\quad 330(\mathrm{M}) 39 \mathrm{NM}$
d. $322(\mathrm{M}) 41 \mathrm{NM}$

124 What is the radial and DME distance from SHA VOR (5243N 00853W) to Birr airport (5304N 00755W)?
a. $\quad 068(\mathrm{M}) 42 \mathrm{NM}$
b. $060(\mathrm{M}) 40 \mathrm{NM}$
c. $\quad 068(\mathrm{M}) 40 \mathrm{NM}$
d. $060(\mathrm{M}) 42 \mathrm{NM}$

125 What is the lat and long of the SHA VOR (5243N 00853W) 239(M)/36 NM radial/ range?
a. 5215 N 00930 W
b. 5220 N 00937 W
c. 5212 N 00930 W
d. 5212 N 00915 W

126 A Lambert conformal conic chart has a constant of the cone of 0.80 .
A straight line course drawn on this chart from $A\left(53^{\circ} \mathrm{N} 004^{\circ} \mathrm{W}\right)$ to $B$ is $080^{\circ}$ at $A$; course at $B$ is $092^{\circ}$. What is the longitude of $B$ ?
a. $\quad 019^{\circ} \mathrm{E}$
b. $\quad 008^{\circ} \mathrm{E}$
c. $\quad 009^{\circ} 36^{\prime} \mathrm{E}$
d. $\quad 011^{\circ} \mathrm{E}$

127 An aircraft at position 0000N/S 16327 W flies a track of $225^{\circ}(\mathrm{T})$ for 70 NM . What is its new position?
a. 0049 N 16238 W
b. 0049S 16238W
c. 0049 N 16416 W
d. 0049 S 16416 W

128 On a Polar Stereographic map, a straight line is drawn from position A (70N $102 \mathrm{~W})$ to position $B(80 \mathrm{~N} 006 \mathrm{E})$. The point of highest latitude along this line occurs at longitude 035 W . What is the straight-line track angle from $B$ to $A$, measured at $B$ ?
a. $\quad 023^{\circ}(\mathrm{T})$
b. $\quad 077^{\circ}(\mathrm{T})$
c. $\quad 229^{\circ}(\mathrm{T})$
d. $\quad 131^{\circ}(\mathrm{T})$

129 Given that the value of ellipticity of the Earth is $1 / 297$ and that the semi-major axis of the Earth, measured at the axis of the Equator is 6378.4 km, what is the semiminor axis of the Earth measured at the axis of the Poles?
a. $\quad 6399.9 \mathrm{~km}$
b. $\quad 6367.0 \mathrm{~km}$
c. $\quad 6378.4 \mathrm{~km}$
d. $\quad 6356.9 \mathrm{~km}$

130 On a chart, meridians at 45 N are shown every 10 degrees apart. This is shown on the chart by a distance of 14 cm . What is the scale?
a. 1:2000000
b. 1:4000000
c. 1:5000000
d. 1:5600000

131 Please refer to Appendix A. Which is the symbol for a NDB?
a. 4
b. $\quad 5$
c. 2
d. $\quad 14$

132 How do rhumb lines (with the exception of meridians) appear on a Polar Stereographic chart?
a. concave to the nearer pole
b. convex to the nearer pole
c. ellipses round the pole
d. straight lines

133 Please refer to Appendix A. What does symbol 9 represent?
a. lit obstacle
b. lighthouse
c. VRP
d. aeronautical ground light

134 What is the chart convergence factor on a Polar Stereographic chart?
a. 0
b. $\quad 1.0$
c. $\quad 0.866$
d. 0.5

135 At 0422 you are 185 NM from a VOR at FL370. You need to descend at a mean descent rate of 1800'/min to be at FL80 overhead the VOR. Your ground speed in the level cruise is currently 320 knots. In the descent your mean $\mathrm{G} / \mathrm{S}$ will be 232 knots. What is the latest time to commence descent?
a. 0437
b. 0441
c. 0445
d. 0451

136 Given: Heading 165(M), Variation 25W, Drift $10^{\circ} \mathrm{R}, \mathrm{G} / \mathrm{S} 360$ knots. At 'A' your relative bearing to an NDB is 325R. Five minutes later, at ' $B$ ', the relative bearing is 280(R). What is the true bearing and distance from ' $B$ ' to the NDB?
a. $\quad 060^{\circ}(\mathrm{T}) 40 \mathrm{NM}$
b. $\quad 105^{\circ}(\mathrm{T}) 30 \mathrm{NM}$
c. $\quad 060^{\circ}(\mathrm{T}) 30 \mathrm{NM}$
d. $\quad 105^{\circ}(\mathrm{T}) 40 \mathrm{NM}$

137 What is the diameter of the Earth?
a. $\quad 40000 \mathrm{~km}$
b. $\quad 12732 \mathrm{~km}$
c. $\quad 21600$ km
d. $\quad 6366$ km

138 An aircraft flies 100 stat.m in 20 minutes. How long does it take to fly 215 NM?
a. $\quad 50 \mathrm{~min}$
b. $\quad 37 \mathrm{~min}$
c. $\quad 57 \mathrm{~min}$
d. $\quad 42 \mathrm{~min}$

141 What is the shortest distance between Point ' $A$ ' (3543N 00841E) and Point ' $B$ ' (5417N 17119W)?
a. $\quad 5400 \mathrm{NM}$
b. 6318 NM
c. $\quad 6557 \mathrm{NM}$
d. 6000 NM

142 On a conformal chart, the standard parallels are $41^{\circ} 20^{\prime} \mathrm{N}$ and $11^{\circ} 40^{\prime} \mathrm{N}$. What is the constant of the cone?
a. 660
b. . 202
c. . 446
d. .895

143 Given: Runway direction $083^{\circ}(\mathrm{M})$, Surface W/V $045 / 35 \mathrm{kt}$. Calculate the effective headwind component.
a. $\quad 29 \mathrm{kt}$
b. $\quad 27 \mathrm{kt}$
c. $\quad 31 \mathrm{kt}$
d. $\quad 34 \mathrm{kt}$

144 Given: TAS $=375 \quad$ Trk $=335^{\circ}(T) \quad W / V=340^{\circ}(T) / 50$
What is heading and Ground speed?
a. $\quad 335^{\circ}(\mathrm{T}) \quad 322$
b. $\quad 335^{\circ}(\mathrm{T}) \quad 318$
c. $\quad 336^{\circ}(\mathrm{T}) \quad 326$
d. $333^{\circ}(\mathrm{T}) \quad 326$

145 Lines of latitude on a chart are always:
a. Great circles
b. Small circles except for the Equator
c. Vertices
d. Meridians

146 On a Lambert chart, the convergence factor is .78585. What is the parallel of tangency?
a. $\quad 51^{\circ} 02^{\prime}$
b. $\quad 51^{\circ} 36^{\prime}$
c. $\quad 51^{\circ} 15^{\prime}$
d. $51^{\circ} 48^{\prime}$

147 On a Lambert's chart the constant of the cone is 0.80 . A is at 53 N 04 W . You plan to fly to B. The initial Lambert's chart straight-line track is 070(T) and the rhumb line track from $A$ to $B$ is $080(T)$. What is the longitude of $B$ ?
a. 021 E
b. 034 W
c. 011 E
d. 015E

148 On which chart projection is it not possible to show the North Pole?
a. Direct Mercator
b. Lambert's
c. Transverse Mercator
d. Polar Stereographic

149 You are at FL150 and the SAT is $-5^{\circ}(\mathrm{C})$. You are over an airport with an elevation of 720 feet. The QNH is 1003 . Assume 27 feet $=1 \mathrm{hPa}$.
What is your true height?
a. $\quad 14300$ feet
b. $\quad 15300$ feet
c. $\quad 14700$ feet
d. 15600 feet

150 What is the formula for conversion angle?
a. Change of longitude $\times$ Sine latitude
b. Change of longitude / $2 \times$ Sine mean longitude
c. Change of longitude / $2 \times \quad$ Sine mean latitude
d. Change of longitude $\times \quad$ Cosine latitude

151 On the Polar Stereographic projection, a great circle appears as:
a. a straight line
b. a curve which becomes more near to a straight line as the latitude increases
c. a curve convex to the nearer pole
d. a curve which can be concave or convex to the nearer pole, depending on the latitude

152 An aircraft departs Guam (13N 145E) at 2300 Standard Time on 30th April local date. Flight time to Los Angeles, California, USA (34N 118W) is 11 hours 15 minutes. What is the California Standard Time and local date of arrival? Assume Summer Time is being kept.
a. $\quad 1715$ ST 30 Apr
b. 1215 ST 01 May
c. $\quad 1315$ ST 01 May
d. 1615 ST 30 Apr

153 What rate of descent is required to maintain a $3.5^{\circ}$ glide slope at a ground speed of 150 knots?
a. $\quad 850 \mathrm{fpm}$
b. $\quad 800 \mathrm{fpm}$
c. $\quad 600 \mathrm{fpm}$
d. $\quad 875 \mathrm{fpm}$

154 What is the meaning of the term 'standard time'?
a. It is another term for UTC
b. It is the time zone system applicable only in the USA.
c. It is an expression for local mean time.
d. It is the time set by the legal authorities for a country or part of a country.

155 On 27 Feb at $52^{\circ} \mathrm{S} 040^{\circ} \mathrm{E}$ Sunrise is at 0243UTC. On the same day at $52^{\circ} \mathrm{S} 035^{\circ} \mathrm{W}$ the time of Sunrise is:
a. 0743 UTC
b. 0243 UTC
c. $\quad 2143$ UTC
d. 0543 UTC.

156 A compass swing is performed in order to correct for:
a. acceleration
b. deviation
c. variation
d. aperiodicity

157 Isogonals are lines of equal:
a. compass deviation
b. magnetic variation
c. wind velocity
d. pressure

158 On a direct Mercator chart, a rhumb line appears as a:
a. small circle concave to the nearer pole
b. straight line
c. curve convex to the nearer pole
d. spiral curve

Given:
IAS 120 kt
FL80
OAT $+20^{\circ}(\mathrm{C})$
What is the TAS?
a. $\quad 141 \mathrm{kt}$
b. $\quad 102 \mathrm{kt}$
c. $\quad 120 \mathrm{kt}$
d. $\quad 132 \mathrm{kt}$

160 The distance between two waypoints is 200 NM.
To calculate compass heading the pilot used $2^{\circ} \mathrm{E}$ magnetic variation instead of $2^{\circ} \mathrm{W}$. Assuming that the forecast W/V applied, what will the off track distance be at the second waypoint?
a. $\quad 14 \mathrm{NM}$
b. $\quad 7 \mathrm{NM}$
c. 0 NM
d. $\quad 21 \mathrm{NM}$

161 Given:
True course $300^{\circ}$
Drift $8^{\circ} \mathrm{R}$
Variation $10^{\circ} \mathrm{W}$
Deviation $-4^{\circ}$
Calculate the compass heading.
a. $\quad 322^{\circ}$
b. $\quad 306^{\circ}$
c. $\quad 278^{\circ}$
d. $294^{\circ}$

Given:
True track $180^{\circ} \quad$ Drift $8^{\circ} \mathrm{R}$
Compass Heading $195^{\circ} \quad$ Deviation $-2^{\circ}$ Calculate the variation.
a. $\quad 21^{\circ} \mathrm{W}$
b. $\quad 25^{\circ} \mathrm{W}$
c. $\quad 5^{\circ} \mathrm{W}$
d. $\quad 9^{\circ} \mathrm{W}$

163 Given the following:
Magnetic heading: $060^{\circ}$
Magnetic variation: $8^{\circ} \mathrm{W}$
Drift angle: $4^{\circ}$ right
What is the true track?
a. $\quad 064^{\circ}$
b. $\quad 056^{\circ}$
c. $072^{\circ}$
d. $\quad 048^{\circ}$

164 Given:
W/V 262/90 kt, Track $234^{\circ}$ and TAS 305 kt, what are the Heading and Groundspeed?
a. $\quad 226^{\circ}$ and 224 kt
b. $\quad 252^{\circ}$ and 214 kt
c. $\quad 242^{\circ}$ and 224 kt
d. $\quad 250^{\circ}$ and 224 kt

165 Given:
Half way between two reporting points the navigation log gives the following information:
TAS 360 kt W/V $330^{\circ} / 80 \mathrm{kt} \quad$ Compass heading $237^{\circ}$
Deviation on this heading - $5^{\circ} \quad$ Variation $19^{\circ} \mathrm{W}$
What is the average ground speed for this leg?
a. $\quad 403 \mathrm{kt}$
b. $\quad 354 \mathrm{kt}$
c. $\quad 373 \mathrm{kt}$
d. $\quad 360 \mathrm{kt}$

166 (For this question use Appendix C)
Complete line 5 of the 'FLIGHT NAVIGATION LOG', position 'J' to 'K'.
What is the $\operatorname{HDG}^{\circ}(\mathrm{M})$ and ETA?
a. HDG $337^{\circ}$ - ETA 1422 UTC
b. HDG $320^{\circ}$ - ETA 1412 UTC
c. HDG $337^{\circ}$ - ETA 1322 UTC
d. HDG $320^{\circ}$ - ETA 1432 UTC

167 During a low level flight 2 parallel roads are crossed at right angles by an aircraft. The time between these roads can be used to check the aircraft:
a. track
b. drift
c. ground speed
d. heading

168 The angle between the true great circle track and the true rhumb line track joining the following points: $A(60 S 165 \mathrm{~W})$ and $B(60 S 177 E)$ at the place of departure $A$, is:
a. $\quad 9^{\circ}$
b. $\quad 15.6^{\circ}$
c. $\quad 5.2^{\circ}$
d. $\quad 7.8^{\circ}$

169 Which of the following indicates an advisory airspace (ADA) boundary?
a.
b.

170 Given the following:
True track: $192^{\circ}$
Magnetic variation: $7^{\circ} \mathrm{E}$
Drift angle: $5^{\circ}$ left
What is the magnetic heading required to maintain the given track?
a. $\quad 180^{\circ}$
b. $190^{\circ}$
c. $194^{\circ}$
d. $\quad 204^{\circ}$

171 Given:
A Polar Stereographic chart whose grid is aligned with the zero meridian. Grid track $344^{\circ}$, longitude $115^{\circ} 00^{\prime} \mathrm{W}$, calculate the true course. (Assume N hemisphere).
a. $099^{\circ}$
b. $\quad 279^{\circ}$
c. $049^{\circ}$
d. $\quad 229^{\circ}$

172 The rhumb line distance between points $A\left(60^{\circ} 00^{\prime} N 002^{\circ} 30^{\prime} E\right)$ and $B\left(60^{\circ} 00^{\prime} N\right.$ $007^{\circ} 30^{\prime} \mathrm{W}$ ) is:
a. $\quad 300 \mathrm{NM}$
b. $\quad 450 \mathrm{NM}$
c. $\quad 600 \mathrm{NM}$
d. $\quad 150 \mathrm{NM}$

173 Given:
TAS $=485 \mathrm{kt}, \mathrm{OAT}=\mathrm{ISA}+10^{\circ} \mathrm{C}$, FL410. Calculate the Mach Number.
a. $\quad 0.87$
b. $\quad 0.825$
c. $\quad 0.90$
d. $\quad 0.85$

174 Fuel flow per hr is 22 US.gal, total fuel on board is 83 imp.gal. What is the endurance?
a. $\quad 2 \mathrm{hr} 15 \mathrm{~min}$
b. $\quad 4 \mathrm{hr} 32 \mathrm{~min}$
c. $\quad 3 \mathrm{hr} 12 \mathrm{~min}$
d. $\quad 3 \mathrm{hr} 53 \mathrm{~min}$

175 Given: Position A is 60N 020W, Position B is 60N 021W, and Position C is 59 N 020W, what are, respectively, the distances from $A$ to $B$ and from $A$ to $C$ ?
a. $\quad 60 \mathrm{NM}$ and 30 NM
b. $\quad 30 \mathrm{NM}$ and 60 NM
c. $\quad 52 \mathrm{NM}$ and 60 NM
d. $\quad 60 \mathrm{NM}$ and 52 NM

176 Given: FL350, Mach 0.80 , OAT $-55^{\circ}(\mathrm{C})$, calculate values for TAS and local speed of sound?
a. $\quad 461$ kt, LSS 296 kt
b. $\quad 461$ kt, LSS 576 kt
c. $\quad 237$ kt, LSS 296 kt
d. $\quad 490$ kt, LSS 461 kt

177 How many nautical miles are travelled in 1 minute 45 seconds at a ground speed of 135 knots?
a. $\quad 2.36$
b. $\quad 3.25$
c. $\quad 39.0$
d. $\quad 3.94$

178 The distance $A$ to $B$ is 90 NM in a straight line. You are 60 NM from $A$ when you fix your position 4 NM to the left of track. What correction do you need to make to arrive at $B$ ?
a. $\quad 4^{\circ}$
b. $\quad 8^{\circ}$
c. $\quad 12^{\circ}$
d. $\quad 10^{\circ}$

179 A Great Circle crosses the Equator at longitude $030^{\circ} \mathrm{W}$. The direction of the GC at Equator is $035^{\circ}(\mathrm{T})$. An aircraft following this Great Circle will reach its highest latitude ( N or S ) at position:
a. 35 S 120 W
b. 55 S 060E
c. $35 \mathrm{~N} \quad 120 \mathrm{~W}$
d. $\quad 55 \mathrm{~N} \quad 060 \mathrm{E}$

180 The heading is $299^{\circ}(\mathrm{G})$. Magnetic variation is $90^{\circ} \mathrm{W}$ and chart convergence is $55^{\circ} \mathrm{W}$. What is magnetic heading?
a. $\quad 154^{\circ}(\mathrm{M})$
b. $\quad 084^{\circ}(\mathrm{M})$
c. $\quad 264^{\circ}(\mathrm{M})$
d. $\quad 334^{\circ}(\mathrm{M})$

181 The orbit of the Earth round the Sun is elliptical. An ellipse has 2 foci. Which of the following is a correct statement?
a. The Earth is positioned at one of the foci.
b. The Sun is positioned at the mid-point of the 2 foci.
c. The Sun is positioned at one of the foci.
d. The Earth is positioned at the mid-point of the 2 foci.

182 An aircraft homing to a VOR/DME loses 2500 feet in 11.1 NM change of DME range. What is the gradient of the slope?
a. $4.1 \%$
b. $3.5 \%$
c. $3.9 \%$
d. $\quad 3.7 \%$

183 An aircraft at position 8500 N 02000 E flies a rhumb line track of $075^{\circ}(\mathrm{T})$. What will be its path over the Earth?
a. The shortest route to a destination
b. A spiral path leading towards the North Pole
c. A great circle route continuing over the pole and then southwards over the
other side
d. An increasing track angle

184 Given: For take-off an aircraft requires a headwind component of at least 15 kt and has a cross-wind limit of 35 kt . The angle between the wind direction and the runway is $60^{\circ}$. Calculate the maximum and minimum allowable wind speeds.
a. $\quad 30 \mathrm{kt}$ and 40 kt
b. $\quad 15 \mathrm{kt}$ and 43 kt
c. $\quad 12 \mathrm{kt}$ and 38 kt
d. $\quad 18 \mathrm{kt}$ and 50 kt

185 An aircraft is cruising at FL350, Temp $-50^{\circ} \mathrm{C}$ and is told to descend to FL80, Temp $-10^{\circ} \mathrm{C}$. If the IAS for the descent was 188 kt , what would be the appropriate TAS?
a. $\quad 260 \mathrm{kt}$
b. $\quad 188 \mathrm{kt}$
c. $\quad 335 \mathrm{kt}$
d. $\quad 224 \mathrm{kt}$

An aircraft has to climb from FL50-10 ${ }^{\circ} \mathrm{C}$ to $\mathrm{FL} 260-25^{\circ} \mathrm{C}$. The IAS for the climb is 180 kt and the WC is +30 kt . If the ROC is $900 \mathrm{ft} / \mathrm{min}$, how many miles will the climb take?
a. $\quad 96 \mathrm{NM}$
b. $\quad 106 \mathrm{NM}$
c. $\quad 83 \mathrm{NM}$
d. $\quad 120 \mathrm{NM}$

187 An aircraft is flying at FL200, the OAT is $0^{\circ} \mathrm{C}$. When the actual air pressure on an airfield at MSL is placed on the subscale of the altimeter the indicated altitude is 19300 ft . What is the aircraft's True Altitude?
a. $\quad 17300 \mathrm{ft}$
b. $\quad 19300 \mathrm{ft}$
c. $\quad 20000 \mathrm{ft}$
d. $\quad 21300 \mathrm{ft}$

Questions

Appendix A to Revision Questions
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Appendix B to Revision Questions


## Appendix C to Revision Questions

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## Answers

c. A common mistake here is to set the W/V, then put 200 up at the 12 o'clock index and just read the drift and ground speed off. But that is the answer for a HEADING of 200. The question asks for a TRACK of 200. You now have to lay off for drift, which will give you a heading of 217 at the top, balanced by 17 degrees of port drift, giving a track of 200.
b. We recommend that you use the CRP-5 for true altitude problems, especially if the question gives you the actual SAT instead of ISA deviation. But if you want to use the formula, the ISA deviation is +30 degrees:

True Alt $=5000+(+30 \times 4 / 1000 \times 5000)=5600$ feet
which is pretty close to answer (b).
d. You would expect the greatest drift when the wind direction makes the greatest angle with from the track or reciprocal direction (the JAA use the word 'course' to mean 'desired track'). The reciprocal track of $040^{\circ}$ is $220^{\circ}$ and answer (d) is the furthest from $220^{\circ}$.
b.
a. You should note that these longitudes are meridian and anti-meridian. This is the over-the-pole case.
a. If your compass heading is $090^{\circ}(\mathrm{C})$, you can fill in the following table:

| T | V | M | D. | C |
| :--- | :---: | :---: | :---: | :---: |
| 100 | 12 E | 088 | 2 W | 090 |

So your true heading is 100 . VOR radials are always magnetic, so you are flying a true track of 082. Your TAS is 160. If you have gone 14 NM in 6 min, your $\mathrm{G} / \mathrm{S}$ is 140 . Now use your CRP-5 as in Chapter 6.
d. This question is solved by scale drawing. Use the scale to draw a line 480 NM along the $110^{\circ} \mathrm{E}$ meridian from the North Pole, then plot a line 300 NM in a direction of $154^{\circ} \mathrm{Grid}$.
d.
c. $\quad$ This is a departure question. A very similar example is given in Chapter 15
c
a
d
b
c.
c. $\quad 74 \mathrm{NM}$ @ the planned G/S of 115 knots $\quad=\quad 38.6 \mathrm{~min}$ 250 NM @ the planned G/S of 115 knots $=130.4 \mathrm{~min}$ After 74 NM , the actual elapsed time $\quad=\quad 40.1 \mathrm{~min}$ Therefore the flight time for 250 NM at the revised G/S $=130.4 \times 40.1 / 38 / 6$ This comes to 135 min . 0900UTC $+135 \mathrm{~min}=1115 \mathrm{UTC}$
c. ETA B and ETA C are not relevant. You fly 30 NM in 17 min . How long will it take you to fly the remaining 20 NM at the same ground speed?
a
c. Plotting question. Solve by measurement on the chart.
a.
d Find the present TAS, apply the wind and get the present G/S (230 knots)
150 NM @ 230 kt G/S is 39 min . You need to arrive 5 minutes later, so your new time to go is 44 min.

The new required G/S will be 150 NM in 44 min, which is 205 kt G/S
The wind should not change, so the new required TAS will be 240
Then either use FL140/-5 ${ }^{\circ}(\mathrm{C})$ in the airspeed window to convert the TAS to 190 kt CAS, or put the old TAS (264) against the old IAS (210), then against 240 TAS, you will see 190.

210 to 190 is a 20 knot reduction in IAS (strictly CAS, but the PEC should not change much in 20 knots).

Answer (d).
c. ETA $Y$ is irrelevant. If it takes you 30 minutes to cover 30 NM, it will take you a further 20 minutes to cover the remaining 20 NM at the same ground speed.
b The QNH is 988 hPa . The datum for Pressure Altitude is 1013 hPa , which is 25 hPa greater. Using 27 feet to one hPa, this equates to a distance of 675 feet. A greater static pressure occurs at a lower pressure level. Therefore the 1013 hPa pressure level is below sea level by 675 feet. The airport is 1000 feet above sea level, which means that it is 1675 feet above the 1013 hPa pressure level.
b The aircraft flies 2950 NM N, which takes it to 4500 N 17822W. Now convert 314 km to NM and then use the departure formula.
d. $\quad$ The true bearing from you TO the headland is $050^{\circ}(\mathrm{T})$. However, you are going to have to plot FROM the headland to the aircraft (the headland is on your map - the aircraft's position is not!)

The aircraft's heading has been measured relative to the direction of True North at the aircraft's position. If there is significant chart convergence between the aircraft's position and the headland's position, the direction of True North will be different at the headland. We must therefore plot the reciprocal bearing using the aircraft's meridian paralleled through the headland's position.
a
d Plotting question. Solve by measurement on the chart.
d. Plotting question. Solve by measurement on the chart.
a
d. Multiply by $15^{\circ}$ per hour. The easiest way of dealing with minutes and seconds is by using the DMS function of a scientific calculator.
c. The combination of the inclination of the ecliptic to the Equator and the orbit of the Earth round the Sun produces a constantly changing declination. The length of the hours of daylight at a given latitude varies with the declination of the Sun.
b On a Lambert chart, the Parallel of Origin is midway between the 2 Standard Parallels. The 'constant of the cone' is simply another term for the sine of the parallel of origin.
c. See Example 1 in Chapter 16, which is the same type of question.
a
b. The scale on a Lambert chart alters as latitude changes but, for a fixed latitude (i.e. along a parallel of latitude), it remains constant.
d
d. The Jeppesen conventions differ slightly from the ICAO ones. The key is given in the introduction to the Jeppesen Student Pilots' Manual.
a
b. Apply $20^{\circ} \mathrm{E}$ variation to $345^{\circ}(\mathrm{M})$ to get $005^{\circ}(\mathrm{T}) .30^{\circ}$ left of this is a true bearing of $335^{\circ}(\mathrm{T})$ from you TO the headland. However, you are going to have to plot FROM the headland to the aircraft (the headland is on your map - the aircraft's position is not!) You therefore plot the reciprocal of $155^{\circ}(\mathrm{T})$.
c. As per the reply to Q3.
d
d
a A $5 \%$ glide slope means that for every 100 knots you go forward horizontally, you go down 5 knots vertically. $5 \%$ of your forward speed of 150 is 7.5 knots vertically. Convert this to feet per min and you will see that option (a) is the nearest
a. 65 NM @ 240 knots will take 16.25 minutes. I need to lose 25000 feet, also in 16.25 minutes. The required ROD is $25000 / 16.25=1338$ feet per minute.
b The Jeppesen conventions differ slightly from the ICAO ones. The key is given in the introduction to the Jeppesen Student Pilots' Manual.
c
b. The lowest latitude at which there is at least one day a year without a Sunset (Mid-summer Day) and one day a year without a Sunrise (Mid-winter Day) is the Arctic Circle (which is $661^{\circ} 2^{\circ} \mathrm{N}$ ). Therefore the Sun will rise and set every day at 62 N and 66 N . The higher of these two is 66 N .
a. (Pressure alt + (ISA dev $\times 120$ ) gives 27560 ; CRP5 gives 27000 )
c. Look up the LMT of Sunrise at 49 N on the 6 th December. There is 3 minutes change between the 4th Dec and the 7th Dec. So, interpolating, the times at 50 N and 45 N are 0742 and 0723 respectively. That is a difference of 19 minutes. One-fifth of that is about 4 minutes, so the LMT of Sunrise at 49 N on the 6th December is 0738 . Now set it out in a table:

|  | Day | Hour | Minute | Time |
| :--- | :---: | :---: | :---: | :---: |
| LMT sunrise at Vancouver | 6 Dec | 07 | 38 | LMT |
| Arc/time $123^{\circ} 30^{\prime}$ W (long. west, UTC best) | - | +08 | 14 | - |
| UTC | 6 Dec | 15 | 52 | UTC |

To calculate arc/time, enter the degrees and minutes in DMS format into your calculator and divide by 15 .
b
a. This is just the 1 in 60 rule. A 3-degree glide path gives you 300 feet per nautical mile. 25 NM @ 300 feet per NM is 7500 feet. That would be for a touch-down at zero feet at the runway threshold. However, height over the threshold is 50 feet, which has to be added.
c. As always, use a table:

|  | Day | Hour | Minute | Time |
| :--- | :---: | :---: | :---: | :--- |
| Standard Time at Queensland | Today | 06 | 00 | ST |
| STD (long. east, UTC least) | - | -10 | 00 | - |
| UTC | Yesterday | 20 | 00 | UTC |
| STD (long. west, UTC best) | - | -10 | 00 | - |
| Standard Time at Hawaii | Yesterday | 1000 | 00 | ST |

a
d

C
57 a

60 d. You have 120 NM horizontal distance to travel at 288 knots. This will take 25 minutes. You need to lose 24000 feet in 25 minutes, which is 960 feet per minute.
a
a The first leg takes it down to the Equator at 170W, the second leg takes it to the Equator at 143 W , the third leg takes it to 27 N 143 W and you then work out the change of longitude of 1620 NM due west departure at 27 N , which is $30^{\circ} 18^{\prime}$ W, i.e. 17318 W .

However, you do not need to do the full calculation. You should realize that a departure of 3000 km will give a greater change of longitude at 27 N than it will at the Equator. Therefore, the answer must be at 27 N , but WEST of the 170 W meridian. There is only one option which fits.
b Plotting question. Solve by measurement on the chart.
a You need to lose 6500 feet at a rate of descent of 1000 feet per minute. This will take 6.5 minutes. Flying for this time at a ground speed of 156 knots gives a distance of 16.9 NM . However, this is 6 NM before the DME, so the total distance to start the descent is 22.9 NM .

C
d. This is a 9-degree change of latitude. You could work this out as 540 NM and then convert it to km, but there is a quicker way. 90 degrees of change of latitude is 10000 km (see pages 28/29, so a 9 degree change of latitude will be 1000 km.
d. The thing to realize is that on a Mercator chart, meridians are drawn as parallel lines. Therefore the total length of the 53 N parallel of latitude will be 133 cm as will be the length of the 305 parallel of latitude.

So now find the departure from 180E to 180W at 30 S .
Dep $=$ ch.long $\times \cos 30$
$=360 \times 60 \times \cos 30=18706 \mathrm{NM}$

You now have a simple scale problem. The Chart length is 133 cm , the Earth Distance is 18706 . This gives you a scale of approximately $1: 26$ million.
a The Sun would be at its Zenith.
b. Find the true heading in the normal way, then apply variation and deviation. First, check that you have balanced the drift. When deviation is given as East or West then you use the DEVIATION EAST COMPASS LEAST RULE. However, deviation (not variation, only deviation) is sometimes quoted as plus or minus-. In this case the rule is that deviation is what you apply to the compass to get magnetic (not the other way round). So EAST deviation is plus and WEST deviation is minus.
d
d
c. $\quad 3 \times 444$ is 1332 km . Divide by 1.852 to get 719.2 NM. This has obviously been approximated to 720 to give $12^{\circ}$ change of latitude, which takes us to latitude $02^{\circ} \mathrm{S}$.
d The average great circle track is the rhumb line track.
d. You should remember that the distance from the Equator to a Pole is 5400 NM ( $90^{\circ}$ change of latitude $\times 60 \mathrm{NM}$ per degree). So the Earth's circumference is $4 \times 5400$, which is 21600 NM. Alternatively, multiply $360^{\circ}$ (a meridian plus the associated anti-meridian) by 60 .
b
a. The rhumb line track from 705 to $70 S$ is along a parallel of latitude, so it will be 090(T). However, the great circle track will 'cut the corner' and take the shortest route. This will be to the south of 090(T). Draw it out on a Lambert's style projection, or try it on a globe.

The conversion angle will be 42 degrees, so this makes the initial great circle track 132(T).
d
b
a
c. $\quad 6^{\circ} \mathrm{S}$ is a greater value of latitude than $4^{\circ} \mathrm{N}$. Therefore, for a given departure, you get a greater change of longitude eastwards. When you come back to $4^{\circ} \mathrm{N}$ again, 600 NM does not take you so far westwards. So you will finish up east of where you started.
b
b
c
a.
d
b. CRP-5
a

85 a. Either use the CRP-5 or remember that there are 5 imperial gallons to 6 US.gal and the imp.gal to litres conversion is 4.55 . Then multiply by .78 to get the weight in kg, i.e.
$380 \times 5 / 6 \times 4.55 \times .78=1123.85$
It is probably easier with the CRP-5.
b
a

C
a. Distance still remaining $=475-190=285 \mathrm{NM}$.

Time to go $=1130-1040=50 \mathrm{~min}$
b. Plotting question. Solve by measurement on the chart. An explanation of the point about 'both DME distances decreasing' is given in the Plotting chapter.
d The Jeppesen conventions differ slightly from the ICAO ones. The key is given in the introduction to the Jeppesen Student Pilots' Manual.
a Plotting question. Solve by measurement on the chart.
b Plotting question. Solve by measurement on the chart.
d Plotting question. Solve by measurement on the chart.
b. The mean great circle is the same as the rhumb line track. The question is asking which pairs of latitudes will give the greatest difference between great circle and rhumb line track, i.e. which will give the greatest conversion angle?

The ca formula is $1 / 2$ ch.long sin mean lat.
If you keep the $1 / 2$ ch.long constant for all cases, then you are asking which pairs of latitudes will give you the greatest sine mean lat.
b The longitudes are the same so, from the given lats and longs, the true bearing of the aircraft from the VOR is $000 / 360$. For VOR, you take the variation at the VOR, because the bearing is measured at the beacon. (For an ADF bearing, it would be the variation at the aircraft, because that is where the variation is measured).
d. The air vector is 5 times as long as the wind vector, so the direction will be nearer 180 than 179. Try drawing the triangle of velocities out as a diagram.
a The combination of the inclination of the ecliptic to the Equator and the orbit of the Earth round the Sun produces a constantly changing declination. The predominant cause of the seasons is the inclination (tilt) of the Earth.
c. Apply $17^{\circ} \mathrm{W}$ variation to $120^{\circ}(\mathrm{M})$ to get $103^{\circ}(\mathrm{T})$ heading. The island is $15^{\circ}(\mathrm{T}) \mathrm{o}$ the left, which makes the true bearing TO the island $088^{\circ}(\mathrm{T})$.
c Radials radiate. That is what the word means. In other words they are, by definition, the magnetic bearing FROM the VOR. On the VOR in the aircraft you have a needle with 2 ends. The sharp end points to the VOR and is the QDM, or mag track to the VOR and the other end points to the QDR, or radial, which is the mag bearing from the VOR.
d Plotting question. Solve by measurement on the chart.
b Plotting question. Solve by measurement on the chart.
a Plotting question. Solve by measurement on the chart.
d Plotting question. Solve by measurement on the chart.
a
c. As the air becomes less dense, the aircraft has to fly faster through it (TAS) to experience the same dynamic pressure (CAS). So the TAS is increasing, which means that the Mach No. would also increase even if there were no temperature change.

However, in addition, temperature normally decreases with increasing altitude. This means that the speed of sound will decrease so, for a given TAS, the Mach No. will increase, giving an additional effect.
a
b
a
a This is just the 1 in 60 rule. A 3-degree glide path gives you 300 feet per nautical mile. Divide 2500 feet by 300 feet and the answer is just over 8 nautical miles.
a. To convert metres into NM, divide by 1852. This gives you NM per second. Now multiply by $60 \times 60$ to convert to NM per hour, i.e. knots.
$80 \times 3600 / 1852=155$ knots
d. Polar navigation can be (and often is) carried out on a Polar Stereographic, a Transverse Mercator or an Oblique Mercator, in fact, but the JAA consider Polar Stereographic to be the preferred answer.

C
a. This is a departure problem. The departure formula is:

Departure $=$ change of longitude (minutes) $\times$ cosine latitude
$6 \mathrm{NM}=$ change of longitude (minutes) $\times$ cosine $58^{\circ} 33^{\prime}$
Change of longitude $=6 / .5218=11.5$ minutes (westwards) from $174^{\circ} 00^{\prime} \mathrm{W}$
a. This is covered in detail in the 'General Navigation Problems' Chapter. The numbers are different but the principle is the same. If the question does not give you any wind information, you can only assume that TAS $=G / S$.
c. Do not forget that a deviation of $-5^{\circ}$ is equivalent to $5^{\circ} \mathrm{W}$.
c Plotting question. Solve by measurement on the chart.
a Plotting question. Solve by measurement on the chart.
b Plotting question. Solve by measurement on the chart.
d. 'Constant of the cone' is just another term for 'sine of the parallel of origin'.

| Chart convergence | $=$ | change of longitude $\times$ sin parallel of origin |  |
| :--- | :--- | :--- | :--- |
| $12^{\circ}$ | $=$ | change of longitude $\times 0.80$ |  |
| Change of longitude | $=$ | $15^{\circ}$ |  |
| $15^{\circ}$ eastwards from $004^{\circ} \mathrm{W}$ is $011^{\circ} \mathrm{E}$. |  |  |  |

d. Resolve 70 NM along a track of 225 into their change of latitude and change of longitude, i.e. $70 \sin 45$ and $70 \cos 45$. This gives 49 NM south and 49 NM west. As you are at the Equator, you can take the 49 NM west as 49 min change of longitude - you do not need the departure formula. So the final position is 49 min south and 48 min west of the original position.
d. The only problem with this question is the language, not the concept. The question is telling us that the polar diameter and the equatorial diameter of the Earth are in the ratio 296:297. The 'semi-major axis of the Earth, measured at the axis of the Equator' is simply half the equatorial diameter and the 'semiminor axis of the Earth measured at the axis of the Poles' is half the Polar diameter. We need to find half the Polar diameter, or the Polar radius.

Polar radius $/ 6378.4=296 / 297$
So Polar radius $=6356.9 \mathrm{~km}$
Alternatively, you could remember from page 3 that the Polar diameter is 43 km shorter than the equatorial diameter. So the radius or semi-diameter will be 21.5 km shorter.
6378.4-21.5 = 6356.9 km . Either method gives you the right answer.
d. This is a combination of a departure and a representative fraction (scale) problem.

| Departure | $=$ change of longitude $\times 10^{\circ} \quad(\times 60$, to get minutes $) \times 0.7071$ |
| ---: | :--- |
|  | $=424.3 \mathrm{NM}$ |
|  | $=$ Chatitude |
| Scale | $=\quad$ Cength $/$ Earth Distance $=14 \mathrm{~cm} /(424.3 \times 1852 \times 100)$ |
|  | $=1 / 5.6$ Million (approx), which is closest to (d). |

c
a
a
b
c There are 2 parts to this profile. You are going to continue to fly at FL370 until the top of descent, and then descend to FL80. The question is asking for the ETA for top of descent.

From FL370 to FL80 is 29000 feet. At 1800 feet per minute, that is 16.1 minutes. Your mean ground speed in the descent is 232 knots so, for 16.1 min, that is 62 NM in the descent.

Your total distance to run is 185 NM . If there are 62 NM is the descent, the high level distance to top of descent is 123 NM. 123 NM @ your level ground speed of 320 is 23 min . Add 23 to 0422 to get 0445 .
c. True heading is 140 , true track is 150 . Relative bearing at A is $35^{\circ}$ left of the nose, ie a true bearing of 105 to the NDB. At B, the NDB is 80 left of the nose, true bearing to the NDB is 060 .

So you now have 3 lines. A track line of 150, and a bearing from A of 105 and a bearing from $B$ of 060 . This gives a right angles isosceles triangle, with the 2 short sides being 30 NM long ( 5 minutes at 360 knots $\mathrm{G} / \mathrm{S}$ ). Draw it out and it become obvious.
b. You are not supposed to remember the diameter of the Earth. But you should know that the circumference of the Earth is 40000 km . Circumference $=2 \pi r$ (or $\pi \mathrm{d}$ ). So divide 40000 km by $\pi$.
a. Use the CRP-5.
d
d Plotting question. Solve by measurement on the chart.
a Plotting question. Solve by measurement on the chart.
c. On a Lambert chart, the parallel of origin is midway between the 2 Standard Parallels, which in this case is $26^{\circ} 30^{\prime}$. Sine $26^{\circ} 30^{\prime}$ is 0.446 .
a. CRP-5 problem.
c
b
d. The 'constant of the cone' is the sine of the parallel of origin (or tangency). Simply use your calculator to find the arc sine of 0.78585 . Use the DMS button if you need to convert the answer from degrees and decimal degrees to degrees and minutes.
a The 'constant of the cone' is another term for the sine of the parallel of origin. The initial straight-line track is 070 and the mean is 080 . Therefore half chart convergence is 10 degrees, so the chart convergence between $A$ and $B$ is 20 degrees.

Chart convergence $=$ change of longitude $\times$ sine parallel of origin
Therefore change of longitude = chart convergence/sin parallel of origin
Change of longitude $=20 / .8=25$ degrees .
25 degrees east of 004 W is 021 E .
a. The North Pole is in the hole at the end of the cylinder.
c. (The correct answer is 14610 , but 14700 was the nearest option).
c
b

|  | Day | Hour | Minute | Time |
| :--- | :---: | :---: | :---: | :---: |
| Standard Time at Guam | 30 Apr | 23 | 00 | ST |
| STD (long east, UTC least) |  | -10 | 00 |  |
| UTC | 30 Apr | 13 | 00 | UTC |
| Flight Time |  | +11 | 15 |  |
| UTC arrival at Los Angeles | 01 May | 00 | 15 | UTC |
| STD (long west, UTC best) |  | -8 | 00 |  |
| Standard Time at LA | 30 Apr | $16^{\star}$ | 15 | ST |

* But California Summer Time rules apply. Add an hour to get 1715.
b Apply $8^{\circ} \mathrm{W}$ variation to $060^{\circ}(\mathrm{M})$ to get $052^{\circ}(\mathrm{T})$. With $4^{\circ}$ starboard drift, the true track is $056^{\circ}(\mathrm{T})$.
c.

C
c. Ground speed is the only of these options which depends on time difference.
d. This is a conversion angle problem:

| Conversion angle | $=1 / 2$ change of longitude |
| ---: | :--- |
|  | $=1 / 2 \times$ |
|  | $=18 \times$ |
|  | $=7.8^{\circ}$ |

b. $192^{\circ}(\mathrm{T})+7 \mathrm{E}$ Var $=185^{\circ}(\mathrm{M})$ ag Track. With $5^{\circ}$ of left drift, you need to aim off $5^{\circ}(\mathrm{T})$ o the right, onto a Mag heading of $190^{\circ}(\mathrm{M})$.
d. The datum meridian is Greenwich, so the aircraft is $115^{\circ}$ West of the datum in the Northern hemisphere. Therefore convergence is $115^{\circ} \mathrm{E}$. Convergence East - True Least. So True track will be 115 degrees less than Grid Track. $344^{\circ}(\mathrm{G})$ $115^{\circ}(\mathrm{C})$ onvergence $=229^{\circ}(\mathrm{T})$.
a. Rhumb line distance between 2 points at the same latitude is the Departure.

| Departure |  | change of longitude (in minutes) |  |  | $\times$ | cos latitude |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | = | $10^{\circ}$ | $\times$ | 60 | $\times$ | $\cos 60^{\circ}$ |
|  | = | 600 | $\times$ | 0.5 | = | 300 NM |

b. Do not forget that ISA at FL410 is still $-56.5^{\circ}(\mathrm{C})$, even though you are higher than 36090 feet pressure alt. So the SAT is $-46.5^{\circ}(\mathrm{C})$. Use the CRP-5.
b. Convert 22 US.gal per hour to imperial gallons on the CRP-5. The answer is about 18.3 imp.gal/hour. Then use either the CRP-5 or your calculator to get the total endurance.
b. $\quad A$ and $B$ are at the same latitude but different longitudes. So it is a departure problem.
Departure change of longitude (in minutes) $\times$ cosine latitude
Departure $=1 \times 60 \times 30 \mathrm{NM}$

A and C are at different latitudes but the same longitude. So it is a simple change of latitude problem. One degree is 60 NM.
b. Use the CRP-5.
d. $\quad 135 \times 1.75 / 60=3.94$

178 c. You need $4^{\circ}$ right to correct for track error angle and parallel track. Then, with 60 NM gone, there are 30 NM to go. Four NM off in 30 gives a closing angle of $8^{\circ}$. Total change $=12^{\circ}$ right.

179 d
180 b
181 c
182 d
183 b
184 a
185 a To calculate the TAS for the descent you need to have the mean altitude and mean temperature with the IAS. Mean altitude is 21500 ft and mean temperature is $-30^{\circ} \mathrm{C}$.
$186 \mathrm{~b} \quad$ For a climb the TAS is calculated using the $2 / 3$ change in altitude and temperature. $2 / 3$ altitude is 21000 ft and temperature is $-20^{\circ} \mathrm{C}$ so the TAS is 242 kt . GS is 272 kt and the time is 23.3 min

187 d ISA Dev $=+25^{\circ} \mathrm{C}$, Temperature Correction $=4 \mathrm{ft} \times 20 \times+25=2000 \mathrm{ft}$. TA $=1 \mathrm{~A}+$ Temp Corr $=19300+2000=21300 \mathrm{ft}$

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[^0]:    * Summer time may be kept in these countries.

    1 Except Broken Hill Area which keeps 09h30m; 2 Except Diego Garcia which keeps 06h
    3 Including West Berlin; 4 Except Kiritimati Island and the Phoenix Islands which keep 10h and 11 h slow on UT

